SOCIAL AGGREGATION WITHOUT THE EXPECTED UTILITY HYPOTHESIS

by

Charles Blackorby, David Donaldson, and Philippe Mongin

NOVEMBER, 2000

Discussion Paper No.: 00-18

DEPARTMENT OF ECONOMICS
THE UNIVERSITY OF BRITISH COLUMBIA
VANCOUVER, CANADA V6T 1Z1

http://web.arts.ubc.ca/econ

Social Aggregation Without the Expected Utility Hypothesis*

by

Charles Blackorby, David Donaldson and Philippe Mongin

November 2000

Charles Blackorby: University of British Columbia and GREQAM
David Donaldson: University of British Columbia
Philippe Mongin: Université de Cergy-Pontoise

* We thank Jean-Yves Jaffray, Yves Sprumont and John Weymark for comments on earlier drafts. Research support from the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged.

December 1, 2000

Abstract

This paper investigates the possibilities for satisfaction of both the *ex-ante* and *ex-post* Pareto principles in a general model in which neither individual nor social preferences necessarily satisfy the Expected Utility Hypothesis. If probabilities are subjective and allowed to vary, three different impossibility results are presented. If probabilities are 'objective' (identical across individuals and the observer), necessary and sufficient conditions on individual and social value functions are found (Theorem 4). The resulting individual value functions are consistent not only with Subjective Expected Utility Theory, but also with some versions of Prospect Theory, Subjectively Weighted Utility Theory and Anticipated Utility Theory. Social preferences are Weighted Generalized Utilitarian and, in the case in which individual preferences satisfy the Generalized Bernoulli Hypothesis, they are Weighted Utilitarian. The objective-probability results for social preferences cast a new light on Harsanyi's Social Aggregation Theorem, which assumes that both individual and social preferences satisfy the Expected Utility Hypothesis.

When a society is faced with the problem of ranking uncertain prospects, the Pareto principle can be applied in two different ways. The *ex-ante* principle requires the existence of a ranking of individual values for the prospects and uses it to compare them. The *ex-post* principle requires the existence of individual rankings of consequences in each possible state, and uses these *ex-post* rankings to compare the prospects.[1]

Harsanyi's Social Aggregation Theorem [1955, 1977] shows that, when individuals agree on probabilities for contingent social states and each individual and a social observer have preferences that satisfy the Expected Utility Hypothesis (von Neumann and Morgenstern [1947]), the *ex-ante* principle implies that the observer's preferences must be Weighted Generalized Utilitarian. That is, the observer must have a value function that is ordinally equivalent to the weighted sum of individual transformed utilities. If, in addition, the individual *ex-ante* utilities satisfy the Bernoulli Hypothesis (Arrow [1972]), which makes each person's *ex-ante* utility *equal* to the expected value of von-Neumann – Morgenstern utilities rather than an increasing transform of it, Weighted Utilitarianism results.[2] The original theorem employed lotteries as social alternatives with fixed utilities in the various states. Some of the subsequent work (for example, Blackorby, Donaldson and Weymark [1998]) has proved the theorem in models in which probabilities are identical across individuals and utilities are state-contingent.

If, however, individual probabilities are subjective rather than 'objective' as above and individuals are sufficiently diverse in their beliefs or in their utilities, the social observer cannot have preferences that satisfy *ex-ante* welfarism. This paradox has been investigated by Broome [1991], Hammond [1981, 1983] and Mongin [1995]. Mongin employs the Bayesian axiomatization of Savage [1954]. Individual diversity is formalized in terms of either affinely independent subjective probabilities or affinely independent utility functions on the consequence set.

Mongin [1998] exploits the observation that these impossibility proofs depend critically on Savage's postulates of state independence. In Savage's model, all consequences are available in all states and their utility values do not depend on the state that is realized. When utilities are state-dependent, a possibility emerges. This resolution of the paradox is unimpressive, however, because it takes the edge off the Bayesian doctrine: subjective probabilities are known to be indeterminate in the pure state-dependent model of subjective expected utility, and subjective probability is, of course, what Bayesianism is about. When restrictions are added to the pure state-dependent model to reinstate the uniqueness of each person's

[1] See Hammond [1981, 1983].

[2] See Blackorby, Bossert and Donaldson [1998], Blackorby, Donaldson and Weymark [1980, 1990, 1998ab], Bossert and Weymark [1996], Broome [1990, 1991], Coulhon and Mongin [1989], De Meyer and Mongin [1995], Fishburn [1984], Hammond [1981, 1983], Mongin [1995, 1998], Mongin and d'Aspremont [1997], Roemer [1996], Sen [1976] and Weymark [1991, 1994].

subjective probabilities, the impossibility reappears, albeit in a more complex form than in the Savage model.

In this paper, we do not pursue state-dependence, but take the crucial step of abandoning the Expected Utility Hypothesis, a suggestion that has been made by Diamond [1967] and Sen [1970] in the course of criticizing Harsanyi. Each person is assumed to have a continuous *ex-ante* value function which depends on probabilities and state-contingent utility levels. These functions are assumed to satisfy several sensitivity properties but, beyond that, no structure is imposed on them. We assume that the observer has a value function that satisfies State-Contingent Utility Aggregation, our version of the *ex-post* Pareto principle, and Value Aggregation, our version of the *ex-ante* principle. State-Contingent Utility Aggregation requires the observer's utility level in any state to be a continuous and increasing function of individual utilities in that state. Value Aggregation requires the observer's value function to be a continuous and increasing function of individual (*ex-ante*) values. We assume that individual state-contingent utility levels can take on any value in an interval which can be different for each person and each state.

In Section 3, probabilities are subjective and this leads to several impossibility theorems. Theorems 1 and 2 show that the observer's preferences cannot simultaneously satisfy State-Contingent Utility Aggregation, Value Aggregation and either Equal-Utility Probability Independence or Probability Aggregation. Equal-Utility Probability Independence requires probabilities not to matter when utility payoffs are the same in all states and Probability Aggregation requires the observer's probabilities to be independent of individual utility levels. In both theorems, each individual's subjective probabilities are allowed to vary across a set of possible values. This assumption is dispensed with in Theorem 3. Without using the expected-utility assumption that is normally part of such a proof, it presents a further impossibility when probabilities are subjective and fixed and it is not the case that all probabilities, including the observer's, are the same. Theorem 3 appeals to the Betting Property, an axiom that is related to Machina and Schmeidler's [1992] Probabilistic Sophistication.

Section 4 considers the case of 'objective' probabilities which are common to all individuals and the observer. In this case, we characterize both individual and social preferences when State-Contingent Utility Aggregation and Value Aggregation are satisfied. The result, which is presented in Theorem 4, shows that individual and social value functions must be very similar, but they do not have to satisfy the Expected Utility Hypothesis. Surprisingly, however, Harsanyi's result about Generalized Utilitarianism is preserved. If, in addition, a generalization of the Bernoulli Hypothesis is imposed on the resulting value functions, Weighted Utilitarianism results.

Our two basic assumptions—Value Aggregation and State-Contingent Utility Aggregation—together require *ex-ante/ex-post* consistency. In the case of social aggregation theorems that employ the Harsanyi-style of assumptions, *ex-ante* welfarism and the assumption that all individuals and the social observer have preferences that satisfy the Expected Utility Hypothesis automatically imply *ex-post* welfarism. By contrast, we assume both *ex-ante* and *ex-post* welfarism and require consistency.

1. Individuals

Suppose that X, a subset of a Euclidean vector space, is a set of social alternatives, $N = \{1, \ldots, n\}$ is a set of individuals with $n \geq 2$ and $M = \{1, \ldots, m\}$ is a set of states of nature with $m \geq 2$. For each $i \in N$, $j \in M$, $U^i_j\colon X \longrightarrow \mathcal{R}$ is person i's utility function. In state j alternative $x_j \in X_j \subseteq X$ is realized and person i's utility payoff is $u^i_j = U^i_j(x_j)$. We define $\mathbf{u}^i = (u^i_1, \ldots, u^i_m)$ and, if utilities are the same in each state with $u^i_j = u$ for all $j \in M$, we write $\mathbf{u}_c = u\mathbf{1}_m = (u, \ldots, u)$. $\mathcal{U}^i_j = U^i_j(X_j)$ is the image of U^i_j and we define $\mathcal{U}^i = \mathcal{U}^i_1 \times \ldots \mathcal{U}^i_m$ for all $i \in N$ and $\mathcal{U}_j = \mathcal{U}^1_j \times \ldots \times \mathcal{U}^n_j$ for all $j \in M$. The set of feasible utility vectors for all n individuals is $\mathcal{U} = \{(\mathbf{u}^1, \ldots, \mathbf{u}^n) \mid \exists (x_1, \ldots, x_m) \in X_1 \times \ldots \times X_m \ni, \forall i \in N, j \in M, u^i_j = U^i_j(x_j)\}$.

We make use of several different assumptions about the set $\mathcal{U}$ in our four theorems.

Domain Assumption A: There exists $\xi = (\xi_1, \ldots, \xi_n) \in \mathcal{R}^n$ such that, for all $i \in N$, $j \in M$, $U^i_j(x_j) = \xi_i$ and $(\xi_1\mathbf{1}_m, \ldots, \xi_n\mathbf{1}_m)$ is an interior point of $\mathcal{U}$.

Domain Assumption B: For all $i \in N$, $j, k \in M$, $\mathcal{U}^i_j = \mathcal{U}^i_k$.

Domain Assumption C: Domain Assumption B holds, for all $i \in N$, $j \in M$, $\mathcal{U}^i_j$ is a nondegenerate interval and $\mathcal{U} = \prod_{i=1}^n \prod_{j=1}^m \mathcal{U}^i_j$.

