THE GAINS FROM TRADE AND POLICY REFORM REVISITED
by
W E Diewert and A D Woodland

Abstract:
The paper considers what are the gains to a country of opening up its borders to international trade. It also consider the more modern and general question: what are the welfare effects of some tax policy change or of some other exogenous shock to the economy? The paper examines two sources of trade gains that have received little examination in the literature: (1) the gains that can accrue to a country from the elimination of excess supplies as a result of a policy move from autarky to free trade and (2) the gains from the introduction of new goods. The paper decomposes an index of welfare change into various consumer and producer substitution functions and additional components that are sources of growth, including technical progress, growth in real imports, growth in endowments and reductions in excess supplies. This identity is similar in some respects to those derived by Ohyama and Grinols and Wong.

Key Words:
Gains from trade; welfare measurement; sources of growth; producer theory; consumer theory; tariff and tax policy.

Journal of Economic Literature Classification System Numbers:
D11, D24, D60, D90, F11, F43, H21, O12, O13, O40.

W.E. Diewert, Department of Economics, University of British Columbia, Vancouver, Canada, V6T 1Z1; Website: http://web.arts.ubc.ca/econ/diewert/hmpgdie.htm Email: diewert@econ.ubc.ca

A.D. Woodland, Department of Econometrics, University of Sydney, Sydney NSW, Australia 2006; Email: A.Woodland@econ.usyd.edu.au

---

1 This is a substantial revision of the paper "The Gains from Trade Revisited", UBC Discussion Paper 87-19, June 1987 by W.E. Diewert. The authors are indebted to C. Blackorby, J.P. Neary for helpful comments on this earlier version. Woodland gratefully acknowledges the hospitality of the Economic Policy Research Unit, University of Copenhagen, where the current version was completed. Diewert acknowledges the financial support of the Social Sciences and Humanities Research Council of Canada.
1. Introduction

In this paper, we consider once again a very old question: what are the gains to a country of opening up its borders to international trade? We also consider the more modern and general question: what are the welfare effects of some tax policy change or of some other exogenous shock to the economy?

The primary purpose of the paper is to provide characterizations of the conditions for welfare improvements in several situations that have received very little attention in the existing literature. The first of these is to exhibit the gains that can accrue to a country from the elimination of excess supplies as a result of a policy move from autarky to free trade. This is a source of gain that has been overlooked in the recent literature, with the exception of Neary and Schweinberger (1986; 428), but has its origins in the ‘vent for surplus’ idea developed by Myint (1958). This source of gains accrues to the country where the autarky equilibrium has some free goods, i.e., goods that are in excess supply, and the opening up of trade eliminates these excess supplies. These gains, which are not the usual consumption and production gains that are discussed by Dixit and Norman (1980; 78) and Woodland (1982; 267), are illustrated in section 4 below using the general methods developed in section 3. The second contribution of the paper is to characterize the conditions under which the introduction of new goods into the economy will generate welfare gains. While the appearance of new goods is a fundamental feature of the modern economy and has been the subject of analysis in various ‘endogenous growth’ models, there has been little attention paid to their role in the gains from trade and policy reform literature. Accordingly, our analysis of the gains from the introduction of new goods in section 4 below is offered as a step in the direction of redressing this deficiency in the literature, as discussed by Romer (1994). Our analysis of this source of welfare gain requires a formulation of the model to allow consumer specific and producer specific ‘taxes’, so the traditional formulation of trade gains is not sufficient for our purposes. The third main area discussed is the extension of our methodology to a large open economy that can influence its terms of trade. Our measure of welfare improvement now includes terms reflecting a shift in the foreign offer set and the optimality (or otherwise) of tariff and tax policies. We develop a new sufficiency condition for a welfare improvement that applies if the country imposes optimal tariffs in the new situation.
The techniques that we use to illustrate the gains from eliminating excess supplies and from the introduction of new goods have a much wider applicability. Thus, in sections 2 and 3 below, we set out a general equilibrium model of an open economy that can be adopted to study a wide range of problems involving the welfare consequences of discrete changes in the economy. Our main task in these sections is the derivation of alternative expressions for the decomposition of a measure of welfare change, which we call the aggregate quasi variation. This variation gives cardinal measures of changes in consumers' welfare between two periods and is evaluated at an arbitrarily given vector of reference prices. Our main special cases of this general expression are the identities for the aggregate quasi-compensating variation and for the aggregate quasi-equivalent variation, evaluated at the initial and second period world market prices respectively. These quasi variations reduce to the usual compensating and equivalent variations originally defined by Hicks (1942), provided that there are no consumer commodity tax distortions in the economy. Our formulae for these quasi variations are analogous to an identity derived by Ohyama (1972; 47), who decomposed aggregate changes in real income into various endowment growth, production growth, tax revenue growth and changes in the terms of trade components. They also bear similarity, but are different from, expressions obtained by Grinols and Wong (1994).

2. A Model of an Open Economy
We assume that there are $N$ internationally traded goods and $M$ domestic goods in the economy. There are $H$ households or consumers, $K$ firms and one (consolidated) government sector in the economy. For period $t \in T \equiv \{0,1\}$ and household $h \in H$, let the consumption vector be $c^h_t = (c^h_T, c^h_N)$, where $c^h_T$ and $c^h_N$ denote the observed period $t$ consumption vectors for household $h$ for internationally traded goods and domestic (internationally nontraded) goods respectively (factor supplies are indexed with negative signs).\(^2\)

Let $y^k_t = (y^k_T, y^k_N)$ denote the observed period $t$ net supply vector for firm $k$ (input demands are indexed with negative signs). The net export vector for the economy in period $t \in T$ is denoted as $x^t = (x^t_T, x^t_N)$, where $x^t_N = 0$ (if $x^t_{Tn} < 0$, then the $n$-th internationally traded

\(^2\) The modern literature on this subject begins with Samuelson (1939) and includes Kemp (1962), Johnson (1965), Ohyama (1972), Kemp and Wan (1972), and Smith (1982). For textbook accounts, see Kemp (1969), Dixit and Norman (1980; 74-80) and Woodland (1982; 256-272).
good is imported into the economy in period \( t \). The vector \( v' = (v'_1, v'_N) \) represents the sum of the economy’s endowments of goods less the government’s net demand vector for goods.

In period \( t \) the market balance conditions for the economy for goods are given by:

\[
\sum_{h \in H} e^{ht} = \sum_{k \in k} y^{kt} - x' + v' - e', \quad t \in T.
\]

The vector \( e' \equiv (e'_1, e'_N) \) is the period \( t \) excess supply vector. We assume, consistent with market equilibrium, that these vectors are non-negative; i.e.,

\[
e' \geq 0 \quad \text{for} \quad t \in T,
\]

and that \( e'_i \) can only be positive if all demanders of the \( i \)-th good face zero prices for this good.

Turning now to the price side of the economy, we assume that the period \( t \) price vector for goods is \( p' = (p'_1, p'_N) \geq 0 \). The price vector for internationally traded goods, \( p'_F \), is given exogenously for this small open economy, while \( p'_N \) is a vector of market prices for domestic goods in period \( t \). In period \( t \in T \), household \( h \) faces the price vector \( p' + t^{hi} \geq 0 \) where \( t^{hi} = (t^{h1}, t^{hN}) \). Vectors \( t^{h1} \) and \( t^{hN} \) should usually be interpreted as vectors of commodity tax distortions faced by household \( h \) in period \( t \), but other interpretations are possible as we shall see later. In period \( t \in T \), firm \( k \) faces the price vector \( p' + \tau^{kt} \geq 0 \) where \( \tau^{kt} = (\tau^{k1}, \tau^{kN}) \). Vectors \( \tau^{k1} \) and \( \tau^{kN} \) should usually be interpreted as vectors of commodity tax distortions faced by firm \( k \) in period \( t \). In the usual case considered in international trade theory, \( t^{h1}_N = \tau^{k1}_N = 0 \) and \( t^{ht}_F = \tau^{kt}_F \equiv \tau' \) so there are no domestic tax distortions and a common tariff vector \( \tau' \) faced by all consumers and producers in period \( t \). However, our general formulation allows us to deal with household and industry specific taxes and subsidies.

