Ratifiability of Efficient Collusive Mechanisms in Second-Price Auctions with Participation Costs

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APRIL 2004

Discussion Paper No.: 04-08
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First Version: September 2003
This Version: April 2004

Abstract

We investigate whether efficient collusive bidding mechanisms are affected by potential information leakage from bidders’ decisions to participate in them within the independent private values setting. We apply the concept of ratifiability introduced by Cramton and Palfrey (1995) and show that when the seller uses a second-price auction with participation costs, the standard efficient cartel mechanisms such as preauction knockouts analyzed in the literature will not be ratified by cartel members. A high-value bidder benefits from vetoing the cartel mechanism since doing so sends a credible signal that she has high value, which in turn discourages other bidders from bidding in the seller’s auction.

JEL Classifications: C72, D44, D82
Keywords: Auctions, collusion, ratifiability

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1 Introduction

Auctions are important and widely employed trading mechanisms. The well-known advantages of using auctions diminish, however, when bidders collude and they have incentives to do so. Bidders problem of finding a way to profitably collude and share the collusive surplus is complicated by the possibility that each has some private information.

Much of the theoretical literature on collusion in auctions treats it as a mechanism design problem with a special focus on feasibility of efficient collusive mechanisms when side-payments are possible.\(^1\) Graham and Marshall (1987) analyze collusion in second-price sealed bid and English auctions with independent private values. For instance, in the context of a second-price auction with a reserve price, they show that a second-price preauction knockout run by an outside agent is an incentive-efficient and durable mechanism in the sense of Holmstrom and Myerson (1983). The knockout balances ex ante budget among bidders. Mailath and Zemsky (1991) consider the case of second-price auctions with a reserve price and heterogeneous bidders and show that for any subset of bidders there exists an incentive compatible, individually rational mechanism that balances budget ex post and achieves ex post efficient collusion. McAfee and McMillan (1992) study collusion in first-price sealed-bid auctions with independent private values and show that efficient collusion among all bidders with ex post budget balancing is possible and can be implemented with a first-price preauction knockout.\(^2\)

All these papers follow the standard mechanism design approach in that when they consider (interim) individual rationality constraints they compare the bidders interim payoff from the mechanism with that from an equilibrium of the status quo, i.e., the seller’s auction. Note, however, that in this bidder collusion problem the status quo is itself a strategic game the outcome of which is affected by bidders’ beliefs about each other. This may create a difference if bidders are making interim voluntary participation decisions, as first pointed out by Cramton and Palfrey (1995) in the general case. Players can make inferences about a bidder’s private information from her choice between the collusive mechanism and the status quo which in turn may affect the outcome of the status quo game if it is played. Cramton

\(^1\)There is a recent literature on repeated auctions. See, for example, Aoyagi (2003), Blume and Heidhues (2002), and Skrzypacz and Hopenhayn (2004).

\(^2\)Also see the recent work by Marshall and Marx (2003) on collusive mechanisms with a focus on those that do not rely on information from the auctioneer.
and Palfrey (1995) refer to this possibility as the *information leakage* problem from participation decisions.\(^3\) To address this problem they consider a two-stage process: Players simultaneously vote (interim) for or against the proposed mechanism. If the mechanism is unanimously *ratified*, then it is implemented; otherwise the status quo game is played under revised beliefs where these satisfy consistency requirements in the spirit of rational expectations.\(^4\)

In this paper we apply the approach of Cramton and Palfrey (1995) to show that in the independent private values environment the standard efficient collusive mechanisms characterized in the literature are *not* ratifiable when the status quo is a second-price auction and there are participation costs, however small they may be.\(^5\) We will show that a high-value bidder can send a credible signal that she has high value by vetoing the cartel mechanism which would increase her payoff from the auction sufficiently that it exceeds her payoff from the efficient cartel. In particular, if a veto by \(i\) leads others to believe that her value is greater than, say, \(v_N\), then there is a continuation equilibrium of the auction such that \(i\) would prefer the status quo to the cartel mechanism iff her value is greater than \(v_N\). This will be the case even though the cartel mechanism is individually rational in the standard sense, i.e., when bidders’ interim payoffs from it is compared to that from the equilibrium of the auction with prior beliefs. Since entry fees and all kinds of participation costs are frequently observed in practice, our finding indicates that collusion may not be as easy as the previous literature has suggested. Moreover, the seller may be able to use entry fees to destabilize potential collusive arrangements among bidders.