Domain assumption A, which is used in Theorems 1 and 3, is the least demanding of the three. It requires the existence of some $(x_1, \ldots, x_n)$ such that each person's utility outcome is the same in each state. Because the vector ξ is assumed to be in the interior of $\mathcal{U}$, all n-person utility vectors in a neighbourhood of $(\xi_1\mathbf{1}_m, \ldots, \xi_n\mathbf{1}_m)$ are also feasible. Domain Assumption B, which is required for Theorem 2, makes the set of feasible utility outcomes independent of state for each individual. It is satisfied if utility functions are state-independent and $X_j = X$ for all $j \in M$. Domain Assumption C, which includes B, is required for Theorem 4 and it ensures that Gorman's theorem can be applied. It is trivially satisfied if X is the set of allocations in a private-goods economy and individual utility functions are continuous and self-interested.

Person i's subjective probability for state j is p^i_j and his or her vector of subjective probabilities is $\mathbf{p}^i = (p^i_1, \ldots, p^i_m)$, $\mathbf{p}^i \in \mathcal{S} = \{\pi \in \mathcal{R}^m_{++} \mid \sum_{j=1}^m \pi_j = 1\}$.[3] Probabilities may

[3] We assume that all subjective probabilities are feasible, but our results hold on more restrictive domains. What is important is that any individual's probabilities are independent of others' probabilities.

also be described by the vector $\tilde{\mathbf{p}}^i := (p_1^i, \ldots, p_{m-1}^i)$, $\tilde{\mathbf{p}}^i \in \tilde{\mathcal{S}} = \{\pi \in \mathcal{R}_{++}^{m-1} \mid \sum_{j=1}^{m-1} \pi_j < 1\}$. A prospect for individual i is the vector $(\mathbf{p}^i, \mathbf{u}^i)$ and a social prospect is the vector $(\mathbf{p}^1, \mathbf{u}^1, \ldots, \mathbf{p}^n, \mathbf{u}^n)$.

Person i ranks prospects with the value function $V^i: \mathcal{S} \times \mathcal{U}^i \longrightarrow \mathcal{R}$ where

$$v^i = V^i(\mathbf{p}^i, \mathbf{u}^i). \tag{1.1}$$

Decision theory suggests several restrictions that V^i may satisfy:

Continuity in Prospects: *V^i is continuous;*

Monotonicity in Probabilities: *For all $\mathbf{u}^i \in \mathcal{U}^i$, all $\hat{\mathbf{p}}^i$, $\check{\mathbf{p}}^i \in \mathcal{S}$ and for all k, $l \in \{1, \ldots, m\}$, if $u_k^i > u_l^i$, $\hat{p}_j^i = \check{p}_j^i$ for all $j \in \{1, \ldots, m\} \setminus \{k, l\}$, $\hat{p}_k^i > \check{p}_k^i$ (and, therefore, $\hat{p}_l^i < \check{p}_l^i$), then $V^i(\hat{\mathbf{p}}^i, \mathbf{u}^i) > V^i(\check{\mathbf{p}}^i, \mathbf{u}^i)$;*

Betting Property: *For all $\mathbf{p}^i \in \mathcal{S}$, all $u_j^i \in \mathcal{U}_k^i$, $u_l^i \in \mathcal{U}_k^i$, $(u, \ldots, u) \in M \setminus \{k, l\}$ such that $u_k^i > u_l^i$, if $p_k^i > p_l^i$, then*

$$V^i(\mathbf{p}^i, u, \ldots, u_k^i, \ldots, u_l^i, \ldots, u) > V^i(\mathbf{p}^i, u, \ldots, u_l^i, \ldots, u_k^i, \ldots, u). \tag{1.2}$$

A reasonable property in addition to the above requires V^i to be increasing in utility levels. Because it is not needed for our theorems and is implied by other axioms, we do not include it. Continuity in Prospects and Monotonicity in Probabilities together imply

Equal-Utility Probability Independence (EUPI): *For all $\mathbf{u}_c \in \mathcal{U}^i$, $V^i(\cdot, \mathbf{u}_c)$ is a constant function.*

Because EUPI is satisfied, without loss of generality we normalize V^i so that

$$V^i(\mathbf{p}, \mathbf{u}_c) = u \tag{1.3}$$

for all $\mathbf{p} \in \mathcal{S}$ and all $\mathbf{u}_c \in \mathcal{U}^i$.

Our notions of a prospect and a value function for i constitute a reduced form for several decision-theoretic models at the same time. It is easy to check that 'objective' Expected Utility Theory as well as some more recent constructions such as Rank-Dependent Utility Theory (RDUT)[4] satisfy all the conditions except for the Betting Property. Subjective Expected Utility Theory (SEUT)[5], as well as some recent nonlinear contributions involving subjective probabilities, can be seen to satisfy all of the assumptions except Monotonicity in Probabilities. Notice that this assumption is a particular application of Monotonicity

[4] See Quiggin [1982].

[5] See Savage [1954], for example.

with Respect to First-Order Stochastic Dominance. The Betting Property is satisfied by a number of non-expected utility constructions and is related to Probabilistic Sophistication in Machina and Schmeidler [1992]. Conceptually, it originates in Ramsey's method of eliciting subjective probabilities from betting attitudes.

2. The Observer

The observer compares social prospects, $(\mathbf{p}^1, \mathbf{u}^1, \ldots, \mathbf{p}^n, \mathbf{u}^n)$ according to a social value function $\phi : \mathcal{S} \times \mathcal{U}^1 \times \ldots \times \mathcal{S} \times \mathcal{U}^n \ (=: \mathcal{D}) \longrightarrow \mathcal{R}$ with image

$$v^0 = \phi(\mathbf{p}^1, \mathbf{u}^1, \ldots, \mathbf{p}^n, \mathbf{u}^n). \tag{2.1}$$

Without loss of generality, it is always possible to write ϕ as

$$\phi(\mathbf{p}^1, \mathbf{u}^1, \ldots, \mathbf{p}^n, \mathbf{u}^n) = V^0(\mathbf{p}^0, \mathbf{u}^0) = V^0(p^0_1, \ldots, p^0_m, u^0_1, \ldots, u^0_m) \tag{2.2}$$

with

$$\mathbf{p}^0 = \tilde{P}^0(\mathbf{p}^1, \mathbf{u}^1, \ldots, \mathbf{p}^n, \mathbf{u}^n) \tag{2.3}$$

and

$$\mathbf{u}^0 = \tilde{U}^0(\mathbf{p}^1, \mathbf{u}^1, \ldots, \mathbf{p}^n, \mathbf{u}^n), \tag{2.4}$$

where $V^0 \colon \mathcal{S}^0 \times \mathcal{U}^0 \longrightarrow \mathcal{R}$, $\tilde{P}^0 \colon \mathcal{D} \longrightarrow \mathcal{S}$, $\tilde{U}^0 \colon \mathcal{D} \longrightarrow \mathcal{R}^m$, $\mathcal{S}^0$ is the image of $\tilde{P}^0$ and $\mathcal{U}^0$ is the image of $\tilde{U}^0$.

Our first condition retains the general idea of the *ex-post* approach without relying on any specific theory such as SEUT.

State-Contingent Utility Aggregation: *There exists a continuous and increasing function* $U^0 \colon \cup_{j=1}^m \mathcal{U}_j \longrightarrow \mathcal{R}$ *such that, for all* $j \in \{1, \ldots, m\}$ *and all* $(\mathbf{p}^1, \mathbf{u}^1, \ldots, \mathbf{p}^n, \mathbf{u}^n) \in \mathcal{D}$,

$$\mathbf{u}^0 = \tilde{U}^0(\mathbf{p}^1, \mathbf{u}^1, \ldots, \mathbf{p}^n, \mathbf{u}^n) = \big(U^0(u^1_1, \ldots, u^n_1), \ldots, U^0(u^1_m, \ldots, u^n_m)\big). \tag{2.5}$$

This condition states that the observer assesses state-contingent vectors of utility levels in a state-independent way. As the definition of V^i indicates, our framework is compatible with state-dependent decision theories (such as state-dependent SEUT) at the individual level, but such flexibility is absent at the observer's level.

Conditions on probability aggregation that are roughly similar to the Pareto condition have sometimes been envisaged in the literature.[6] However, the *ex-post* school of welfare economics is usually agnostic about the origin of social probabilities. The impossibility

[6] See Genest and Zidek [1986] and Section 2 in Mongin [1995].

result of Section 3 can be made independent of their mode of formation if one is suspicious of any particular probability aggregation axiom, and the positive results of Section 4 take the social probability, $\mathbf{p}^0$, as well as the individual probabilities, $(\mathbf{p}^1, \ldots, \mathbf{p}^n)$, to be exogenously determined.

Welfare economics has also followed an altogether different approach from that implied by State-Contingent Utility Aggregation. In contrast with the *ex-post* theorists, those of the *ex-ante* school apply the Strong Pareto Principle directly to the individual evaluations of prospects. The next condition retains the guiding idea of the *ex-ante* approach without adhering to any specific decision-theoretic hypothesis. Let $\mathcal{M}^i$ be the image of V^i.

Value Aggregation: *There exists a continuous and increasing function* $W\colon \mathcal{M}^1 \times \ldots \times \mathcal{M}^n \longrightarrow \mathcal{R}$ *such that*

$$\phi(\mathbf{p}^1, \mathbf{u}^1, \ldots, \mathbf{p}^n, \mathbf{u}^n) = W(v^1, \ldots, v^n) = W\big(V^1(\mathbf{p}^1, \mathbf{u}^1), \ldots, V^n(\mathbf{p}^n, \mathbf{u}^n)\big). \tag{2.6}$$

If a social value function satisfies State-Contingent Utility Aggregation and Value Aggregation, we call it *consistent* for the reasons given in the introduction. In parallel with earlier investigations, we now enquire whether there exists a consistent social value function.

3. The Impossibility of Consistent Social Aggregation When Subjective Probabilities Can Differ

This section is divided into three subsections, in each of which a single additional assumption is placed on the value function of the observer. The first subsection imposes Equal-Utility Probability Independence, the second requires the probabilities of the observer to depend only upon the probabilities of the agents and the third requires the Betting Property to hold for the observer. Each of these assumptions leads to an impossibility if consistency is also required. These results lead us to abandon the idea of independent subjective probabilities in Section 4.

