A NOTE ON THE NEUTRALITY OF MONEY: A "NO-SPECULATION" FORMULATION

by

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June 2000

Discussion Paper No.: 01-06

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A Note On The Neutrality of Money: A "No-Speculation" Formulation

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Abstract

This note shows that the neutrality of the medium of exchange – money – may be viewed as a no-speculation result: every ex-ante Pareto efficient allocation is monetary neutral. No monetary uncertainty or policy can lead to a change in the real variables of the economy, even in the presence of asymmetric information. The similarities and differences between this result and Lucas' work on the neutrality of money are discussed.

1 Introduction

> "Money is not, properly speaking, one of the subjects of commerce; but only the instrument which men have agreed upon to facilitate exchange of one commodity for another. It is none of the wheels of trade: it is the oil which renders the motion of the wheels more smooth and easy. If we consider any one kingdom by itself, it is evident, that the greater or less plenty of money is of no consequence." (Hume, 1752)

Contrary to Hume's conjecture, Lucas' pioneering work on rational expectations ([8], [9]) showed that even in this environment there may be a relationship between inflation and unemployment, based on asymmetric information among agents in the economy. This relationship, however, is not exploitable for policy purposes: the monetary authority can not manipulate the unemployment level via its monetary policy. Lucas' work followed the works of Friedman [4] and Phelps [11] which explained the limitations the monetary authority faces under adaptive expectation.

On a different track of economic theory, a rich microeconomic theoretical literature has evolved, investigating the possibility of speculative trade. Kreps [7] was the first to derive a formal result of no-speculation in a rational expectation environment. Milgrom and Stokey [10] proved that there exist no trade that is commonly known to Pareto dominate an ex-ante efficient allocation. They also showed that if agents trade through a market mechanism, this result is supported through a fully revealing rational expectation equilibrium.

The goal of this work is to explore the relationship between these two branches of Economic theory. I present a model in which the neutrality of money emerges naturally via a no-trade argument. It allows for a deeper understanding of the assumptions that drive the neutrality result. Following Hume, I assume that money serves solely as a medium of exchange. Outside money alone will necessarily be a liability of the monetary authority, and hence a net asset of the private sector. In order to isolate the "medium of exchange" function of money, IOU notes that are issued by agents are introduced. The "outside money" serves only to back-up the inside money, and is not held by individuals for trade or store of value purposes. The monetary authority, that holds the outside money serves as a clearing center for the bills written by the agents. A no-betting (among agents who share a common

prior) argument yields the "money neutrality" of all ex-ante Pareto efficient allocations. A no-speculation theorem proves that those allocations are robust to asymmetries in information, when rationality is common knowledge, that is in a rational expectation environment[1]. The note concludes with a discussion on the relation between this observation and the work of Lucas. It clarifies why, in many models, money is not neutral. Specifically, it shows that Lucas' result of non-neutrality of money is a consequence of his modeling strategy of money as a store of value, and not only of the asymmetric information concerning the real shocks in the economy.

2 The Framework

Consider an exchange economy[2] with K goods and I agents. The initial allocation in the economy is denoted by $e = (e^1, ..., e^I)$. The uncertainty in the economy stems from two factors: A "physical" factor, which may influence the demand and the supply functions (e.g. through endowments), denoted by $\theta \in \Theta$; and a monetary uncertainty, denoted by $\lambda \in \Lambda$, which stems from the fact that at the time of signing a contract, agents do not know the realized (outside) monetary supply which will prevail in the economy when the physical side of the contracts is to be executed. A state of the world is $\omega = (\theta, \lambda) \in \Omega \subseteq \Theta \times \Lambda$ (it is assumed that Ω is finite). The initial allocation, e, depends only on the physical uncertainty, that is e is a function from Θ to $\mathfrak{R}_+^{IK}$

Contracts between the parties in the economy are contingent upon the realization of uncertainty (physical and monetary). They are denominated in monetary terms, and therefore have a real side and a monetary side. Each agent may sign contracts with the $(I-1)$ other agents, over the K commodities. Each contract specifies the types and quantities of goods to be exchanged, and the amount of money to be transacted. The physical transactions are executed after the uncertainty has been resolved . Thus, for each

[1] This note focuses on the characterization of allocations which may result from a variety of mechanisms, and not on the mechanism that may lead to those allocations. In spite of this, the competitive mechanism (which supports the result) is mentioned, mainly to gain intuition.