Competitive optimizing behaviour on the part of consumers and producers is assumed in both periods. Thus, we assume that household \( h \)'s preferences over various combinations of internationally traded goods and domestic goods can be represented by means of the utility

---

3 We let the number of households and the set of households to be identified by the same notation. Firms and goods are similarly treated.
function \( f^h \). The utility level attained in period \( t \) by household \( h \) is \( u^{ht} \equiv f^h(c^{ht}) \), for \( t \in T \), \( h \in H \), where \( c^{ht} = (c_{ht}^1, \ldots, c_{ht}^N) \) is the observed period \( t \) consumption vector for household \( h \). Expenditure minimizing behaviour for household \( h \) in each period is assumed, whence

\[
\begin{align*}
\min_{c} & \left\{ (p' + t_{ht}) \cdot c : f^h(c) \geq u^{ht} \right\} \\
\equiv & \ (p' + t_{ht}) \cdot c^{ht} \\
\end{align*}
\]

(3)

where \( p' \cdot c \) denotes the inner product of the vectors \( p' \) and \( c \), and \( m^h \) is the expenditure function for household \( h \).

Turning now to producers, we assume that firm \( k \in K \) has a feasible set of net output vectors, \( S^{kt} \), in period \( t \in T \). The assumption of profit maximizing behaviour for firm \( k \) in period \( t \) may be represented as

\[
\max_{y} \left\{ (p' + \tau_{kt}) \cdot y : y \in S^{kt} \right\} \equiv \ (p' + \tau_{kt}) \cdot y^{kt} \\
\]

(4)

where \( (4) \) defines the profit function \( \pi^k \) for firm \( k \) and \( y^{kt} \) is the firm \( k \) observed period \( t \) net output vector.

### 3. Welfare Identities

#### 3.1. Producer and Consumer Substitution Functions

Before we define our main welfare identities, it is useful to detour briefly and define various consumer and producer substitution functions. These functions characterize the gains from substitution that consumers and producers experience as a result of price changes in the economy. These functions are then used to help decompose aggregate welfare changes into easily interpreted components. They are evaluated at an arbitrarily given reference price vector, specific choices of which will be considered further below.

Consider the problem of minimizing the expenditure required for household \( h \) to attain its period \( r \) utility level, \( u^{hr} \), but using the price vector \( p \) instead of the actual period \( r \) prices

---

4 This function was introduced to the economics literature by Hicks (1946; 331). The mathematical properties of expenditure functions are discussed in Diewert (1982; 553-556). We assume that the minima in (3) exist.

5 Hicks (1946; 319-325), Gorman (1968), McFadden (1978) and Diewert (1973) discuss properties of profit functions. We assume that the maxima in (4) exist.
faced by household $h$, $p^r + t^{hr}$.

Define the household $h$ substitution function $s_{hr}(p)$ ($hr$ refers to the utility level attained by household $h$ in period $r$) by

$$s_{hr}(p) \equiv p \cdot c^{hr} - m^h(u^{hr}, p) \geq 0 \quad r \in T, \ h \in H,$$

where the inequality follows since $c^{hr}$ is feasible for the minimization problem (3) but is not necessarily optimal. This expression gives the cost, at the reference price vector $p$, of the consumption vector actually consumed in period $r$ minus the minimum expenditure needed to attain period $r$ utility at this reference price vector. It therefore represents a measure of the gains from substitution around the period $r$ indifference curve.

Turning to the producer side of the model, consider a hypothetical sector $k$ profit maximization problem where firm $k$ faces the price vectors $p$ and has available the period $r$ technology set $S^{kr}$. Define the firm $k$ substitution function $\sigma_{kr}(p)$ ($kr$ refers to the firm $k$ technology set in period $r$, $S^{kr}$) by

$$\sigma_{kr}(p) \equiv \pi^k(p; S^{kr}) - p \cdot y^{kr} \geq 0 \quad r \in T, \ k \in K,$$

where the inequality follows since $y^{kr}$ is feasible for the maximization problem but is not necessarily optimal. This expression is the profit attained in period $r$ at reference price $p$ minus the profit (at this same reference price vector) attained using the actual period $r$ production point. It therefore represents a measure of the gains from substitution around the period $r$ transformation frontier for firm $k$.

There is one additional set of definitions that we require in subsequent sections. We first relate the firm $k$ production possibility set in period $0$, $S^{k0}$, to the firm $k$ production possibility set in period $1$, $S^{k1}$, by assuming that there is no technological regress, i.e., that

$$S^{k0} \text{ is a subset of } S^{k1}, \ k \in K.$$

Using this assumption, it is easy to see that

$$\pi^k(p; S^{k1}) \equiv \max_y \{p \cdot y : y \in S^{k1}\} \geq \max_y \{p \cdot y : y \in S^{k0}\} \equiv \pi^k(p; S^{k0}).$$

---

$^6$ While our emphasis is on discrete changes, it is interesting to observe that the consumer substitution function may be approximated to the second order around the consumer price vector in period $r$ as $s_{hr}(p) \approx -0.5 p \cdot \nabla^2 m^h(u^{hr}, p + t^{hr}) \cdot p \geq 0$. If consumer preferences have been econometrically estimated, then estimates of the consumer substitution functions can be obtained.

$^7$ The producer substitution function may be approximated to the second order in a similar way to the consumption substitution function, thus allowing an estimate to be obtained from econometric estimates of the profit function.
Define the firm \( k \) technological progress function \( \alpha_k \), using the reference prices \( p \), by
\[
\alpha_k(p) = \pi_k^r(p, S_{k}^r) - \pi_k^r(p, S_{k}^0) \geq 0 \quad r \in T, k \in K,
\]
where the inequality follows from the inequality (8). It can be seen that \( \alpha_k(p) \) is a measure of the expansion in firm \( k \)'s production possibilities set going from period \( 0 \) to \( 1 \).

It is useful to record the situations where the various functions defined above take zero values. For future reference, we note that:
\[
s_{hr}(p) \equiv p \cdot c^{hr} - m^h(u^{hr}, p) = 0 \quad \text{if} \quad p = p_{hr}^r \equiv p^r + t^{hr}; \quad h \in H, r \in T
\]
\[
\sigma_{kr}(p) \equiv \pi_k^r(p, S^{kr}) - p \cdot y^{kr} = 0 \quad \text{if} \quad p = p_{kr}^r \equiv p^r + \tau^{kr}; \quad k \in K, r \in T
\]
\[
\alpha_k(p) \equiv \pi_k^r(p, S^{k1}) - \pi_k^r(p, S^{k0}) = 0 \quad \text{if} \quad S^{k1} = S^{k0}; \quad k \in K, r \in T.
\]
Thus, the consumer substitution function is zero if it is evaluated at the period \( r \) price vector facing that consumer, while the producer substitution function is zero if it is evaluated at the period \( r \) price vector facing that producer. If the technology does not change then the technology progress function is zero for any reference price vector.