3.1. Equal-Utility Probability Independence for the Observer

We begin this subsection by imposing Equal-Utility Probability Independence on the observer's value function.

Equal-Utility Probability Independence for the Observer: $V^0(\cdot, \mathbf{u}^0_c)$ *is a constant function for all* $\mathbf{u}^0_c \in \mathcal{U}^0$.

Theorem 1 shows that consistency is impossible when this axiom is satisfied.

Theorem 1: *Suppose that Domain Assumption A is satisfied. If individual value functions satisfy Continuity and Monotonicity in Probabilities, there is no social value function that satisfies Equal-Utility Probability Independence for the Observer, State-Contingent Utility Aggregation and Value Aggregation.*

Proof: Choose $(\xi_1, \ldots, \xi_n)$ as in Domain Assumption A. There exists $\eta > 0$ such that

$$\begin{aligned} U^0(\xi_1 + \eta, \xi_2, \ldots, \xi_n) &> U^0(\xi_1, \xi_2, \ldots, \xi_n), \\ U^0(\xi_1, \xi_2 + \eta, \ldots, \xi_n) &> U^0(\xi_1, \xi_2, \ldots, \xi_n), \\ &\vdots \\ U^0(\xi_1, \xi_2, \ldots, \xi_n + \eta) &> U^0(\xi_1, \xi_2, \ldots, \xi_n). \end{aligned} \tag{3.1}$$

Let $\bar{y}$ be the smallest value on the left side of (3.1) and pick $\bar{u}^0$ so that $\bar{y} > \bar{u}^0 > U^0(\xi_1, \xi_2, \ldots, \xi_n)$. Because U^0 is continuous and increasing there exist $\epsilon_1, \ldots, \epsilon_n > 0$ such that

$$\begin{aligned} \bar{u}^0 &= U^0(\xi_1 + \epsilon_1, \xi_2, \ldots, \xi_n) \\ &= U^0(\xi_1, \xi_2 + \epsilon_2, \ldots, \xi_n) \\ &\vdots \\ &= U^0(\xi_1, \xi_2, \ldots, \xi_n + \epsilon_n). \end{aligned} \tag{3.2}$$

First suppose that $2 \leq m \leq n$. Given $(\mathbf{p}^1, \ldots, \mathbf{p}^n)$ construct social prospect $\hat{\mathbf{z}} = (\mathbf{p}^1, \hat{\mathbf{u}}^1, \ldots, \mathbf{p}^n, \hat{\mathbf{u}}^n)$ with

$$\begin{aligned} \hat{\mathbf{u}}^1 &= (\xi_1 + \epsilon_1, \xi_1, \ldots, \xi_1) \\ &\vdots \\ \hat{\mathbf{u}}^m &= (\xi_m, \ldots, \xi_m, \xi_m + \epsilon_m) \end{aligned} \tag{3.3}$$

and

$$\hat{\mathbf{u}}^i = (\xi_i, \ldots, \xi_i) \tag{3.4}$$

for $m < i \leq n$. From (3.2) and (2.2)–(2.4) we see that

$$\phi(\hat{\mathbf{z}}) = V^0(\tilde{P}^0(\hat{\mathbf{z}}), \bar{u}^0, \ldots, \bar{u}^0) \tag{3.5}$$

and the EUPI property for the observer implies that $\phi(\hat{\mathbf{z}})$ is independent of the individual probabilities. Now suppose that $\mathbf{p}^1$ is changed to $\hat{\mathbf{p}}^1 \in \mathcal{S}$ where, for some $k \neq 1$,

$$\hat{p}^1_1 = p^1_1 + \pi \tag{3.6}$$

and

$$\hat{p}^1_k = p^1_k - \pi \tag{3.7}$$

for some $\pi > 0$, and the rest of the probabilities are unchanged. Call this modified prospect $\hat{\mathbf{z}}'$. Monotonicity in Probabilities for V^1 and Value Aggregation imply that

$$\phi(\hat{\mathbf{z}}') > \phi(\hat{\mathbf{z}}), \tag{3.8}$$

a contradiction.

If $n < m$ modify the prospect $\hat{\mathbf{z}}$ by adding as many of the elements of $(\xi_1 + \epsilon_1, \xi_1, \ldots)$ as needed to $\hat{\mathbf{u}}^1$ (repeating if necessary), as many of the elements of $(\xi_2, \xi_2 + \epsilon_2, \xi_2, \ldots)$ to $\hat{\mathbf{u}}^2$ (repeating if necessary), and so on, until each vector has dimension m and then proceed as above. ■

3.2. Probability Aggregation

The impossibility result of Theorem 1 needed no detailed model of probability aggregation. The following axiom is a defensible restriction on the value function of the observer.

Probability Aggregation: *There exists a function* $P^0 \colon \mathcal{S}^n \longrightarrow \mathcal{S}^0$ *such that, for all* $(\mathbf{p}^1, \mathbf{u}^1, \ldots, \mathbf{p}^n, \mathbf{u}^n) \in \mathcal{D}$*,*

$$\begin{aligned} \mathbf{p}^0 = \tilde{P}(\mathbf{p}^1, \mathbf{u}^1, \ldots, \mathbf{p}^n, \mathbf{u}^n) &= P^0(\mathbf{p}^1, \ldots, \mathbf{p}^n) \\ &= \big(P^0_1(\mathbf{p}^1, \ldots, \mathbf{p}^n), \ldots, P^0_m(\mathbf{p}^1, \ldots, \mathbf{p}^n)\big). \end{aligned} \tag{3.9}$$

Thus, the observer's probabilities depend only on individual probabilities, not on individual utility levels. However, any component may depend on all of the individual probabilities. This condition is weaker than the assumption which is sometimes made in the literature that aggregate probabilities depend on individual probabilities, component by component.[7] This axiom can be used, in conjunction with State-Contingent Utility Aggregation and Value Aggregation, to produce another impossibility theorem.

Theorem 2: *Suppose that Domain Assumption B is satisfied. If individual value functions satisfy Continuity and Monotonicity In Probabilities, there is no social value function that satisfies Probability Aggregation, State-Contingent Utility Aggregation and Value Aggregation.*

[7] See Genest and Zidek [1986] and Mongin [1997] for a criticism.

Proof: If the value function of the observer satisfies EUPI we are done. By State-Contingent Utility Aggregation and Probability Aggregation, P^0 must be sensitive to individual probabilities. Suppose that EUPI does not hold for the observer. Then there exist $\mathbf{p}^0$, $\mathbf{q}^0 \in \mathcal{S}^0$ with $\mathbf{p}^0 \neq \mathbf{q}^0$ and $\mathbf{u}_c^0 \in \mathcal{U}^0$ such that

$$V^0(\mathbf{p}^0, \mathbf{u}_c^0) \neq V^0(\mathbf{q}^0, \mathbf{u}_c^0). \tag{3.10}$$

Consequently, there exist $(\mathbf{p}^1, \ldots, \mathbf{p}^n)$ and $(\mathbf{q}^1, \ldots, \mathbf{q}^n)$ in $\mathcal{S}^n$ such that $\mathbf{p}^0 = P^0(\mathbf{p}^1, \ldots, \mathbf{p}^n)$, $\mathbf{q}^0 = P^0(\mathbf{q}^1, \ldots, \mathbf{q}^n)$, and $\mathbf{p}^i \neq \mathbf{q}^i$ for some i. Letting $u^0 = U^0(y^1, \ldots, y^n)$ and using Domain Assumption B, this implies that

$$W\big(V^1(\mathbf{p}^1, \mathbf{y}_c^1), \ldots, V^n(\mathbf{p}^n, \mathbf{y}_c^n)\big) \neq W\big(V^1(\mathbf{q}^1, \mathbf{y}_c^1), \ldots, V^n(\mathbf{q}^n, \mathbf{y}_c^n)\big) \tag{3.11}$$

which contradicts EUPI for individuals, which follows from Continuity and Monotonicity in Probabilities. ■

The notion that separate aggregation of individual utilities and individual probabilities involve difficulties can be traced back to an early paper by Hylland and Zeckhauser [1979] who, however, only considered SEUT for both the individuals and the observer.

3.3. The Betting Property for the Observer

In this subsection we assume that both individual probabilities and those of the observer are fixed, and show that, if any two individual probabilities differ, a contradiction results. The theorem statement makes use of the following condition.