We use second-price auctions rather than first-price, since, even when bidders are ex ante symmetric, we need to contemplate cases when a veto creates asymmetry, and calculating expected payoffs for first-price auctions becomes problematic. We also want the equilibrium in the auction to be responsive to players’ beliefs about each other’s values, and this is where the participation cost plays a role.

\(^3\)There is also the possibility of information leakage from the selection of the mechanism, i.e., the informed seller’s problem; see, for example Myerson (1983) and Maskin and Tirole (1990), and, in a bargaining context, Yilankaya (1999).

\(^4\)The refinement is based on Grossman and Perry (1986).

\(^5\)In the analysis below, “participation cost” can be replaced by “entry fee” without any change in the formal model or results. Accordingly, we will use both interpretations in our discussions.
In the next section we briefly describe the setup, discuss the concept of ratifiability and present our main finding. Section 3 provides some discussions. Except where they contribute to the exposition, proofs are relegated to the Appendix.

2 The Model and Main Result

A single indivisible object is being sold by a seller who uses a sealed-bid second-price auction with no reserve price. Consider a standard independent private values environment. There are \( n \geq 2 \) risk-neutral (potential) bidders. Bidders’ values for the object are independent draws from the same cumulative distribution function (c.d.f.) \( F(.) \) with continuous density \( f(.) \) and support \([0, 1] \). Bidder \( i \)'s value, \( v_i \), is her private information. There is a bidder participation cost, or entry fee, common to all bidders, denoted by \( c \in (0, 1) \). Bidders know their values before they decide whether to participate in the auction and must pay \( c \) in order to be able to submit a bid; they do not know others’ participation decisions when they make theirs. All of the above is common knowledge.

2.1 The Status Quo: Non-cooperative Play

The status quo game we consider is the second-price auction. Let the feasible action set for any type of bidder be: \( \{\text{No}\} \cup [0, \infty) \), where “No” denotes not participating; bidder \( i \) incurs the participation cost iff her action is different from “No”. Let \( b_i(.) \) denote \( i \)'s strategy.

In the presence of a participation cost, it is not a weakly dominant strategy for a bidder to always bid her value in the seller’s auction. However, if a bidder finds participating optimal, then she cannot do better than bidding her value. Let \( G(y) = F(y)^{n-1} \) denote the c.d.f. of the highest valuation of \( n - 1 \) bidders. It is easy to see that there exists a unique (up to changes for measure zero set of values) symmetric equilibrium, where each bidder’s strategy is (suppressing subscripts)

\[ G(y) = F(y)^{n-1} \]

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6We assume zero reserve price to keep the discussion and notation simple. The analysis and results hold when there is a binding reserve price.

7See, for example, Matthews (1995).
\[ b^*(v) = \begin{cases} 
  \text{No} & \text{if } v \leq v_0 \\
  v & \text{if } v > v_0 
\end{cases}, \]

where \( G(v_0)v_0 = c \). The expected payoff of a type-\( v \) bidder in this equilibrium is given by

\[ U^s(v) = \begin{cases} 
  G(v_0)v + \int_{v_0}^v (v - y)dG(y) - c & \text{if } v \leq v_0 \\
  \int_{v_0}^v G(y)dy & \text{if } v > v_0 
\end{cases}. \]

or, after using integration by parts

\[ U^s(v) = \begin{cases} 
  \text{No} & \text{if } v \leq v_0 \\
  \int_{v_0}^v G(y)dy & \text{if } v > v_0 
\end{cases}. \]