[2] The assumption of no production is done for the sake of simplicity only. If the production set for every good is closed and the aggregate production set admits the regular conditions (strict convexity, no free-lunch, irreversibility and inaction) the result will still hold.

agent, a contract is a function from Ω to $\mathcal{C}^i$, where an element of $\mathcal{C}^i$ is a $K \times 2$ matrix of consequences. A typical consequence is the pair (t^i, m^i), such that $t^i = (t^i_1, \ldots, t^i_K)$ is agents i's vector of net trades, and $m^i = (m^i_1, \ldots, m^i_K)$ is the vector of monetary values of these net trades[3]. The set of all possible contracts for agent i is denoted by $\mathcal{A}^i$, and is characterized by real feasibility, that is:

$$e^i(\omega) + t^i(\omega) \in \mathfrak{R}^K_+ \ \forall \ \omega \in \Omega \tag{1}$$

and a monetary constraint (to be detailed below). The set of all possible contracts is denoted by $\mathcal{A} = (\mathcal{A}^1, ..., \mathcal{A}^I)$. Thus $\mathcal{A}$ may be represented as $(\mathcal{T}, \mathcal{M})$ where $\mathcal{T}$ represents the real side of the transactions, with typical element $t(\omega) = \left(t^1(\omega), \ldots, t^I(\omega)\right)$, and $\mathcal{M}$ represents the monetary side with typical element $m(\omega) = (m^1(\omega), \ldots, m^I(\omega))$. To be a feasible net trade, $t \in \mathcal{T}$ has to abide by the aggregate feasibility constraint:

$$\sum_{i=1}^{I} t^i(\omega) \in \mathfrak{R}^K_- \ \forall \ \omega \in \Omega. \tag{2}$$

Agent i's utility function is defined over the real side of the transaction, $\mathcal{T}^i(\omega)$. Hence Pareto efficiency depends only on the real side of the trade. Note, however, that the real side is a function of the state of the world, and might depend on the monetary uncertainty. The focus of this note is the ex-ante and interim dependence (or independence) of $\mathcal{T}$ on the monetary uncertainty.

Definition 1 *A trade t is* ex-ante Pareto efficient *if there does not exist another feasible trade t' such that $e^i + t' \succsim^i e^i + t$ for all $i \in \mathcal{I}$ with strict preference for at least one agent.*

All agents maximize their expected payoff and are strictly risk averse[4]. Agents may have different prior beliefs over Ω. We assume Ω is fully supported

[3] In summing up transactions of agent i over all other agents, I implicitly assume a market mechanism. This simplification is done only to identify the current work with previous works on speculative trade, and can be easily relaxed.

[4] We assume a state independent utility function, but assuming dependence on θ would not change the observation.

in every agent's belief and that the beliefs of all agents over λ conditional on θ, are identical.

Agent i's private information is described by a partition Π^i over Ω. He knows event H at ω if $\Pi^i(\omega) \subseteq H$. The event "$i$ knows H happened" is: $\{\omega : \Pi^i(\omega) \subseteq H\}$. An event E is self evident if at the time of happening all agents know it, that is for all $i \in \mathcal{I}$: $E = \{\omega : \Pi^i(\omega) \subseteq E\}$. An event D will be *Common Knowledge* at ω if all agents know D, all know that all know D, etc. This infinite sequence of conditions is equivalent to the single condition that there exists a self-evident event E such that $\omega \in E \subseteq D$ (Aumann [1]).