Betting Property for the Observer: V^0 *satisfies the Betting Property.*

Theorem 3: *Suppose that Domain Assumption A is satisfied and that* $\mathbf{p}^0$ *and* $(\mathbf{p}^1, \ldots, \mathbf{p}^n)$ *are fixed and that individual value functions satisfy Continuity and the Betting Property.*

(i) If $\mathbf{p}^i \neq \mathbf{p}^h$ *for individuals* i, $h \in \{1, \ldots, n\}$, *there is no social value function that satisfies State-Contingent Utility Aggregation and Value Aggregation.*

(ii) If there exists $i \in \{1, \ldots, n\}$ *such that* $\mathbf{p}^i \neq \mathbf{p}^0$, *there is no social value function that satisfies the Betting Property for the Observer, State-Contingent Utility Aggregation and Value Aggregation.*

Proof: (i) Without loss of generality, let $i = 1$, $h = 2$, $p_1^1 > p_2^1$, and $p_1^2 < p_2^2$. Take $(\xi_1, \ldots, \xi_n)$ as in the proof of Theorem 1 and let

$$\bar{u}^0 = U^0(\xi_1 + \epsilon_1, \xi_2, \ldots, \xi_n) = U^0(\xi_1, \xi_2 + \epsilon_2, \ldots, \xi_n). \tag{3.12}$$

Construct prospects $\tilde{\mathbf{z}} = (\mathbf{p}^1, \tilde{\mathbf{u}}^1, \ldots, \mathbf{p}^n, \tilde{\mathbf{u}}^n)$ and $\check{\mathbf{z}} = (\mathbf{p}^1, \check{\mathbf{u}}^1, \ldots, \mathbf{p}^n, \check{\mathbf{u}}^n)$ with

$$\begin{aligned} \tilde{\mathbf{u}}^1 &= (\xi_1 + \epsilon_1, \xi_1, \ldots, \xi_1) \\ \tilde{\mathbf{u}}^2 &= (\xi_2, \xi_2 + \epsilon_2, \ldots, \xi_2) \end{aligned} \tag{3.13}$$

and

$$\tilde{\mathbf{u}}^i = (\xi_i, \ldots, \xi_i) \tag{3.14}$$

for $2 < i \leq n$, and

$$\begin{aligned} \check{\mathbf{u}}^1 &= (\xi_1, \xi_1 + \epsilon_1, \ldots, \xi_1) \\ \check{\mathbf{u}}^2 &= (\xi_2 + \epsilon_2, \xi_2, \ldots, \xi_2) \end{aligned} \tag{3.15}$$

and

$$\check{\mathbf{u}}^i = (\xi_i, \ldots, \xi_i) \tag{3.16}$$

for $2 < i \leq n$. The Betting Property for V^i implies that

$$V^1(\mathbf{p}^1, \xi_1 + \epsilon_1, \xi_1, \ldots, \xi_1) > V^1(\mathbf{p}^1, \xi_1, \xi_1 + \epsilon_1, \ldots, \xi_1) \tag{3.17}$$

and

$$V^2(\mathbf{p}^2, \xi_2, \xi_2 + \epsilon_2, \xi_2, \ldots, \xi_2) > V^2(\mathbf{p}^2, \xi_2 + \epsilon_2, \xi_2, \ldots, \xi_2) \tag{3.18}$$

so that Value Aggregation yields $\phi(\tilde{\mathbf{z}}) > \phi(\check{\mathbf{z}})$. However, State-Contingent Utility Aggregation and (3.12) imply that

$$\begin{aligned} \phi(\tilde{\mathbf{z}}) &= V^0(\mathbf{p}^0, U^0(\xi_1 + \epsilon_1, \xi_2, \ldots, \xi_n), U^0(\xi_1, \xi_2 + \epsilon_2, \ldots, \xi_n), \ldots, U^0(\xi_1, \xi_2, \ldots, \xi_n)) \\ &= V^0(\mathbf{p}^0, U^0(\xi_1, \xi_2 + \epsilon_2, \ldots, \xi_n), U^0(\xi_1 + \epsilon_1, \xi_2, \ldots, \xi_n), \ldots, U^0(\xi_1, \xi_2, \ldots, \xi_n)) \\ &= \phi(\check{\mathbf{z}}), \end{aligned} \tag{3.19}$$

a contradiction that establishes the first part of the theorem.

(ii) Now suppose that $\mathbf{p}^i \neq \mathbf{p}^0$ for some $i \in \{1, \ldots, n\}$. Given (i), we need only consider the case in which $\mathbf{p}^1 = \mathbf{p}^2 = \ldots = \mathbf{p}^n$. Let $p_1^1 > p_2^1$ and $p_1^0 < p_2^0$ and construct the prospects $\tilde{\mathbf{z}} = (\mathbf{p}^1, \tilde{\mathbf{u}}^1, \ldots, \mathbf{p}^n, \tilde{\mathbf{u}}^n)$ and $\check{\mathbf{z}} = (\mathbf{p}^1, \check{\mathbf{u}}^1, \ldots, \mathbf{p}^n, \check{\mathbf{u}}^n)$ with

$$\tilde{\mathbf{u}}^1 = (\xi_1 + \epsilon_1, \xi_1, \ldots, \xi_1) \tag{3.20}$$

and

$$\tilde{\mathbf{u}}^i = (\xi_i, \ldots, \xi_i) \tag{3.21}$$

for $2 \leq i \leq n$, and

$$\check{\mathbf{u}}^1 = (\xi_1, \xi_1 + \epsilon_1, \ldots, \xi_1) \tag{3.22}$$

and

$$\check{\mathbf{u}}^i = (\xi_i, \ldots, \xi_i) \tag{3.23}$$

for $2 \le i \le n$. The Betting Property for V^1 and Value Aggregation imply that

$$\begin{aligned} &\phi(\tilde{\mathbf{z}}) \\ &= W\big(V^1(\mathbf{p}^1, \xi_1 + \epsilon_1, \xi_1, \ldots, \xi_1), \ldots, V^i(\mathbf{p}^i, \xi_i \ldots, \xi_i), \ldots\big) \\ &> W(V^1(\mathbf{p}^1, \xi_1, \xi_1 + \epsilon_1, \xi_1, \ldots, \xi_1), \ldots, V^i(\mathbf{p}^i, \xi_i \ldots, \xi_i), \ldots) \\ &= \phi(\check{\mathbf{z}}). \end{aligned} \tag{3.24}$$

However, State-Contingent Utility Aggregation and the Betting Property for the Observer imply that

$$\begin{aligned} \phi(\check{\mathbf{z}}) &= V^0\big(\mathbf{p}^0, U^0(\xi_1, \xi_2, \ldots, \xi_n), U^0(\xi_1 + \epsilon_1, \xi_2, \ldots, \xi_n), U^0(\xi_1, \xi_2, \ldots, \xi_n), \ldots\big) \\ &> V^0\big(\mathbf{p}^0, U^0(\xi_1 + \epsilon_1, \xi_2, \ldots, \xi_n), U^0(\xi_1, \xi_2, \ldots, \xi_n), U^0(\xi_1, \xi_2, \ldots, \xi_n), \ldots\big) \\ &= \phi(\tilde{\mathbf{z}}), \end{aligned} \tag{3.25}$$

a contradiction. ■

Observe that the decision-theoretic properties that have been used to derive an impossibility are not the same when probabilities are allowed to vary and when they are fixed. Monotonicity in Probabilities (in effect Stochastic Dominance) and Equal-Utility Probability Independence for the Observer are needed in one case, and the Betting Property (for individuals and the observer) in the other. This is how it should be. When probabilities are allowed to vary, we are implicitly working with a lottery model, as in von-Neumann – Morgenstern and in 'objective' non-expected-utility constructions, where the lotteries are the objects of choice. When we fix probabilities and invoke the Betting Property, we are implicitly working within a subjective-probability framework, as in Savage and in subjective non-expected-utility constructions.

Theorem 3 implies that individuals' and the observer's probabilities must be the same in order to avoid a contradiction. The consequences of the consistency hypothesis for individuals' and the observer's utility values when all probabilities coincide are addressed in the following section.

Here we present a simple application of Theorem 3. Suppose that the individuals are consumers endowed with continuous, selfish, and nonsatiable preferences on $\mathcal{R}^t_+$, where t is the number of commodities. Each consumer has one such preference for each state, so that his or her preferences can be represented by a continuous, nonconstant utility function. The observer is endowed with continuous preferences on the set of individual allocations which are state-independent and are represented by a state-independent function. By further endowing the consumers and the observer with subjective probabilities and value functions, we can define preferences over uncertain allocations and require an *ex-ante* version of the Strong Pareto Principle. The Pareto-Indifference part generates the W function in our Value Aggregation condition, while the increasingness property follows from the remaining part. On the other hand, the *ex-post* version of the Strong Pareto Principle implies the State-Contingent Utility Aggregation condition. Theorem 3 shows that there is no social-welfare function that simultaneously satisfies both the *ex-ante* and *ex-post* requirements. In essence, this is Theorem 2 in Hammond [1982] without the expected-utility assumption.

Theorem 3 may also be compared with other existing results, especially the impossibility theorems of Mongin [1995][8] for Savage's framework of SEUT. There, impossibilities are derived not only for the Strong Pareto Principle, but also for the Weak Pareto Principle, and even for Pareto Indifference alone. The later two conditions are shown to lead to dictatorial social aggregation instead of a logical impossibility. The comparison is imperfect because the flexibility required to prove these results in the framework of Savage can only be found in the probability values and not in the utility values. Unlike in the present paper, the latter typically do not range through an interval.

Disregarding the diversity of technical frameworks, two general observations appear to be in order when Theorem 3 is compared with existing results. First, its proof exhibits a structural feature that underlies all previously available proofs. At some stage in these arguments, the *ex-ante* Pareto principle must be applied twice, once in a natural way when the individuals agree on how to rank probabilities and utilities, and once in a less natural way when they disagree on both rankings but this disagreement happens to cancel out. This structural feature is illustrated by Example 3 in Mongin [1995] and recurs in the two-stage division of our proof here. The other, more important observation is that impossibilities are driven by much weaker decision-theoretic assumptions than one would have expected. The impossibility of consistent social aggregation is a disturbingly robust conclusion.

4. Social Aggregation with 'Objective' Probabilities

In this section, we investigate social aggregation when all individuals and the observer have probabilities that coincide. We refer to probabilities such as these as 'objective' and require State-Contingent Utility Aggregation and Value Aggregation to hold for all such probabilities and all vectors of individual utilities.

If $\mathbf{p}^i = \mathbf{p}^0 = \mathbf{p}$ for all $i \in \{1, \ldots, n\}$, State-Contingent Utility Aggregation and Value Aggregation together imply

$$\begin{aligned} V^0(\mathbf{p}, \mathbf{u}^0) &= V^0\big(\mathbf{p}, U^0(u_1^1, \ldots, u_1^n), \ldots, U^0(u_m^1, \ldots, u_m^n)\big) \\ &= W\big(V^1(\mathbf{p}, \mathbf{u}^1), \ldots, V^n(\mathbf{p}, \mathbf{u}^n)\big). \end{aligned} \tag{4.1}$$

[8] Propositions 5 and 7 in particular.

Theorem 4 provides necessary and sufficient conditions for the observer's preferences to satisfy State-Contingent Utility Aggregation and Value Aggregation. In the theorem statement, $\overset{\text{o}}{=}$ means 'is ordinally equivalent to'.