The consumer and producer substitution functions and the technical change function have been defined above in terms of an arbitrary reference price vector. If the price vector \( p \) is chosen to be the market price vector for periods \( 0 \) or \( 1 \), the substitution functions (functions) takes specific values denoted as
\[
s_{hr}^t \equiv s_{hr}(p^t), \quad \sigma_{kr}^t \equiv \sigma_{kr}(p^t), \quad \alpha_k^t \equiv \alpha_k(p^t) \quad r, t \in T, h \in H, k \in K.
\]
These specific substitution functions will be used extensively below.

Now we are ready to derive our main results.

### 3.2. The Basic Identities
Our results concern aggregate monetary measures of welfare change. These are first defined and then expressed in alternative ways that dis-aggregate the welfare change into important components that are readily interpreted. These measures and dis-aggregation identities are all expressed in terms of an arbitrary reference price vector. Special cases of these welfare measures and identities of particular interest, obtained by replacing the reference price vector by a particular observable price vector (such as the market price vector), are then given special consideration.
Define the money metric change in utility for household $h$ going from period 0 to 1 using the reference prices, $p$, by

$$V_h^k(p) \equiv m^h(u^{h_1}, p) - m^h(u^{h_0}, p), \quad h \in H.$$  

If we choose $p = p_c^{h_1} \equiv p^1 + t^{h_1}$ then (12) becomes Hicks' (1942; 128) compensating variation for household $h$, and if we choose $p = p_c^{h_0} \equiv p^0 + t^{h_0}$ then (12) reduces to Hicks' (1942; 128) equivalent variation for household $h$. Each of these money metric measures is a valid measure of individual welfare change.

To handle an economy consisting of many households, we consider an aggregation of household variation measures. Thus, the aggregate variation $V(p)$ is defined by

$$V(p) \equiv \sum_{h \in H} V_h^k(p) \equiv \sum_{h \in H} \{m^h(u^{h_1}, p) - m^h(u^{h_0}, p)\},$$

which is the sum of households’ money metric changes in utility. If all consumers face the same price vector, $p = p_c^i$, the functions $V^h(p^0_c)$ and $V^h(p^1_c)$ are the aggregate equivalent variation and the aggregate compensating variation, respectively.

This expression for the aggregate variation may be rewritten in several different ways that allow for interesting economic interpretations. Using (5), the aggregate variation may be expressed as the identity:

$$V(p) \equiv \sum_{h \in H} V_h^k(p) = \sum_{h \in H} s_{h_0}(p) - \sum_{h \in H} s_{h_1}(p) + \sum_{k \in K} p \cdot (c^{h_1} - c^{h_0}).$$

If we now replace the aggregate consumption vectors by equivalent vectors using the material balance equations in (1), we obtain the following identity:

$$V(p) \equiv \sum_{h \in H} s_{h_0}(p) - \sum_{h \in H} s_{h_1}(p) + \sum_{k \in K} p \cdot (y^{k_1} - y^{k_0})$$

$$- p_T \cdot (x_T^1 - x_T^0) + p \cdot (v^1 - v^0) - p \cdot (e^1 - e^0).$$

Now eliminate the production vector $y^{k_1}$ from (15) using definition (6). We thus obtain the third identity:

$$V(p) = \sum_{h \in H} s_{h_0}(p) - \sum_{h \in H} s_{h_1}(p) + \sum_{k \in K} \sigma_{k_0}(p) - \sum_{k \in K} \sigma_{k_1}(p) + \sum_{k \in K} \alpha_k(p)$$

$$- p_T \cdot (x_T^1 - x_T^0) + p \cdot (v^1 - v^0) - p \cdot (e^1 - e^0).$$

---

8 The term is due to Samuelson (1974).
9 This aggregate measure of welfare change is valid if lump sum transfers between the government and consumers are permitted.
Expressions (14)-(16) provide three interesting identities for the aggregate variation. The first identity shows that the aggregate variation equals a difference of non-negative consumer substitution functions plus an index of consumption growth, using as weights the reference prices. The second identity replaces the index of consumption growth by indices of growth in its various components: output, net imports (the negative of net exports), endowments and excess demands (negative of excess supplies). In the third identity, the aggregate variation of output growth on the right hand side of (15) has been replaced by the following three sets of terms: (i) a non-negative sum of producer substitution functions, \( \sum_{k \in K} \sigma_{k_0}(p) \); (ii) a non-positive sum of producer substitution functions, \( -\sum_{k \in K} \sigma_{k_1}(p) \); and (iii) a sum of non-negative technical progress effects, \( \sum_{k \in K} \alpha_k(p) \).

The identity (16) is one of the main results in the paper. In a one consumer economy, it decomposes an index of welfare change into various consumer and producer substitution functions and additional components that are sources of growth, including technical progress, growth in real imports, growth in endowments and reductions in excess supplies. This identity is similar in some respects to one derived by Ohyama (1972; 47) and discussed further below. However, Ohyama deals exclusively with consumer prices and does not explicitly define the consumer and producer substitution functions. Our identity is also similar to an expression derived by Grinols and Wong (1991; 431-434) as an exact measure of welfare change. While Grinols and Wong also define consumer and producer substitution functions, they use domestic market prices for evaluation. As will be seen below, it is convenient to deal with the general identity above, and then consider particular choices of reference prices to establish particular results.

Having established several expressions for the aggregate variation function, we now consider two special cases formed by choosing two particular price vectors to evaluate the function. In particular, we shall find it convenient to sometimes work with the market equilibrium reference prices \( p^0 \) and \( p^1 \). Recall that \( p^*_T \) and \( p^j_T \) are the international price vectors that

---

10 Our identity has no hypothetical tax revenue terms. However, tax distortions do play a role in (16) in the definitions of the producer and consumer substitution terms.

11 At other times it will be convenient to work with consumer prices.
the economy faces in periods 0 and 1 respectively, while \( p^0_N \) and \( p^1_N \) are the market prices for domestic goods in periods 0 and 1. We shall call \( V^h( p^0 ) \) the quasi-equivalent variation and \( V^h( p^1 ) \) the quasi-compensating variation for household \( h \), since, if there are no consumer tax distortions, these two quasi variations reduce to the corresponding Hicksian variations.

By setting the arbitrary reference price vector \( p \) equal to \( p^0 \) or \( p^1 \), we also obtain the aggregate quasi-equivalent and aggregate quasi-compensating variations, respectively. First, choose the period 1 world price vector \( p^1 \). In this case, \( V(p^1) \) is called the aggregate quasi-compensating variation, which measures the change in expenditure need to obtain the period 1 utility levels using period 1 prices for the consumers. Consequently, the period 1 price vector is used to evaluate the substitution functions and as weights in the various growth measures. Specifically, the various indices of growth in consumption, output, net export, endowments and excess supplies that appear in identities (14)-(16) above are evaluated at the period 1 market price vector \( p^1 \). Therefore, they are interpreted as Paasche quantity indices written in difference form rather than the usual ratio form.\(^{12}\) Thus, for example, equation (16) then expresses the aggregate quasi-compensating variation as a difference of non-negative consumer substitution functions plus an index of consumption growth, using the period 1 vector \( p^1 \) as price weights. Hicks (1942; 128) called an index like \( p^1 \cdot (c^{h1} - c^{h0}) \) the Paasche variation of consumption growth for household \( h \). Thus, the last set of functions on the right hand side of (14) becomes the economy’s aggregate Paasche variation of consumption growth.