2.1 The Monetary Discount Mechanism

Our goal is to analyze the effect of asymmetric information on the interaction between the real and the monetary sectors of the economy when money serves only as a *medium of exchange* and *not* as a store of value. The monetary authority holds all of the "outside" money, λ. The monetary part of the transaction is executed before the realization of ω is known. The demand for money is derived from the need to finance the positive net trades. Individuals can issue IOU notes. A unit of the issued notes is a liability to supply one unit of account of outside money ex-post. The role of the monetary authority (or the banking center) is to serve as a clearing center for the notes issued by the agents.
The monetary constraint facing agent i is:

$$\sum_{\Omega} \sum_{k=1}^{K} m_k^i(\omega) = 0 \tag{3}$$

The absolute value of transactions is determined by the ex-post outside money. The discount factor between the notes and the outside money is not modeled, thus allowing an extra degree of freedom to the monetary authority to pin down absolute prices in terms of inside money.

2.1.1 The Monetary Supply

The randomness of the monetary supply, λ, could be interpreted in two ways: the first is to assume there is some exogenous uncertainty concerning the monetary supply. Thus it may be viewed as a "sunspot" uncertainty. Hence

this work analyses whether this monetary sunspot can affect the real side of the economy. The second interpretation, identifies λ with the uncertain monetary *policy*. The monetary authority may decide on λ as a function of the information it receives. If, for example, the monetary authority thinks it has better information over θ than individual agents in the economy, it may try to use this information to achieve its own objectives (e.g. more equal distribution of resources). Note that in this case the monetary policy is measurable with respect to the information available to the agents.

2.2 Neutrality of Money

A contract is monetary neutral if its real side does not depend on the money supply:

Definition 2 *A contract* $a=(t,m)\in\mathcal{A}$ *is* monetary neutral *if* t*:* $\Theta\rightarrow\Re^{KI}$.

Lemma 3 *In the framework described above, the set of Pareto efficient trades is spanned by the set of monetary neutral contracts.*

Proof. The proof is similar to Cass and Shell's [2] proposition 3: agents who share a common prior (conditional on the realization of θ) and are strictly risk averse would not be engaged in mutual bets. ■

Note that Lemma 3 is independent of the trade mechanism. In particular, it holds true for a competitive equilibrium in an Arrow-Debreu contingent markets economy. We may conclude from it that once a Pareto efficient allocation in the set of monetary neutral contracts is reached, opening markets for trade contingent on λ will not lead to further trade. If markets for θ were not complete, then it might well be that dependence on λ could serve a substitute for the missing markets.

We now prove that if the initial allocation is Pareto optimal (and therefore monetary neutral) then any asymmetries in information, including any monetary policy, can not lead rational agents to recontract.

Theorem 4 *Under the assumptions above, if* e *is ex-ante Pareto-efficient allocation then any asymmetric information, including any information concerning the monetary supply, can not lead the agents to a trade which is commonly known to be Pareto improving.*
Formally: There does not exist a state $\omega^*=(\theta^*,\lambda^*)$ *and a contract* $a=(t,m)$ *with* $t\neq\mathbf{0}$*, such that:*

$$D = \left\{ \omega : e + t \succsim^{i}_{\Pi^i(\omega)} e \ \forall \ i \in \mathcal{I} \ \textit{with strict preference for at least one agent} \right\}$$

is common knowledge at ω^*.

Proof. The proof is basically an adaptation of Milgrom and Stokey's [10] Theorem 1 to the monetary mechanism in the economy. Assume it is common knowledge at ω^* that there exists a contract a which is physically and monetarily feasible, and accepted by all agents. Therefore, there exists a self evident event E such that $\omega^* \in E \subseteq D$. For all agents and all $\omega \in E$: $e^i + t^i \succsim^{i}_{\Pi^i(\omega)} e^i$ with strict preference for agent j. For every $i \in I$ the physical feasibility (1) and monetary feasibility (3) are satisfied and the trade t satisfies the feasibility constraint (2). Define the composite contract:

$$a' = \begin{cases} a = (t, m) & \text{if } \omega \in E \\ 0 = (0, 0) & \text{otherwise} \end{cases}$$