Theorem 4: *Suppose that Domain Assumption C is satisfied,* $\mathbf{p}^i = \mathbf{p}^0 = \mathbf{p}$ *for all* $i \in \{1, \ldots, n\}$ *and individual value functions satisfy Continuity and Monotonicity in Probabilities.* V^0 *satisfies State-Contingent Utility Aggregation and Value Aggregation if and only if, for all* $(\mathbf{u}^1, \ldots \mathbf{u}^n) \in \mathcal{U}$*:*

$$v^i = V^i(\mathbf{p}, \mathbf{u}^i) = g^{i^{-1}}\left(\sum_{j=1}^{m} a_j(\mathbf{p}) g^i(u^i_j)\right) \tag{4.2}$$

for all $i \in \{1, \ldots, n\}$*;*

$$u^0_j = U^0(u^1_j, \ldots, u^n_j) \overset{\text{o}}{=} \sum_{i=1}^{n} w_i g^i(u^i_j) \tag{4.3}$$

for all $j \in \{1, \ldots, m\}$*;*

$$W(v^1, \ldots, v^n) \overset{\text{o}}{=} \sum_{i=1}^{n} w_i g^i(v^i); \tag{4.4}$$

and

$$V^0(\mathbf{p}, \mathbf{u}^0) \overset{\text{o}}{=} \sum_{i=1}^{n}\sum_{j=1}^{m} w_i a_j(\mathbf{p}) g^i(u^i_j) \tag{4.5}$$

where, for all $j \in \{1, \ldots, m\}$*,* a_j *is continuous,* $a_j(\mathbf{p}) > 0$ *and* $\sum_{j=1}^m a_j(\mathbf{p}) = 1$ *for all* $\mathbf{p} \in \mathcal{S}$*, and, for all* $i \in \{1, \ldots, n\}$*,* $w_i > 0$*,* $\sum_{i=1}^n w_i = 1$ *and* g^i *is continuous and increasing.*

Proof: Consider the interior of $\mathcal{U}$. Continuity of U^0 and W imply that V^0 is continuous. Conditional on $\mathbf{p}$, (4.1) implies that the variables in each row and column of Table 2 are separable from their complements.

u^1_1	$\ldots$	u^1_m
$\vdots$		$\vdots$
u^n_1	$\ldots$	u^n_m

Table 2

Consequently, by Gorman's overlapping theorem,[9]

$$V^0(\mathbf{p}, \mathbf{u}^0) = \overset{*}{V}{}^0\left(\sum_{i=1}^{n}\sum_{j=1}^{m} \sigma_j^i(\mathbf{p}, u_j^i), \mathbf{p}\right), \tag{4.6}$$

where $\overset{*}{V}{}^0$ is continuous and increasing in its first argument and each σ_j^i is continuous and increasing in its second argument.

Equation (4.6) implies that, for every $\mathbf{p}$, $\sum_{i=1}^n \sigma_j^i(\mathbf{p}, u_j^i)$ is ordinally equivalent to $U^0(u_j^1, \ldots, u_j^n)$ and, therefore, that

$$F_j\left(\sum_{i=1}^{n} \sigma_j^i(\mathbf{p}, u_j^i), \mathbf{p}\right) = U^0(u_j^i, \ldots, u_j^n) \tag{4.7}$$

where F_j is continuous and increasing in its first argument. Because (4.7) holds for all $\mathbf{p}$, $\mathbf{p}$ may be set equal to some arbitrary probability vector such as $\bar{\mathbf{p}} = (1/m, \ldots, 1/m)$. Defining $\bar{\sigma}_j^i(\cdot) = \sigma_j^i(\bar{\mathbf{p}}, \cdot)$ and $\bar{F}_j(\cdot) = F_j(\cdot, \bar{\mathbf{p}})$, (4.7) implies

$$\bar{F}_j\left(\sum_{i=1}^{n} \bar{\sigma}_j^i(u_j^i)\right) = U^0(u_j^i, \ldots, u_j^n). \tag{4.8}$$

(4.7) and (4.8) together imply

$$F_j\left(\sum_{i=1}^{n} \sigma_j^i(\mathbf{p}, u_j^i), \mathbf{p}\right) = \bar{F}_j\left(\sum_{i=1}^{n} \bar{\sigma}_j^i(u_j^i)\right). \tag{4.9}$$

Define $z_j^i = \bar{\sigma}_j^i(u_j^i)$ and $\hat{\sigma}_j^i(\mathbf{p}, z_j^i) = \sigma_j^i(\mathbf{p}, \bar{\sigma}_j^{i^{-1}}(z_j^i)) = \sigma_j^i(\mathbf{p}, u_j^i)$ to get

$$F_j\left(\sum_{i=1}^{n} \hat{\sigma}_j^i(\mathbf{p}, z_j^i), \mathbf{p}\right) = \bar{F}_j\left(\sum_{i=1}^{n} z_j^i\right). \tag{4.10}$$

This implies

$$\sum_{i=1}^{n} \hat{\sigma}_j^i(\mathbf{p}, z_j^i) = G_j\left(\sum_{i=1}^{n} z_j^i, \mathbf{p}\right) \tag{4.11}$$

where G_j is continuous and increasing in its first argument. For each $\mathbf{p}$, (4.11) is a Pexider equation (Eichhorn [1978]) whose solution is

$$\hat{\sigma}_j^i(\mathbf{p}, z_j^i) = \bar{a}_j(\mathbf{p}) z_j^i + b_j^i(\mathbf{p}), \tag{4.12}$$

which implies

$$\sigma_j^i(\mathbf{p}, u_j^i) = \bar{a}_j(\mathbf{p}) \bar{\sigma}_j^i(u_j^i) + b_j^i(\mathbf{p}). \tag{4.13}$$

9 See the appendix.

$\bar{a}_j(\mathbf{p}) > 0$ because $\hat{\sigma}^i_j$ is increasing in its second argument and $\bar{\sigma}^i_j$ is increasing. Given equation (4.13), (4.6) can be rewritten as

$$V^0(\mathbf{p}, \mathbf{u}^0) = \overset{*}{V}{}^0\bigg(\sum_{i=1}^{n} \sum_{j=1}^{m} \Big(\bar{a}_j(\mathbf{p}) \bar{\sigma}^i_j(u^i_j) + b^i_j(\mathbf{p}) \Big), \mathbf{p} \bigg). \tag{4.14}$$

(4.8) implies

$$U^0(\xi_1, \ldots, \xi_n) = \bar{F}_j\bigg(\sum_{i=1}^{n} \bar{\sigma}^i_j(\xi_i) \bigg) \tag{4.15}$$

for all $(\xi_1, \ldots, \xi_n) \in \cup_{j=1}^m \mathcal{U}_j$. The right side of (4.15) is independent of j because the left side is and, setting $j = 1$,

$$U^0(\xi_1, \ldots, \xi_n) = \bar{F}_1\bigg(\sum_{i=1}^{n} \bar{\sigma}^i_1(\xi_i) \bigg). \tag{4.16}$$

Defining $\overset{*}{U} = \bar{F}_1$ and $\bar{g}^i = \bar{\sigma}^i_1$,

$$U^0(\xi_1, \ldots, \xi_n) = \overset{*}{U}\bigg(\sum_{i=1}^{n} \bar{g}^i(\xi_i) \bigg). \tag{4.17}$$

(4.8) and (4.17) imply

$$\bar{F}_j\bigg(\sum_{i=1}^{n} \bar{\sigma}^i_j(\xi_i) \bigg) = \overset{*}{U}\bigg(\sum_{i=1}^{n} \bar{g}^i(\xi_i) \bigg) \tag{4.18}$$

for each j. Defining $y_i = \bar{g}^i(\xi_i)$, $\hat{\sigma}^i_j(y_i) = \bar{\sigma}^i_j(\bar{g}^{i^{-1}}(y_i)) = \bar{\sigma}^i_j(\xi_i)$, and $\overset{*}{U}_j = \bar{F}_j^{-1} \circ \overset{*}{U}$, (4.18) becomes

$$\sum_{i=1}^{n} \hat{\sigma}^i_j(y_i) = \overset{*}{U}_j\bigg(\sum_{i=1}^{n} y_i \bigg), \tag{4.19}$$

a Pexider equation (Eichhorn [1978]) whose solution is

$$\hat{\sigma}^i_j(y_i) = c_j y_i + d^i_j, \tag{4.20}$$

which implies

$$\bar{\sigma}^i_j(\xi_i) = c_j \bar{g}^i(\xi_i) + d^i_j \tag{4.21}$$

where c_j, $d^i_j \in \mathcal{R}$. Because $\bar{\sigma}^i_j$ and $\bar{g}^i$ are increasing functions, $c_j > 0$. (In addition, because $\bar{\sigma}^i_1 = \bar{g}^i$, $c_1 = 1$ and $d^i_1 = 0$.) Without loss of generality, c_j and d^i_j can be absorbed into the functions $\bar{a}_j$ and b^i_j respectively. Given that, equation (4.14) can be rewritten as

$$V^0(\mathbf{p}, \mathbf{u}^0) = \overset{*}{V}{}^0\bigg(\sum_{i=1}^{n} \sum_{j=1}^{m} \Big(\bar{a}_j(\mathbf{p}) \bar{g}^i(u^i_j) + b^i_j(\mathbf{p}) \Big), \mathbf{p} \bigg) \tag{4.22}$$

which implies, using (4.1), that

$$V^i(\mathbf{p},\mathbf{u}^i) = \overset{*}{V}{}^i\left(\sum_{j=1}^m \Big(\bar{a}_j(\mathbf{p})\bar{g}^i(u_j^i) + b_j^i(\mathbf{p})\Big), \mathbf{p}\right). \tag{4.23}$$