Similarly, the term \( \sum_{k \in K} p^1 \cdot (y^{k1} - y^{k0}) \) is the aggregate Paasche variation of net output growth, \(- p^1 \cdot (x^{i1} - x^{i0}) \) is minus the Paasche variation of net export growth or plus the Paasche variation of net import growth (\(- x^i \) is a net import vector for period \( t \)), \( p^1 \cdot (v^i - v^0) \) is the Paasche variation of endowment growth, and \(- p^1 \cdot (e^i - e^0) \) is minus the Paasche variation of excess supply growth. Thus, if excess supplies diminish going from period 0 to 1, this last term will be non-negative.

\(^{12}\) The Paasche index of consumption growth in its normal ratio form is \( (p^1 \cdot c^{h1})/(p^1 \cdot c^{h0}) \). Thus the variation concept replaces ratios with differences.
Second, the above derivations can be repeated with small modifications for the aggregate quasi-equivalent variation, $V(p^0)$, where $p^0$ is the vector of period 0 market prices of internationally traded and domestic goods. Consequently, the period 0 price vector is used to evaluate the substitution functions and as weights in the various growth measures. Specifically, the various indices of growth in consumption, output, net export, endowments and excess supplies that appear in identities (14)-(16) above are evaluated at the period 0 market price vector $p$. Therefore they are interpreted as Laspeyres quantity indices written in difference form. Thus, the term $\sum_{h=H} p^0 \cdot (c^h - c^0)$ is the aggregate Laspeyres variation of consumption growth, $p^0 \cdot (y^1 - y^0)$ is the Laspeyres variation of production growth, $-p^0 \cdot (x^1 - x^0)$ is the Laspeyres variation of net import growth, $p^0 \cdot (v^1 - v^0)$ is the Laspeyres variation of endowment growth and $-p^0 \cdot (e^1 - e^0)$ is minus the Laspeyres variation of excess supply growth.

In the following sections, we shall illustrate some uses of the identities established above.

4. Welfare Comparisons
The identities defined above allow a welfare comparison between any two situations facing a small open economy. These situations might be differentiated by the economic policies undertaken by the government, such a difference in the tariff vector or a difference in domestic taxation policy or by the move from autarky to free trade. Alternatively, the two situations might arise through technical change or a change in the terms of trade. Finally, the different situations might occur because of the introduction of new goods into the economy. Whatever the source of discrete change, our identities may be used to decompose the welfare effects into basic component parts that are readily interpreted. In the following sub-sections, we use these identities to illustrate the well-known gains from trade, the gains from the elimination of excess supplies and the gains from the introduction of new goods into the economy. The final sub-section extends the welfare analysis to a large open economy.
4.1 The Traditional Gains from Trade

For illustrative purposes, we first consider the well-known gains to consumer welfare arising from a movement from autarky to free trade.

Let period 0 represent the autarky equilibrium for an economy and let period 1 represent the free trade equilibrium for the economy. Assume for simplicity that there are no commodity tax or tariff distortions in the economy, so that

\[(17) \quad t^h = 0, \quad \pi^h = 0, \quad \text{for} \quad h \in H, k \in K, r \in T,\]

whence all consumers and producers face the same price vectors, \(p^r\). In period 0, we assume that \(x^0 = 0\) and that \(p^0\) is the autarky price vector, and define the period 0 vector of prohibitive tariffs as \(t^0 \equiv p^0 - p^1\).

If we further assume balanced trade in each period so that \(b^1 = b^0 = 0\), no technical progress, no change in endowments and no free goods, then the aggregate variations, evaluated at international prices \(p^1\) and \(p^0\) respectively, are:

\[(18) \quad V(p^1) = \sum_{h \in H} s^1_{h0} + \sum_{k \in K} \sigma^1_{k0},\]

and

\[(19) \quad V(p^0) = -\sum_{h \in H} s^0_{h1} - \sum_{k \in K} \sigma^0_{k1} - p^0 \cdot x^1.\]

These expressions illustrate the well-known components of the gains from trade. The first expression (18) provides a decomposition of trade gains into gains from each consumer altering his consumption vector in response to the price changes (consumer substitution effect) and gains from each producer altering her production vector in response to the price changes (producer substitution effect). These substitution effects are based upon free trade prices. If autarky prices are used the substitution effects enter negatively and are supplemented by a Laspeyres’ volume of trade effect \(-p^0 \cdot x^1\), as indicated in the second expression (19) above.

If we further assume \(N = 2\) (only two internationally traded goods), \(M = 0\) (no domestic goods), \(H = 1\) (only one household) and \(K = 1\) (only one firm), then the gains from trade measures (18) and (19) can be geometrically illustrated. In Figure 1, the frontier of the
The economy's production possibility set is represented by the curve $C'C$. The pre-trade and post-trade indifference curves for the single household are the curves through the consumption vectors $c^{h0}$ and $c^{h1}$ respectively, where $h = 1$. The economy's pre-trade production vector is $y^{k0} = c^{h0}$ and the post-trade production vector is $y^{k1}$, where $k = 1$. The economy's period 1 net export vector is $x^t = y^{k1} - c^{h1}$ and the corresponding net import vector is $-x^t = c^{h1} - y^{k1}$. Note that good 1 is exported and good 2 is imported.

If we measure the gains and losses in terms of good 1, it can be seen that the distance $AB$ equals the consumer substitution function $s^t_{h0}$ and $BD$ equals the producer substitution function $\sigma^t_{k0}$. Thus $V(p^t) = \text{compensating variation AD}$ = the consumer substitution gain $AB$ plus the producer substitution gain $BC$. This is the traditional geometry illustrating the gains from trade.\(^\text{13}\)

However, the equivalent variation $V(p^0)$ may also be used to measure the gains from trade. From Figure 1, the equivalent variation is $V(p^0) = FG$. This consists of $p^0 \cdot (-x^t)$, net imports valued at period 0 prices given by the distance $EH$, less the consumer substitution function $s^0_{h1}$ given by the distance $GH$, less the producer substitution function $\sigma^0_{k1}$ given by the distance $EF$. The fact that not all terms in (19) have the same sign explains why (18) is normally used to illustrate the gains from trade rather than (19).

### 4.2 The Gains from Eliminating Excess Supplies

In this section, we use the welfare decomposition developed above to characterize the gains from trade that can arise from the elimination of excess supplies. Some goods that may be in excess supply, and hence have zero prices, in the autarky equilibrium may command positive prices in a free trade situation due to strong foreign demand. The result is that there is a welfare gain arising from this elimination of the excess supply.

Assume that autarky equilibrium occurs in period $t=0$ and that free trade occurs in period $t=1$. Furthermore, assume for simplicity that there are no taxes before or after trade, no

---

\(^{13}\) See Woodland's (1982) diagram on page 259 and his discussion on page 267. Dixit and Norman (1980; 78) have the decomposition of the gains formula, but no diagram. Johnson (1965) has a breakdown of the gains from trade into producer and consumer components, but his decomposition is different. Kemp (1969; 259) also speaks of producer and consumer substitution gains.
technical change and no endowment changes. We now consider the case where some goods are free in the autarky equilibrium, so that there are excess supplies of some commodities in the period $t=0$ equilibrium, but that there are no excess supplies in the free trade equilibrium, so that $e^I = 0$. Under these conditions, we have $p^0 \cdot e^0 = 0$ and the aggregate quasi-equivalent variation may be expressed as:

$V( p^I ) = \sum_{k \in H} s^I_{k0} + \sum_{k \in K} \sigma^I_{k0} + p^I \cdot e^0$.