Since a is feasible, so is a'. Since for every agent, E is the union of the disjoint events $\Pi^i(\omega)$ for $\omega \in E$, on which t^i is weakly preferred to the no-trade, then: $e^i + t^i \succsim^{i}_{E} e^i$ for every agent, with strict inequality for agent j. As a direct consequence of the Sure Thing Principle, a' ex-ante Pareto dominates the no-trade. This contradicts the assumption that e was ex-ante Pareto efficient. ■

The intuition behind the result is simple: the ex-ante allocation exhausted all mutual benefits of trade. This allocation is monetary neutral since given the real uncertainties in the economy agents agree on the distribution of the monetary supply. Consider the case in which the monetary authority is trying to achieve an objective, based on some signal profile. Therefore the monetary policy is a function of the signals. Since agents could have contracted on any signal, they could have contracted on any possible λ (but chose not too). At the interim stage, any private information which will be advantageous for one agent, will result in decrease in the welfare of other agents. Therefore there exist no trade from which all agents can rationally expect to gain.

3 Discussion: A comparison with Lucas' Model

Lucas' [8] seminal paper on rational expectations is the first monetary model that explicitly incorporates asymmetric information. I compare and explain the differences between Lucas' model and the no-trade formulation.

A major methodological difference lies in the function of money. Lucas' model builds on Samuelson's [12] overlapping generations model. In this model money is the individual's only store of value between the work period of life and later periods[5]. In our model money serves only as a medium of exchange and individuals are constrained from using it as a store of value. If individuals would hold money, then the stochastic structure of λ would influence their indirect payoff, and the monetary uncertainty would become "payoff relevant." In such an environment, Lemma 3 and Theorem 4 do not hold.

A second major difference concerns the ex-ante allocation. In Lucas' model there is no ex-ante round of trade and the initial allocation is not Pareto efficient. Instead, Lucas studies a rational expectations equilibrium, that is, an allocation and a price vector such that every agent uses all information available in the market, including information conveyed by prices, in making optimal decisions. If there is no noise in the economy, the only rational expectations equilibrium is a fully-revealing one, i.e., an Arrow-Debreu equilibrium in an economy in which all agents know all the signals and markets are complete (theorem 2 in Lucas [8]). The result is that money is neutral even in the short run. However, if there is real uncertainty (noise) in the economy on which agents hold asymmetric information, i.e. - uncertain demand, then prices are not a sufficient statistic for an agent to recover the vector of signals, and there does not exist a fully revealing equilibrium[6]. In this case there is a tradeoff between inflation and unemployment, but the government can not manipulate the unemployment level by monetary policy, because only unexpected changes in the money supply will change the equilibrium. The only rational expectation equilibrium in the no-speculation model is a fully revealing one, basically since all ex-ante benefits of trade

[5]In the Samuelson model money may be viewed as an intergenerational medium of exchange too (Cass and Yaari interpretation [3]). However, this interpretation is limited since exchange opportunities between generations are constrained by the the lack of market in which the first and the "last" generation (as in [3]) can trade.

[6]This is generically the case when the signal's space dimension is higher than the price's space dimension [5].

were exhausted. If markets for the real uncertainty (θ) are missing, then the ex-ante efficient allocation would not be (in general) monetary neutral, and asymmetric information may lead agents to trade.

A third (technical) difference is the structure of contracts. In Lucas' work all contracts are for spot markets, but in making their decision agents "look forward" to next period markets, which will determine their demand for balances and their production this period. In our model, contracts are contingent upon the realization of uncertainty, and enable the agents to contract directly on uncertain events.

4 Conclusion

This work tries to relate the no-speculation and the neutrality of money literatures, thus allowing a new perspective on the neutrality of money. If money serves solely as a medium of exchange, then its neutrality relative to asymmetric information may be viewed as a no-speculation result. Hence, the conditions of ex-ante efficiency, common prior, expected utility and common knowledge of rationality which are sufficient for the latter, would suffice for the former too. Economies in which money serves as a store of value, or one of the conditions above is not satisfied, might display non-neutrality and enable the monetary authority to affect the real allocation, through a speculative mechanism.

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