Equal-Utility Probability Independence, which follows from Continuity and Monotonicity in Probabilities, implies $V^i(\mathbf{p},\mathbf{u}_c) = u$ (equation (1.3)), and

$$\overset{*}{V}{}^i\left(\sum_{j=1}^m \bar{a}_j(\mathbf{p})\bar{g}^i(u) + \sum_{j=1}^m b_j^i(\mathbf{p}), \mathbf{p}\right) = u. \tag{4.24}$$

Let $t = \sum_{j=1}^m \bar{a}_j(\mathbf{p})\bar{g}^i(u) + \sum_{j=1}^m b_j^i(\mathbf{p})$ and solve for u to get

$$\overset{*}{V}{}^i(t,\mathbf{p}) = \bar{g}^{i^{-1}}\left(\frac{t - \sum_{j=1}^m b_j^i(\mathbf{p})}{\sum_{k=1}^m \bar{a}_k(\mathbf{p})}\right). \tag{4.25}$$

Using (4.23), this implies

$$\begin{aligned} v^i = V^i(\mathbf{p},\mathbf{u}^i) &= \bar{g}^{i^{-1}}\left(\frac{\sum_{j=1}^m \bar{a}_j(\mathbf{p})\bar{g}^i(u_j^i) + \sum_{j=1}^m b_j^i(\mathbf{p}) - \sum_{j=1}^m b_j^i(\mathbf{p})}{\sum_{k=1}^m \bar{a}_k(\mathbf{p})}\right) \\ &= \bar{g}^{i^{-1}}\left(\sum_{j=1}^m \frac{\bar{a}_j(\mathbf{p})}{\sum_{k=1}^m \bar{a}_k(\mathbf{p})}\bar{g}^i(u_j^i)\right). \end{aligned} \tag{4.26}$$

Defining $a_j(\mathbf{p}) = \bar{a}_j(\mathbf{p})/\sum_{k=1}^m \bar{a}_k(\mathbf{p})$,

$$V^i(\mathbf{p},\mathbf{u}^i) = \bar{g}^{i^{-1}}\left(\sum_{j=1}^m a_j(\mathbf{p})\bar{g}^i(u_j^i)\right). \tag{4.27}$$

Now choose $(w_1,\ldots,w_n) \in \mathcal{R}^n_{++}$ such that $\sum_{i=1}^n w_i = 1$ and define $g^i(u_j^i) = \bar{g}^i(u_j^i)/w_i$. Because $g^{i^{-1}}(t) = \bar{g}^{i^{-1}}(w_i t)$,

$$V^i(\mathbf{p},\mathbf{u}^i) = g^{i^{-1}}\left(\sum_{j=1}^m a_j(\mathbf{p})g^i(u_j^i)\right) \tag{4.28}$$

which is (4.2). a_j is continuous because V^i is, and $a_j(\mathbf{p}) > 0$ because $\bar{a}_j(\mathbf{p}) > 0$. Using (4.17) and the definition of g^i,

$$U^0(u_j^1,\ldots,u_j^n) = \overset{*}{U}\left(\sum_{i=1}^n w_i g^i(u_j^i)\right) \tag{4.29}$$

which implies (4.3). Using (4.22), (4.26) and Value Aggregation ((2.6)),

$$\begin{aligned} V^0(\mathbf{p},\mathbf{u}^0) &= \overset{*}{V}{}^0\left(\sum_{i=1}^{n}\sum_{k=1}^{m}\bar{a}_k(\mathbf{p})\bar{g}^i(v^i) + \sum_{i=1}^{n}\sum_{j=1}^{m} b_j^i(\mathbf{p}), \mathbf{p}\right) \\ &= \overset{*}{V}{}^0\left(\sum_{j=1}^{m}\bar{a}_j(\mathbf{p})\left(\sum_{i=1}^{n}\bar{g}^i(v^i)\right) + \sum_{i=1}^{n}\sum_{j=1}^{m} b_j^i(\mathbf{p}), \mathbf{p}\right) \\ &= W(v^1,\ldots,v^n). \end{aligned} \tag{4.30}$$

Set $\mathbf{u}^i = \bar{u}^i \mathbf{1_m}$ for all $i \in \{1,\ldots,n\}$ so that $v^i = \bar{u}^i$ to get

$$\overset{*}{V}{}^0\left(\sum_{j=1}^{m}\bar{a}_j(\mathbf{p})\left(\sum_{i=1}^{n}\bar{g}^i(\bar{u}^i)\right) + \sum_{i=1}^{n}\sum_{j=1}^{m} b_j^i(\mathbf{p}), \mathbf{p}\right) = W(\bar{u}^1,\ldots,\bar{u}^n). \tag{4.31}$$

Because W is independent of $\mathbf{p}$ and $(\bar{u}^1,\ldots,\bar{u}^n)$ can be moved independently of $\mathbf{p}$, the left side of (4.31) is also independent of $\mathbf{p}$ and, as a consequence, (4.30) implies

$$W(v^1,\ldots,v^n) \overset{\text{o}}{=} \sum_{i=1}^{n}\bar{g}^i(v^i) = \sum_{i=1}^{n} w_i g^i(v^i) \tag{4.32}$$

which is (4.4). (4.5) follows from (4.21) and (4.19). Sufficiency is immediate. ■

The assumption that $\mathcal{U}$ is a product set (in Domain Assumption C) is essential for Theorem 4 to be true globally. In general, $\mathcal{U} \subseteq \prod_{i=1}^{n}\prod_{j=1}^{m}\mathcal{U}_j^i$ and it can be a proper subset. Even in that case, however, the result of Theorem 4 is true locally. That is, at any $(\bar{\mathbf{u}}^1,\ldots,\bar{\mathbf{u}}^n)$ in the interior of $\mathcal{U}$, there is some $\varepsilon > 0$ such that equations (4.2)–(4.5) are true in an ε-neighbourhood of $\bar{u}$. Several writers[10] have investigated the conditions under which local additivity implies global additivity on the interior of $\mathcal{U}$. If $\mathcal{U}$ is convex, a sufficient condition is that the indifference sets of $\Psi^0(\mathbf{p},\cdot)$ given by $\Psi^0(\mathbf{p},\mathbf{u}^1,\ldots,\mathbf{u}^n) = W(V^1,(\mathbf{p},\mathbf{u}^1),\ldots,V^n(\mathbf{p},\mathbf{u}^n))$ (from (4.1)) must be connected for every value of $\mathbf{p}$.[11] We cannot, however reasonably expect this to hold unless $\mathcal{U}$ is a product set. Even without the global result of Theorem 4, however, the comparisons of the next section are valid.

It is possible to write equations (4.3)–(4.5) without the 'weights' $w_1,\ldots,w_n$ by incorporating them into the functions $g^1,\ldots,g^n$. This move does not change the individual utility functions, as the proof shows. Thus the weights are not unique. We have chosen the more complicated presentation because we think it is reasonable to distinguish the observer's preferences from the individual utility functions.

10 See Wakker [1993] and the references therein.

11 See Theorem 2.2 in Wakker [1993] and the discussion that follows it.

Because the function U^0 is not normalized, Theorem 4 does not include a characterization of the function V^0 in terms of the observer's utility vector $(u_1^0, \ldots, u_m^0)$. One possible normalization requires the observer's utility to be the same as individual utility levels when they are all equal. Using (4.3), this requires

$$u_j^0 = U^0(u_j^1, \ldots, u_j^n) = g^{0^{-1}}\left(\sum_{i=1}^{n} w_i g^i(u_j^i)\right), \tag{4.33}$$

where g^0 is defined by

$$g^0(t) = \sum_{i=1}^{n} w_i g^i(t). \tag{4.34}$$

Using (4.5), this implies

$$V^0(\mathbf{p}, \mathbf{u}^0) \overset{\text{o}}{=} g^{0^{-1}}\left(\sum_{j=1}^{m} a_j(\mathbf{p}) g^0(u_j^0)\right) \tag{4.35}$$

which has the same functional form form as V^i, $i \in \{1, \ldots, n\}$. Thus, when V^0, suitably normalized, satisfies State-Contingent Utility Aggregation and Value Aggregation, it satisfies all the properties that the individual value functions do.

Anonymity is not used in Theorem 4 and it can be imposed in two ways. It can be applied to the function W or to the function U^0. Defining the functions $\breve{g}^i$ by $\breve{g}^i(u) = w_i g^i(u)$ for all $i \in \{1, \ldots, n\}$, Anonymity requires, in both cases, that

$$\breve{g}^i(u) = \breve{g}^1(u) + \gamma_i \tag{4.36}$$

for all i. If this condition is satisfied,

$$g^{i^{-1}}\left(\sum_{j=1}^{m} a_j(\mathbf{p}) g^i(u_j^i)\right) = \breve{g}^{-1}\left(\sum_{j=1}^{m} a_j(\mathbf{p}) \breve{g}(u_j^i)\right) \tag{4.37}$$

where $\breve{g} = \breve{g}^1$, and the individual value functions must be identical.

The value function V^i satisfies the Expected Utility Hypothesis if and only if

$$V^i(\mathbf{p}, \mathbf{u}^i) = g^{i^{-1}}\left(\sum_{j=1}^{m} p_j g^i(u_j^i)\right). \tag{4.38}$$

The function $a_j(\mathbf{p}) = p_j$ and it depends on the j^{th} probability alone. $g^i(u_j^i)$ is person i's von-Neumann – Morgenstern utility in state j. The function g^i expresses i's attitude to utility risk and, if g^i is affine, i is risk-neutral in that sense. Value functions of that type are sometimes called Bernoulli value functions, in which case,

$$V^i(\mathbf{p}, \mathbf{u}^i) = \sum_{j=1}^{m} p_j u_j^i. \tag{4.39}$$

This is the case most commonly investigated, but is is not the only one. See Blackorby, Donaldson and Weymark [1980, 1998ab], Mongin and d'Aspremont [1998, Section 5.3], Roemer [1996], Sen [1976] and Weymark [1991] for discussions. In the Bernoulli case, preferences over utility prospects (but not over non-utility aspects of payoffs) must be identical, and Anonymity requires equal weights. In addition, if any individual has a Bernoulli value function, Anonymity requires each person to have one.