We see that the traditional consumer and producer gains, given by the first two non-negative terms, are now augmented by an additional term that reflects the disappearance of free goods. If the vector of excess supplies in autarky, $e^0$, contains positive elements and the corresponding elements of the free trade price vector, $p^I$, are positive then the last term in (20) will be positive, hence providing a sufficient condition for a welfare improvement. Thus we have the following proposition: A sufficient condition for a welfare gain is that there is some good that is in excess supply in autarky, but trades at a positive price under free trade.

This can be an important source of gains that seems to have been overlooked in the traditional trade literature, with the exception of the paper by Neary and Schweinberger (1986; 428). They point out the potential gains from trade due to previously free factors being positively priced in free trade. However, our formulation applies equally to the elimination of surpluses in either factors or goods. It also applies to any change in circumstance or policy, not just to the move from autarky to free trade. Accordingly, it provides a formalization and generalization of (at least one aspect of) the idea of a ‘vent for surplus’ gain from trade as initially conceived by Adam Smith and further developed by Myint (1958) and Findlay (1970; 70-76).

The gains from trade identity is illustrated using Figure 2 for the special case where $N = 2$, $M = 0$, $H = I$ and $K = I$. The economy’s production possibility set is represented by the curve D’D. The autarky consumption vector is $c^{K0}$ and the autarky production vector is $y^{K0}$. It can be seen that good $I$ is in excess supply in the autarky equilibrium and its price is

---

14 Think of transportation improvement that allows the natural resources of a previously isolated region to be exploited.
zero. When the region is opened up to trade at positive prices, the production vector becomes $y^k$ and the consumption vector becomes $c^h$.

The measure of trading gains, $V(p^1)$, uses the international prices under free trade, $p^1$, as reference prices. If we measure the gains in terms of good $l$, we have that $V(p^1) = AE$. This consists of $s^{l}_{ho}$, the consumption substitution gain $AB$, plus $\sigma^l_{k0}$, the production substitution gain $CE$, plus $p^1 \cdot e^0$, the elimination of excess supply gain $BC$.

Turning to the equivalent variation measure of trading gains represented by (19), which uses the autarky prices as reference prices, we find that we can no longer measure gains and losses in terms of good $l$ because good $l$ is a free good in the autarky equilibrium. However, if we measure the gains in terms of good $2$, we find that $V(p^0) = D'E' = p^0 \cdot (-x^l)$. This consists of the net import vector valued at autarky prices given by the distance $C'F'$, less $s^0_{hj}$, the consumer substitution function $E'F'$, less $\sigma^0_{kj}$, the producer substitution function $C'D'$. Note that the excess supply elimination gain does not explicitly show up in this decomposition of the gains from trade.

4.3 The Gains from the Introduction of New Goods

Nowadays goods are being introduced into most economies at an increasingly rapid pace. Despite this empirical fact, the literature on the role of new goods in international trade is rather sparse. A discussion of why this may be so, and of the importance of developing the theoretical analysis of new goods in the analysis of trade and trade policy, has been provided recently by Romer (1994). In addition, Feenstra (1994) provides an analysis of the role that new goods play in empirical research in international trade, specifically in the measurement of price indices. In this sub-section, we indicate how our gains from trade framework may be adapted to deal with the introduction of a new good into an open economy.

While we do not explain the creation of new goods, we model their introduction as a result of profit and utility maximizing choices: new goods are introduced when current prices yield positive production, while previous prices did not. Our characterization of the gains from the introduction of new goods in terms of the aggregate quasi variation developed above thus requires us to determine shadow prices for new goods before they were ‘introduced’. These
shadow prices are constructed using the concept of restricted profit and expenditure functions and, since they are firm and consumer specific, our general formulation, involving firm and consumer specific taxes, is required for the analysis to proceed.

In period 1, a new internationally traded good, say good 1, is introduced into the world economy. We assume for simplicity that there are no commodity tax distortions in either period so that, in particular, we have:

\[(21) \quad t^{h1} = 0, \quad \pi^{k1} = 0, \quad h \in H; \quad k \in K.\]

The vectors of period 1 equilibrium prices are \(p^{1}_I \equiv (p^{1}_1, \tilde{p}^{1}_I)\) for internationally traded goods and \(p^{1}_N\) for domestic goods, where we define \(\tilde{p}^{1}_I \equiv (p^{1}_2, \ldots, p^{1}_N)\) to be the vector of period 1 international prices excluding the new good. The vectors of period 0 equilibrium prices are \(\tilde{p}^{0}_I \equiv (p^{0}_2, \ldots, p^{0}_N)\) for traded goods and \(p^{0}_N\) for domestic goods.

We assume that consumers have preferences over the new good even before it is introduced but they are restricted to consume zero units of it in period 0. We now look for a shadow or virtual price for good 1 that will just induce household \(h\) to consume zero units of the new good in period 0.\(^{15}\) How can we find these shadow prices?

For \(h \in H\), define the restricted expenditure function for household \(h\) as\(^{16}\)

\[(22) \quad \tilde{m}^h( u^h, c_j, \tilde{p} ) \equiv \min \left\{ \tilde{p} \cdot \tilde{c} : f^h( c_j, \tilde{c} ) \geq u^h \right\},\]

where \(f^h\) is the household \(h\) utility function, \(\tilde{p}\) and \(\tilde{c}\) are vectors of prices and quantities, \(c_j\) is consumption of the new good and \(u^h\) is a reference utility level. We assume that the observed period 0 consumption vector \(\tilde{c}^{h0}\) for household \(h\) solves (22) when \(u^h = u^{h0}, c_j = 0, \tilde{p} = \tilde{p}^{0};\) i.e. we have

\[(23) \quad \tilde{m}^h( u^{h0}, 0, \tilde{p}^{0} ) = \tilde{p}^{0} \cdot \tilde{c}^{0}, \quad h \in H.\]

Assuming that the derivative (from the right) exists, the appropriate household \(h\) shadow price for good 1 in period 0 may be defined as\(^{17}\)

\(^{15}\) The basic idea is due to Hicks (1940). Also see Neary and Roberts (1980), who introduced the term ‘virtual price’ for the shadow price that induces zero consumption.

\(^{16}\) For the properties of restricted expenditure functions, see Diewert (1986; 170-176).

\(^{17}\) To see why the shadow prices defined by (24) do the job, consider the following period 0 expenditure minimization problem for household \(h\):

\[\text{The Gains from Trade and Policy Reform Revisited} \quad 27/04/01\]
(24) \( p_i^{h0} \equiv \frac{\partial \tilde{m}^h(u^{h0},0,\tilde{p}^0)}{\partial c_i}, \quad h \in H \).