We now compare the result of Theorem 4 with various proposals that have been made for individual value functions. The value functions presented are taken from Machina [1991] and rewritten in our notation. We consider seven families of value functions, each of which includes expected utility as a special case.

1. Prospect Theory

Proposed by Edwards [1955, 1962] and by Kahneman and Tversky [1979], Prospect Theory requires

$$V^i(\mathbf{p}, \mathbf{u}^i) \overset{\circ}{=} \sum_{j=1}^{m} \pi^i(p_j) g^i(u_j^i), \tag{4.40}$$

where π^i is increasing and continuous and $\pi^i(\mathbf{p}) > 0$ for all $\mathbf{p} \in \mathcal{S}$. It is well-known Prospect Theory does not, in general, satisfy monotonicity with respect to first-order stochastic dominance for $m > 2$. In addition, it does not always satisfy Equal-Utility Probability Independence (EUPI). For that, $\sum_{j=1}^{m} \pi^i(p_j)$ must be a (positive) constant which can, without loss of generality, be chosen to be 1. If $m \geq 3$, this requires π^i to be affine with $\pi^i(p_j) = \alpha_i p_j + \delta_i$, a requirement that trivializes the theory. To see the formal point involved, note that

$$\sum_{j=1}^{m-1} \pi^i(p_j) = 1 - \pi^i\left(1 - \sum_{j=1}^{m-1} p_j\right) =: F\left(\sum_{j=1}^{m-1} p_j\right), \tag{4.41}$$

a Pexider equation whose solution is (Eichhorn [1978]) is $\pi^i(p_j) = \alpha_i p_j + \delta_i$. Because $\sum_{j=1}^{m} \pi(p_j) = 1$, $\delta_i = (1 - \alpha_i)/m$ and

$$\pi^i(p_j) = \alpha_i p_j + \frac{1 - \alpha_i}{m}. \tag{4.42}$$

To ensure that π^i is increasing and that $\pi^i(p_j) > 0$ for all $p_j \in (0, 1)$, it must be true that $0 < \alpha_i \leq 1$. $\alpha_i = 1$ corresponds to the Expected Utility Hypothesis.

To satisfy the result of Theorem 4, which requires $a_j(\mathbf{p})$ to be independent of i, π^i must a be independent of i and, therefore, $\alpha_i = \alpha$, $0 < \alpha \leq 1$. Each person's deviation from the Expected Utility Hypothesis must be the same.

2. Rank-Dependent (Anticipated) Utility

The Rank-Dependent (Anticipated) Utility value function of Quiggin [1982] is

$$V^i(\mathbf{p}, \mathbf{u}^i) \overset{\circ}{=} \sum_{j=1}^{m} H^i_j(\mathbf{p}) g^i(u^i_j) \tag{4.43}$$

where

$$H^i_j(\mathbf{p}) = \begin{cases} h^i(p_1) - h^i(0), & \text{if } j = 1, \\ h^i\Big(\sum_{k=1}^{j} p_k\Big) - h^i\Big(\sum_{k=1}^{j-1} p_k\Big), & \text{otherwise,} \end{cases} \tag{4.44}$$

and h^i is increasing. Equation (4.2) requires the function h^i to be independent of i. Because $\sum_{j=1}^{m} H^i_j(\mathbf{p}) = h^i(1) - h^i(0)$, EUPI is satisfied.

3. Subjectively Weighted Utility

This family of value functions (Karmarkar [1978, 1979]) requires

$$V^i(\mathbf{p}, \mathbf{u}^i) \overset{\circ}{=} \sum_{j=1}^{m} \frac{\pi^i(p_j)}{\sum_{k=1}^{m} \pi^i(p_k)} g^i(u^i_j). \tag{4.45}$$

This proposal satisfies EUPI and is consistent with the result of Theorem 4 as long as π^i is independent of i.

The next four models do not satisfy the conditions of the theorem.

4. Weighted Utility

Chew [1983], Chew and MacCrimmon [1979] and Fishburn [1983] have proposed the Weighted Utility value function

$$V^i(\mathbf{p}, \mathbf{u}^i) \overset{\circ}{=} \sum_{j=1}^{m} \left(\frac{p_j \tau^i(u^i_j)}{\sum_{k=1}^{m} p_k \tau^i(u^i_k)} \right) g^i(u^i_j), \tag{4.46}$$

and it cannot satisfy the multiplicative separability required by Theorem 4 unless the function τ^i is independent of u^i_j, in which case it satisfies the Expected Utility Hypothesis.

5. General Quadratic

Chew, Epstein and Segal [1988] proposed the General Quadratic value function

$$V^i(\mathbf{p},\mathbf{u}^i) \overset{\circ}{=} \sum_{j=1}^{m}\sum_{k=1}^{m} p_i p_j f^i(u_j^i, u_k^i). \tag{4.47}$$

It does not satisfy the additive separability required by Theorem 4 and, in addition, does not satisfy EUPI.

6. Optimism/Pessimism

Hey [1994] proposed a family of value functions he called Optimism/Pessimism. The value function for person i is

$$V^i(\mathbf{p},\mathbf{u}^i) \overset{\circ}{=} \sum_{j=1}^{m} h^i(p_j,\mathbf{u}^i) g^i(u_j^i). \tag{4.48}$$

If this function is to satisfy the multiplicative separability required by Theorem 4, h^i must be independent of $\mathbf{u}^i$ and, in that case, Optimism/Pessimism is equivalent to Prospect Theory. The comments above apply.

7. Ordinal Independence

The value function for Ordinal Independence (Segal [1984], Green and Jullien [1988]) is

$$V^i(\mathbf{p},\mathbf{u}^i) \overset{\circ}{=} \sum_{j=1}^{m} H_j^i(\mathbf{p}) f^i\left(u_j^i, \sum_{k=1}^{j} p_k\right) \tag{4.49}$$

where H_j^i is defined by (4.44). In order to satisfy the multiplicative separability requirement of Theorem 4, f^i must be independent of its second argument and, in that case, Ordinal Independence is identical to Anticipated Utility.

The major examples of non-expected-utility proposals in the list above that can satisfy equation (4.19) are Subjectively Weighted Utility Theory and Rank-Dependent (Anticipated) Utility Theory, provided that the probability-distorting function is the same for all individuals. Although this is a stringent requirement, we have established that consistent social aggregation is possible without satisfaction of the Expected-Utility Hypothesis.

5. Conclusion

Without the Expected Utility Hypothesis, the social observer's preferences cannot satisfy both *ex-ante* and *ex-post* welfarism (Theorems 1–3). If probabilities are 'objective', possibilities for social aggregation exist and are described in Theorem 4. Individual value functions are given by

$$v^i = V^i(\mathbf{p}, \mathbf{u}^i) = g^{i^{-1}}\left(\sum_{j=1}^{m} a_j(\mathbf{p}) g^i(u_j^i)\right), \tag{5.1}$$

and the observer's value function is ordinally equivalent to

$$\sum_{i=1}^{n} w_i g^i(v^i) = \sum_{i=1}^{n} \sum_{j=1}^{m} w_i a_j(\mathbf{p}) g^i(u_j^i). \tag{5.2}$$

In this equation, the utility levels $(u_1^i, \ldots, u_m^i)$ are person i's actual utility levels in the m states and the transformed utilities $(g^i(u_1^i), \ldots, g^i(u_m^i))$ correspond to his or her von-Neumann – Morgenstern (vNM) utilities.

It may be instructive to rewrite (5.1) in terms of the names of alternatives. For the prospect $(x_1, \ldots, x_m) \in X^m$, person i's utility level in state j is $U_j^i(x_j)$. Writing $\overset{\circ}{U}{}_j^i(x_j) = g^i(U^i(x_j))$, $\overset{\circ}{U}{}^i$ corresponds to i's vNM utility function. Theorem 4 shows that the state-contingent utility function of the observer is ordinally equivalent to $\sum_{i=1}^{n} w_i \overset{\circ}{U}{}_j^i(x_j)$, and the observer's value function is ordinally equivalent to $\sum_{i=1}^{n} \sum_{j=1}^{m} w_i a_j(\mathbf{p}) \overset{\circ}{U}{}_j^i(x_j)$. This value function has the Weighted Utilitarian form, but it is in fact Weighted Generalized Utilitarianism unless the utility functions $\overset{\circ}{U}{}_j^i$ and U_j^i are cardinally equivalent (either one is an increasing affine transform of the other). In this case, we say that value functions satisfy the Generalized Bernoulli Hypothesis.

The most important consequence of Theorem 4 is, therefore, that support for Weighted Generalized Utilitarianism and Weighted Utilitarianism are the same with and without the Expected Utility Hypothesis. It is true, however, that Theorem 4 requires both the *ex-ante* and *ex-post* Pareto principles, and it is also true that its proof requires that the state-contingent utilities can be varied.

The individual and social value functions in (5.1) and (5.2) can be interpreted in different ways. Suppose, first, that $p'_j = a_j(\mathbf{p})$ is interpreted as a subjectively distorted probability that differs from the 'objective' probability p_j. Then all individuals and the observer have preferences that satisfy the Expected Utility Hypothesis with identically distorted subjective probabilities that are functionally related to the 'objective' ones. Alternatively, the preferences represented by $V^1, \ldots, V^n$ and V^0 in (5.1) and (5.2) can be thought of as generalizing the preferences of Subjectively Weighted Utility Theory and Rank-Dependent (Anticipated) Utility Theory, two important constructions in contemporary decision theory.

APPENDIX: Gorman's Overlapping Theorem

For each $i \in N = \{1, \ldots, n\}$, let $\mathcal{I}_i$ be a nondegenerate interval and suppose that $F : \mathcal{I}_1 \times \ldots \times \mathcal{I}_n \longrightarrow \mathcal{R}$ is a continuous and increasing function. Let $N^r \subseteq N$ be such that $\emptyset \neq N^r \neq N$ and let $N^c = N \setminus N^r$. Furthermore, let X^r be the subvector of $X \in \mathcal{I}_1 \times \ldots \times \mathcal{I}_n$ corresponding to N^r and let X^c be the subvector of X corresponding to N^c.

The set of variables N^r is (strictly) separable in F from its complement if and only if there exist continuous and increasing functions $F^r : \prod_{i \in N^r} \mathcal{I}_i \longrightarrow \mathcal{R}$ and $F^0 : A^r \times \prod_{i \in N^c} \longrightarrow \mathcal{R}$ such that

$$F(X) = F^0(F^r(X^r), X^c)$$

for all $X \in \prod_{i \in N} \mathcal{I}_i$, where $A^r := F^r\big(\prod_{i \in N^r} \mathcal{I}_i\big)$, the image of F^r.

The following is the version of the overlapping theorem which is relevant for our purposes (Gorman [1968] actually proves a stronger result):

Theorem: *If N^r and N^s are nonempty and (strictly) separable in F from their respective complements in N, and $N^r \cap N^s$, $N^r \setminus N^s$, $N^s \setminus N^r$ are nonempty, then there exist continuous and increasing functions $F^1 : \prod_{i \in N^1} \mathcal{I}_i \longrightarrow \mathcal{R}$, $F^2 : \prod_{i \in N^2} \mathcal{I}_i \longrightarrow \mathcal{R}$, $F^3 : \prod_{i \in N^3} \mathcal{I}_i \longrightarrow \mathcal{R}$, and $F^0 : A^{r,s} \times \prod_{i \in N^c} \mathcal{I}_i \longrightarrow \mathcal{R}$ such that*

$$F(X) = F^0(F^1(X^1) + F^2(X^2) + F^3(X^3), X^c)$$

for all $X \in \mathcal{R}^n$, where $N^1 := N^r \setminus N^s$, $N^2 := N^r \cap N^s$, $N^3 := N^s \setminus N^r$, $N^c := N \setminus (N^r \cup N^s)$, and $A^{r,s} := F^1\big(\prod_{i \in N^1} \mathcal{I}_i\big) + F^1\big(\prod_{i \in N^2} \mathcal{I}_i\big) + F^3\big(\prod_{i \in N^3} \mathcal{I}_i\big)$.

REFERENCES

Arrow, K., 1972, Exposition of the theory of choice under uncertainty, in C. McGuire and R. Radner, eds., *Decision and Organization,* North Holland, Amsterdam, 19–55.

Blackorby, C., W. Bossert and D. Donaldson, 1998, Uncertainty and critical level population principles, *Journal of Population Economics* **11**, 1–20.

Blackorby, C., D. Donaldson and J. Weymark, 1980, On John Harsanyi's defences of utilitarianism, Discussion Paper No. 8013, Center for Operations Research and Econometrics, Université Catholique de Louvain.

Blackorby, C., D. Donaldson and J. Weymark, 1990, A welfarist proof of Arrow's theorem, *Recherches Economique de Louvain* **56**, 259–286.

Blackorby, C., D. Donaldson and J. Weymark, 1998 (1998a), Social aggregation and the expected utility hypothesis, U.B.C. Department of Economics Discussion Paper No. 97–05, forthcoming in M. Salles and J. Weymark, eds., *Justice, Political Liberalism, and Utilitarianism: Themes from Harsanyi and Rawls,* Cambridge University Press, Cambridge.

Blackorby, C., D. Donaldson and J. Weymark, 1998 (1998b), Harsanyi's social aggregation theorem for state-contingent alternatives, U.B.C. Department of Economics Discussion Paper No. 96–26, 1996, forthcoming in the *Journal of Mathematical Economics.*

Blackorby, C., D. Primont, and R. Russell, 1978, *Duality, Separability and Functional Structure: Theory and Economic Applications,* North-Holland/American Elsevier, Amsterdam/New York.

Bossert, W., and J. Weymark, 1996, Utility in social choice, in S. Barberà, P. Hammond and C. Seidl, eds., *Handbook of Utility Theory,* Vol. 2. Kluwer Academic Publishers, Boston, forthcoming.

Broome, J., 1990, Bolker-Jeffrey expected utility theory and axiomatic utilitarianism, *Review of Economic Studies* **57**, 477–502.

Broome, J., 1991, *Weighing Goods,* Basil Blackwell, Oxford.

Chew, S., 1983, A generalization of the quasilinear mean with applications to the measurement of inequality and decision theory resolving the Allais paradox, *Econometrica* **51**, 1065–1092.

Chew, S. and K. MacCrimmon, 1979, Alpha-nu choice theory: a generalization of expected utility theory, manuscript, Faculty of Commerce and Business Administration, University of British Columbia.

Chew, S., L. Epstein and U. Segal, 1988, Mixture symmetric utility theory, manuscript, Department of Economics, University of Toronto.

Coulhon, T. and P. Mongin, 1989, Social choice theory in the case of von Neumann-Morgenstern utilities, *Social Choice and Welfare* **6**, 175–187.

De Meyer, B. and P. Mongin, A note on affine aggregation, *Economics Letters* **47**, 177–183.

Diamond, P., 1967, Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility, *Journal of Political Economy* **75**, 765–766.

Edwards, W., 1955, The prediction of decisions among bets, *Journal of Experimental Psychology* **50**, 201–214.

Edwards, W., 1962, Subjective probabilities inferred from decisions, *Psychological Review* **69**, 109–135.

Eichhorn, W., 1978, *Functional Equations in Economics,* Addison-Wesley, Reading, Massachusets.

Fishburn, P., 1983, Transitive measurable utility, *Journal of Economic Theory* **31**, 293–317.

Fishburn, P., 1984, On Harsanyi's utilitarian cardinal welfare theorem, *Theory and Decision* **17**, 21–28.

Genest, C and J. Zidek, Combining probability distributions: A critique and an annotated bibliography, *Statistical Science*, **1**, 1986, 114-148.

Gorman, W., 1968, The structure of utility functions, *Review of Economic Studies* **35**, 369–390.

Green, J., and B. Jullien, 1988, Ordinal independence in non-linear utility theory, *Journal of Risk and Uncertainty* **1**, 355–387.

Hammond, P., 1981, *Ex-ante* and *ex-post* welfare optimality under uncertainty, *Economica* **48**, 235–250.

Hammond, P., 1983, Ex-post optimality as a dynamically consistent objective for collective choice under uncertainty, in P. Pattanaik and M. Salles, eds., *Social Choice and Welfare,* North-Holland, Amsterdam, 175–205.

Harsanyi, J, 1955, Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility, *Journal of Political Economy* **63**, 309–321.

Harsanyi, J., 1977, *Rational Behavior and Bargaining Equilibrium in Games and Social Situations,* Cambridge University Press, Cambridge.

Hey, J., 1984, The economics of optimism and pessimism: a definition and some applications, *Kyklos* **37**, 181–205.

Hylland, A. and R. Zeckhauser, 1979, The impossibility of Bayesian group decision making with separate aggregation of beliefs and values, *Econometrica* **47**, 1321-1336.

Kahneman, D., and A. Tversky, 1979, Prospect theory: an analysis of decision under risk, *Econometrica* **47**, 263–291.

Karmarkar, U., 1978, Subjectively expected utility: a descriptive extension of the expected utility model, *Organizational Behavior and Human Performance* **21**, 61–72.

Karmarkar, U., 1979, Subjectively weighted utility and the Allais paradox, *Organizational Behavior and Human Performance* **24**, 67–72.

Machina, M., 1991, Dynamic consistency and non-expected utility, in M. Bacharach and S. Hurley, eds., *Foundations of Decision Theory,* Basil Blackwell, Oxford.

Machina, M. and D. Schmeidler, 1992, A more robust definition of subjective probability, *Econometrica* **60**, 745–780.

Mongin, P., 1995, Consistent Bayesian aggregation, *Journal of Economic Theory* **66**, 313–351.

Mongin, P., 1997, Spurious unanimity and the Pareto principle, THEMA, Centre National de la Recherche Scientifique Université de Cergy-Pontoise, unpublished manuscript.

Mongin, P., 1998, The paradox of the Bayesian experts and state-dependent utility theory, *Journal of Mathematical Economics* **29**, 331–361.

Mongin, P. and C. d'Aspremont, 1998, Utility theory in ethics, in S. Barberà, P. Hammond and C. Seidl, eds., *Handbook of Utility Theory,* Vol. 1, Kluwer Academic Publishers, Boston, 373-482.

Quiggin, J., 1982, A theory of anticipated utility, *Journal of Economic Behavior and Organization* **3**, 323–343.

Roemer, J., 1996, *Theories of Distributive Justice,* Harvard University Press, Cambridge, Massachusetts.

Savage, L., 1954, *The Foundations of Statistics,* John Wiley and Sons, New York.

Segal, U., 1984, Nonlinear decision weights with the independence axiom, manuscript, Department of Economics, University of California, Los Angeles.

Sen, A., 1970, *Collective Choice and Individual Welfare,* Holden-Day, San Francisco.

Sen, A., 1976, Welfare inequalities and Rawlsian axiomatics, *Theory and Decision* **7**, 243–262.

von Neumann, J. and O. Morgenstern, 1947, *Theory of Games and Economic Behavior,* Second Edition, Princeton University Press, Princeton.

Wakker, P., 1993, From local to global additive representation, *Journal of Mathematical Economics* **22**, 523–545.

Weymark, J., 1991, A reconsideration of the Harsanyi-Sen debate on utilitarianism, in J. Elster and J. Roemer, eds., *Interpersonal Comparisons of Well-Being,* Cambridge University Press, Cambridge, 255–320.

Weymark, J., 1994, Harsanyi's social aggregation theorem with alternative Pareto principles, in W. Eichhorn, ed., *Models and Measurement of Welfare and Inequality,* Springer-Verlag, Berlin, 869–887.

∘ ∘ ∘