Using expression (24) for the appropriate household \( h \) shadow price for good \( l \) in period 0, the household specific distortion vectors are defined by choosing the reference price \( p_i^{l0} \equiv p_i^l \) for the new good, and ‘taxes’ according to

\[
(25) \quad t_i^{h0} \equiv p_i^{h0} - p_i^{l0}, \quad t_i^{h0} \equiv 0, \quad i \neq 1, i \in N, h \in H.
\]

On the producers’ side of the economy, we assume that the technology includes the new good even before it is introduced but producers are restricted to produce zero units of it in period 0. Accordingly, we look for a period 0 shadow price for good \( l \) that would induce firm \( k \) to supply a zero quantity of the new good. To do this we define the restricted profit function for firm \( k \in K \), as

\[
(26) \quad \tilde{\pi}^k(y_i, \tilde{p}, S^{k0}) \equiv \max_y \{ \tilde{p} \cdot \tilde{y} : (y_i, \tilde{y}) \in S^{k0} \},
\]

where \( \tilde{y} \) is a vector of quantities excluding the first good. We assume that the observed period 0 production vector \( \tilde{y}^{k0} \) for firm \( k \) solves (26) when \( y_i = 0 \) and \( \tilde{p} = \tilde{p}^0 \); i.e. we have

\[
(27) \quad \tilde{\pi}^k(y_i, \tilde{p}, S^{k0}) = \tilde{p}^0 \cdot \tilde{y}^0, \quad k \in K.
\]

Assuming that the derivative (from the right) exists, the appropriate firm \( k \) shadow or virtual price for good \( l \) in period 0 may be defined as

\[
(28) \quad w_i^{k0} = -\partial \tilde{\pi}^k(0, \tilde{p}, S^{k0})/\partial y_1, \quad k \in K.
\]

\[
m^h(u^{h0}, p_{i}^{h0}, \tilde{p}^0) \equiv \min_{y_i, \tilde{y}} \left\{ p_i^{h0}c_i + \tilde{p}^0 \cdot \tilde{c}^0 : f^h(c_i, \tilde{c}) \geq u^{h0} \right\}
\]

\[
= \min_{y_i} \left\{ p_i^{h0}c_i + \tilde{m}^h(u^{h0}, c_i, \tilde{p}^0) \right\}
\]

\[
= p_i^{h0}c_i^{h0} + \tilde{p}^0 \cdot \tilde{c}^{h0} = \tilde{p}^0 \cdot \tilde{c}^{h0},
\]

which uses the definition of the restricted expenditure function, (22). It should be evident that the first order necessary condition for a solution to this problem is satisfied by \( c_i = 0 \).

18 For the properties of restricted profit functions, see Gorman (1968), Diewert (1973) and McFadden (1978).

19 To see why the shadow prices defined by (28) do the job, consider the following period 0 profit maximization problem for firm \( k \):

\[
\pi^k(w_i^{k0}, \tilde{p}^0, S^{k0}) \equiv \max_y \{ w_i^{k0}y_i + \tilde{p} \cdot \tilde{y} : (y_i, \tilde{y}) \in S^{k0} \}
\]

\[
= \max_y \{ w_i^{k0}y_i + \tilde{\pi}^k(y_i, \tilde{p}, S^{k0}) \}
\]

\[
= w_i^{k0}y_i^{k0} + \tilde{p} \cdot \tilde{y}^{k0} = \tilde{p} \cdot \tilde{y}^{k0},
\]

using the definition of the restricted profit function, (26). It should be evident that the first order necessary condition for a solution to this problem is satisfied by \( y_i = 0 \).
Using the shadow prices defined by (28), the firm specific distortion vectors are defined by using the reference price \( p^1_i \) for the new good, and ‘taxes’ according to

\[
(29) \quad \tau_1^{k0} \equiv p_1^{k0} - p^0_i, \quad \tau_i^{k0} \equiv 0, \quad i \neq 1, i \in N, k \in K.
\]

It should be noted that the assumption that the technology sets in period 0 include the new commodity is not necessary for our analysis. For example, consider the case where there are only two outputs in the economy in period 1 and the new commodity is not producible in period 0. In this case, the output production possibilities sets would simply be line segments along the \( y_2 \) axis emanating from the origin; i.e., no units of \( y_1 \) would be producible in period 0. In this case, we simply set \( y_1 = 0 \) in definitions (26) and (27). Since the derivatives defined by (28) would not exist in this case, we simply replace the \( w_j^{k0} \) defined in (28) by the reference price \( p^0_i \). With these alternative definitions, the firm specific “taxes” \( \tau_1^{k0} \) defined by (29) all become zero. This is as it should be, since under the assumption that the technology for producing the new good in period 0 did not exist, there are no producer substitution effects in period 0.

With these household and firm specific distortions defined, the reference prices determine the welfare effects in the usual way, as is now illustrated. To focus exclusively on the introduction of a new good we assume no technical progress so that \( S^{k0} = S^{k1} \) for each \( k \), no endowment change and no excess supplies in each period. Then expression (18) for the aggregate variation, evaluated at period 1 world prices, reduces to

\[
(30) \quad V(p^1) = \sum_{b \in H} s_{b0}^1 + \sum_{k \in K} \sigma_{k0}^1 - p^1 \cdot (x^1 - x^0),
\]

and expression (19) for the aggregate variation, evaluated at period 0 world prices, reduces to

\[
(31) \quad V(p^0) = \sum_{b \in H} s_{b0}^0 - \sum_{b \in H} s_{b1}^0 + \sum_{k \in K} \sigma_{k0}^1 - \sum_{k \in K} \sigma_{k1}^0 - p^0 \cdot (x^1 - x^0).
\]

If, further, there is a constant trade balance in the two periods \( (b^0 = b^1) \) and international prices are constant \( (p^0 = p^1) \), then the terms \(- p^1 \cdot (x^1 - x^0)\) and \(- p^0 \cdot (x^1 - x^0)\) equal 0. Under these conditions, (30) shows that the country will unambiguously gain from the introduction of the new good if any of the producer or consumer substitution functions are positive. Accordingly, we have established the following proposition: The positivity of any of the producer or consumer substitution functions in the aggregate quasi-compensating...
variation (30) is sufficient for a welfare improvement as a result of the introduction of a new good.

The special case where \( N = 2, \ M = 0, \ H = 1, \ K = 1, \ p^0_1 = p^1_1 = 0 \) and \( b^0_0 = b^1_0 = 0 \) is illustrated in Figure 3. The initial consumption vector is \( c^{h0}_0 = y^{k0}_0 \), where no units of good 1 are consumed or produced. When the new good is introduced, production moves to \( y^{k1}_1 \) and consumption moves to \( c^{h1}_1 \). The period 1, prices are \( p^1_1 \) and \( p^1_2 \) and the lines \( C'C, D'D \) and \( B'B \) have slopes equal to \( -p^1_1 / p^1_2 \). The slope of the line \( C'A \) equals \( -p^h0_1 / p^0_2 \) and the slope of the line \( C'E \) equals \( -w^1_0 / p^0_2 \). Under our simplifying assumptions, (30) becomes

\[
V( p^1 ) = s^1_0 + \sigma^1_0 = BC + CD = BD,
\]

while (31) becomes

\[
V( p^0 ) = s^0_0 - s^0_1 + \sigma^0_0 - \sigma^0_1 = s^0_0 - 0 + \sigma^0_0 - 0 = BC + CD = BD.
\]

Thus, due to the simplicity of the model, our two measures of gain coincide in this case.

4.4 Extension to a Large Open Economy

We now extend the ideas developed above to a large open economy that can influence its terms of trade by its tariff and tax policies. First, we add terms to our expressions for welfare gains to deal with changes in the world trade environment and with the optimality of the tariff policy. Second, we use these expressions to develop a new sufficiency condition for a welfare improvement.

To this end, we define the net trade value function

\[
\beta( p, X ) \equiv \max \left\{ p \cdot z : z \in X \right\},
\]

which is the maximum value of net imports, \( z \), (foreign country net exports) that can be attained at reference price vector \( p \) when the country faces the foreign country’s offer set \( X \). This function is analogous to a profit function, recognizing that \( X \) may be interpreted as the ‘production possibility set from trade’. Using this function, we define

\[
\delta( p ) = \beta( p, X^1 ) - \beta( p, X^0 )
\]

as the difference in the maximum values arising from a shift in the foreign offer set. This function will be zero if there is no change in the foreign offer set, and will be non-negative if the offer set is enlarged. Finally, we define
Function \( \gamma_r(p) \) is the maximum value of attainable net imports minus the cost, at the reference prices \( p \), of the actual net import vector \( \mathbf{z}^r \) in period \( r \). It therefore measures the gains from substitution around the foreign offer set and is called the \textit{trade substitution function}. If there are no consumer specific taxes, optimality of the home country’s tariff vector implies that the (common) consumer price vector, \( \mathbf{p}^r_C \), satisfies the equation \( \gamma_r(p^r_C) = 0 \), which is the usual first-order characterization of optimal tariffs.

Using these definitions, the aggregate variation may be expressed alternatively as

\[
V(p) = \sum_{l \in \mathcal{H}} s_{h_0}(p) - \sum_{l \in \mathcal{H}} s_{h_1}(p) + \sum_{k \in K} p \cdot (y^{k_1} - y^{k_0}) + p \cdot (v^l - v^0) - p \cdot (e^l - e^0) + \gamma_0(p) - \gamma_1(p) + \delta(p) \tag{37}
\]

and

\[
V(p) = \sum_{l \in \mathcal{H}} s_{h_0}(p) - \sum_{l \in \mathcal{H}} s_{h_1}(p) + \sum_{k \in K} \sigma_{h_0}(p) - \sum_{k \in K} \sigma_{h_1}(p) + \sum_{k \in K} \alpha_{h}(p) + p \cdot (v^l - v^0) - p \cdot (e^l - e^0) + \gamma_0(p) - \gamma_1(p) + \delta(p) \tag{38}
\]

Assuming for simplicity that there is no change in technologies, endowments, excess supplies or in the foreign offer set, expression (37) reduces to

\[
V(p) = \sum_{l \in \mathcal{H}} s_{h_0}(p) - \sum_{l \in \mathcal{H}} s_{h_1}(p) + \sum_{k \in K} p \cdot (y^{k_1} - y^{k_0}) + \gamma_0(p) - \gamma_1(p) \tag{39}
\]

If, in addition, it is assumed that tariffs are optimal in period \( 1 \) (and that there are no consumer specific domestic taxes), a further interesting simplification may be established. To obtain this simplification, we choose consumer prices as reference prices for the evaluation of the aggregate variation. Then, since the optimality of tariffs in period \( 1 \) implies that \( \gamma_1(p^1_C) = 0 \), where \( p^1_C \) is the consumer price vector in period \( 1 \), expression (39) reduces to

\[
V(p^1_C) = \sum_{l \in \mathcal{H}} s_{h_0}(p^1_C) + \sum_{k \in K} p^1_C \cdot (y^{k_1} - y^{k_0}) + \gamma_0(p^1_C) \tag{40}
\]

Since the first and third terms on the right hand side of this expression are non-negative, a sufficient condition for a welfare improvement is that

\[
\sum_{k \in K} p^1_C \cdot (y^{k_1} - y^{k_0}) > 0 \tag{41}
\]
This result appears to be new. It establishes the following proposition: *If a country imposes optimal tariffs following some exogenous change, a sufficient condition for a welfare improvement is that the value of production, at the period 1 consumer price vector, increases.*

The assumptions behind the scenes are important. The crucial one is that the country imposes optimal tariffs against the rest of the world in period 1. If tariffs are not optimal, then \( \gamma_t(p_c^1) > 0 \) and hence the result does not follow. Notice, also, that if consumer and producer prices are equal in period 1, sufficiency condition (41) for a welfare improvement automatically holds.

### 5. The Welfare Effects of Policy and Other Changes

#### 5.1. Introduction

The purpose of this section is to show that various results from the literature can be simply obtained using our framework developed above. In particular, we show that many of the results derived by Ohyama (1972) in his classic analysis of the gains from trade and more recently by Wong (1991) may be obtained from our expressions when evaluated at appropriate reference prices. While this demonstration therefore illustrates the generality of our approach, it has the added advantage of shedding light on how the different results in the literature relate to each other.

Our welfare measure, given by the aggregate variation \( V(p) \), is a function of the reference price vector \( p \), which can be chosen arbitrarily. Whatever choice is made, the aggregate variation measures the net welfare gain (assuming the existence of lump sum transfers) of the change in circumstances in moving from period 0 to period 1. The source of the changed circumstances could be any exogenous shift in endowment, technology, world prices or taxes. Different propositions regarding the welfare change are obtained depending upon the choice of the reference price vector and the choice of identity that is examined.

#### 5.2 Ohyama’s Results on Trade Gains

To illustrate this point more clearly, we now show that many of the results in Ohyama’s (1972) path-breaking paper are obtained from our identities using the consumer price vector in period 1 as the reference price vector. To this end, we assume (as does Ohyama) that taxes are not household or firm specific and write the consumer price vector as \( p_c^1 = p^1 + t^1 \geq 0 \).
We now examine some of the implications of our identities obtained by setting the reference price vector equal to the (common) price vector facing consumers. If we set \( p = p^1_c \), then

\[
 s_{h1}(p^1_c) = p^1_c \cdot c^{hr} - m^h(u^{hr}, p^1_c) = 0 \quad h \in H ,
\]

and so equation (14) reduces to

\[
 V(p^1_c) = \sum_{h \in H} s_{h0}(p^1_c) + \sum_{h \in H} p^1_c \cdot (c^{hl} - c^{h0}).
\]

Since the first term on the right hand side is necessarily nonnegative, it follows that a sufficient (but not necessary) condition for a welfare gain, \( V(p^1_c) > 0 \), is that \( \sum_{h \in H} p^1_c \cdot (c^{hl} - c^{h0}) > 0 \). Thus, welfare improves if the value, at period 1 consumer prices, of the aggregate consumption bundle increases. This is basically a revealed preference argument and constitutes the first main result of Ohyama, contained in his Lemma 2. Using the balance conditions (1), the expression for the change in the value of consumption may be expressed in terms of individual components and this leads directly to Ohyama’s Theorem 1.

If it is further assumed that there is no technical change, no endowment change and no change in excess supplies (zero, say) then identity (16) reduces to

\[
 V(p^1_c) = \sum_{h \in H} s_{h0}(p^1_c) + \sum_{k \in K} \sigma_{k0}(p^1_c) - \sum_{k \in K} \sigma_{k1}(p^1_c) - p^1_{CT} \cdot (x_t^1 - x_t^0),
\]

which may be used to obtain Ohyama’s Theorem 2, comprising three special cases. Ohyama’s case (i) assumes no consumer or producer taxes, whence consumer and producer prices (denoted \( p^1_c = p^1_p \)). In this case,

\[
 \sigma_{k1}(p^1_c) = \sigma_{k1}(p^1_p) = 0 \quad \text{and} \quad \sigma_{k0}(p^1_c) = \sigma_{k0}(p^1_p) \geq 0 \quad \text{for all} \quad k \in K .
\]

Thus, since the first two terms in (44) are non-negative and the third is zero, a sufficient condition for a welfare gain is that \( -p^1_{CT} \cdot (x_t^1 - x_t^0) > 0 \). Welfare improves if the value, at consumer prices, of net exports declines (or, of net imports increases).

Ohyama’s case (ii) assumes that trade and production taxes are zero, whence producer prices equal world prices for traded goods (\( p^1_p = p^1 \)). In this case, the expression for the change in the value of consumption may be written as
Since the first term is non-negative, Ohyama’s sufficient condition for a welfare improvement is that
\[
(46) \quad - p^i_T \cdot (x^i_T - x^0_T) - \sum_{h \in H} (p^i_C - p^i) \cdot (c^{h1} - c^{h0}) > 0.
\]

Ohyama’s case (iii) assumes that trade and consumption taxes are zero, whence consumer prices equal world prices for traded goods \((p^i_C = p^i)\). In this case, the expression for the change in the value of consumption may be written as
\[
(47) \quad \sum_{h \in H} p^i_C \cdot (c^{h1} - c^{h0}) = \sum_{k \in K} p^i_C \cdot (y^{k1} - y^{k0}) - p^i_{CT} \cdot (x^i_T - x^0_T)
\]
\[
= \sum_{k \in K} p^i_S \cdot (y^{k1} - y^{k0}) + \sum_{k \in K} (p^i - p^i_S) \cdot (y^{k1} - y^{k0}) - p^i_T \cdot (x^i_T - x^0_T)
\]
\[
= \sum_{k \in K} \sigma_{k1} (p^i_S) + \sum_{k \in K} (p^i - p^i_S) \cdot (y^{k1} - y^{k0}) - p^i_T \cdot (x^i_T - x^0_T) > 0.
\]

Since the first term is non-negative, Ohyama’s sufficient condition for a welfare improvement is that
\[
(48) \quad \sum_{k \in K} (p^i - p^i_S) \cdot (y^{k1} - y^{k0}) - p^i_T \cdot (x^i_T - x^0_T) > 0.
\]

Propositions 1-4 of Ohyama apply to a comparison of trade with autarky. They follow from inequalities similar to those presented above.

Our conclusion is that the results of Ohyama may be obtained by expressing the aggregate variation \(V(p^i_C)\) in various interesting forms. The crucial point is that the aggregate variation is evaluated at the (common) consumer price vector for period \(I\).

### 5.3 Wong’s Results on Trade and Policy Reform Gains

We now consider some implications of evaluating the aggregate variation at the world price vector (at the market price vector for non-traded goods). In doing so, we establish some of Wong’s results on welfare gains.
First, we note that the aggregate variation evaluated at the world price vector in period 1 (the aggregate quasi-variation) may be expressed as

\[(49) \quad V(p^1) = \sum_{h \in H} s_{h0}(p^1) - \sum_{h \in H} s_{h1}(p^1) + \sum_{h \in H} \sum_{k \in K} \sigma_{k0}(p^1) p^1 + \sum_{k \in K} \sigma_{k1}(p^1) + \sum_{k \in K} \alpha_k(p^1) - p^1 \cdot (x^1 - x^0) + p^1 \cdot (v^1 - v^0) - p^1 \cdot (e^1 - e^0).\]

Since consumer prices may be different from world prices, the first two terms may be positive and hence sum to either a positive or negative value. Thus, we get Wong’s (1991; 59-60) result that the condition \(\sum_{h \in H} p^1 \cdot (c_{h1}^1 - c_{h0}^1) > 0\) is neither a necessary nor a sufficient condition for a welfare improvement.

Consideration of several special cases leads to some of Wong’s other propositions. First, if consumer prices equal world prices in period 1 \((p_c^1 = p^1)\) then the second term in (49) is zero and so the condition \(\sum_{h \in H} p^1 \cdot (c_{h1}^1 - c_{h0}^1) > 0\) now become a sufficient condition for a welfare improvement.20 Second, we note that the aggregate variation evaluated at the world price vector in period 1 may be expressed as

\[(50) \quad V(p^1) = \sum_{h \in H} s_{h0}(p^1) - \sum_{h \in H} s_{h1}(p^1) + \sum_{k \in K} \sigma_{k0}(p^1) - \sum_{k \in K} \sigma_{k1}(p^1) + \sum_{k \in K} \alpha_k(p^1) - p^1 \cdot (x^1 - x^0) + p^1 \cdot (v^1 - v^0) - p^1 \cdot (e^1 - e^0).\]

If there are no taxes (so that consumer and producer prices equal world prices) in period 1, this expression reduces to

\[(51) \quad V(p^1) = \sum_{h \in H} s_{h0}(p^1) + \sum_{k \in K} \sigma_{k0}(p^1) + \sum_{k \in K} \alpha_k(p^1) - p^1 \cdot (x^1 - x^0) + p^1 \cdot (v^1 - v^0) - p^1 \cdot (e^1 - e^0)\]

and, if there is no technical change, endowment change or excess supply change, to

\[(52) \quad V(p^1) = \sum_{h \in H} s_{h0}(p^1) + \sum_{k \in K} \sigma_{k0}(p^1) - p^1 \cdot (x^1 - x^0).\]

Since the first two terms are non-negative, it follows that the condition \(- p^1 \cdot (x^1 - x^0) > 0\) is sufficient for a welfare improvement. This is Wong’s Proposition 6. If it is also assumed that \(b^1 = p^1 \cdot x^1 = 0\) (balance of trade in period 1), then this condition may be written as

\[(53) \quad - p^1 \cdot (x^1 - x^0) = - p^1 \cdot x^0 = (p^1 - p^0) \cdot x^0 > 0.\]

Accordingly, the sufficiency condition for Wong’s proposition is a Laspeyres’ terms of trade improvement.

---

20 However, the assumption that producer prices equal world prices does not lead to a clear-cut result.
In summary, it is evident from the preceding discussion that many of the different results on welfare improvements that appear in the literature arise from the use of different reference prices in the measure of aggregate variation and from the use of different (but equivalent) expressions for this variation. Our general formulation, expressed in terms of an arbitrary reference price vector, is potentially useful in allowing these different results to be put into perspective.

6. Conclusion
The basic welfare identities developed above have been shown to be useful in measuring not only the gains from trade and from policy reform, but also the gains (or losses) from other sources of economic change. Our paper considered some of these, including, for example, the gains from reducing excess supplies and from introducing a new internationally traded product. However, the welfare measures and identities we have presented have much wider applicability and can, in principle, be used to measure the welfare effects of a discrete change in any policy instrument or other exogenous variable in our model. Moreover, they are useful in synthesizing what may appear to be disparate results on welfare and trade that have appeared in the literature, as illustrated by our discussion of the results established by Ohyama and Wong.

However, our model and analysis are subject to a number of limitations, which must be kept in mind in evaluating our approach to discrete welfare change. In particular, (i) we are limited to an analysis of ex post data, (ii) the same households are assumed to exist in both periods, (iii) constancy of tastes is assumed, (iv) competitive price taking behaviour is assumed and (v) the model is static. Finally, in the context of many consumers, it should be noted that our aggregate variation measures might not be good measures of aggregate welfare change if welfare of the economy depends upon the distribution of income.

21 If a household existed in only one of the two periods under consideration, we would have to absorb its non-zero consumption vectors into the appropriate net endowment vectors.
22 However, markup monopolists or monopsonists can be accommodated in our model.
23 Accordingly, there is no modeling of saving and investment behaviour, no expectations about future prices are formed, and so on.
References


Hicks, J R (1940), "The Valuation of the Social Income", *Economica* 7, 105-124.

Hicks, J R (1942), "Consumers' Surplus and Index-Numbers", *The Review of Economic Studies* 9, 126-137.


Figure 1: The Gains from Free Trade
Figure 2: The Gains from Elimination of Excess Supplies
Figure 3: The Gains from New Goods