### 2.2 The Efficient All-Inclusive Cartel Mechanism

Suppose transfers among bidders are possible. Bidders may design a collusive bidding mechanism outside of the seller’s auction. If the all-inclusive cartel mechanism is ex post efficient (i.e., the bidder with the highest value acquires the object if this value is greater than the participation cost), then it does not matter whether there is a participation cost or a reserve price in the seller’s auction provided that their magnitudes are the same. Therefore, it follows from Mailath and Zemsky (1991) and McAfee and McMillan (1992) that there exists an incentive compatible, ex post budget balancing, and ex post efficient all-inclusive cartel mechanism \( m \) yielding the following unique payoff (when bidders are treated symmetrically) to a bidder of type \( v \)

\[ U^m(v) = \begin{cases} 
  \pi + \int_c^v G(y)dy & \text{if } v > c 
\end{cases}, \]

where

\[ \pi = \int_c^1 \left[ y - \frac{1 - F(y)}{f(y)} - c \right]G(y)dF(y). \]

There may be “reasonable” asymmetric equilibria. Tan and Yilankaya (2003) show that concavity (respectively, strict convexity) of \( F(.) \) is a sufficient condition for uniqueness (respectively, multiplicity) of equilibrium within the class of cutoff strategy profiles (where each bidder bids her value if it is greater than a cutoff, does not participate otherwise). Since we focus on the ratifiability issue in this note, we will concentrate on symmetric treatment of (symmetric) bidders throughout.
Since $v_0 > c$ and $\pi > 0$, it follows that

$$U^m(v) > U^s(v)$$

for all $v \in [0, 1]$. That is, the efficient cartel mechanism $m$ is also interim individually rational with respect to the symmetric equilibrium payoff in the seller’s auction.

Moreover, $m$ can be implemented by a first-price or second-price preauction knockout in which all bidders compete for the right to be the sole bidder in the seller’s auction. In the case of a first-price knockout auction, the bidder with the highest bid wins and pays her bid, which will be shared equally among the losers.

### 2.3 Ratifying the Cartel Mechanism

The main objective of this paper is to study whether the efficient cartel mechanism described above is affected if we allow the possibility of learning from bidders’ decisions to participate in them. The status quo game is the second-price auction with participation costs, and so its outcome depends on bidders’ beliefs about others’ values which in turn may be affected by their participation decisions in the mechanism. Following Cramton and Palfrey (1995), we consider a two-stage ratification game. In the first stage, bidders simultaneously vote for or against the efficient cartel mechanism. In the second stage, if the cartel mechanism is unanimously accepted then it is implemented; otherwise, bidders participate in the second-price auction knowing who had vetoed the cartel mechanism, and thus having updated beliefs about vetoers’ values. Since the cartel mechanism is incentive compatible and individually rational (with passive beliefs), its ratification in the first stage combined with truthful revelation by all bidders in the second stage is a sequential equilibrium outcome.

Cramton and Palfrey (1995) propose a further refinement of equilibria in the two-stage game. The cartel mechanism is ratifiable, if in addition, other bidders’ beliefs about the vetoer’s value satisfy certain consistency requirements. In particular, if bidder $i$ vetoes, then the rest of the bidders

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9Following much of the literature on collusion from a mechanism design perspective, suppose that an “uninformed third party” proposes the cartel mechanism.

10Since we are looking for an equilibrium in which bidders unanimously ratify the cartel mechanism, we only need to be concerned about unilateral deviations.
try to rationalize \( i \)'s veto decision by identifying a set of types for \( i \) (the veto set) that could have benefitted from the veto. The veto set is credible if types in it prefer the non-cooperative play in the seller’s auction (with the updated belief that \( i \)'s type belongs to the veto set) to the cartel mechanism and types that are not in the veto set prefer the cartel mechanism. With this refinement, the cartel mechanism is ratifiable if either there is no credible veto set for any player or there exists one (and corresponding equilibrium play at the auction) where all the types in the veto set are actually indifferent between the cartel mechanism and the non-cooperative play in the seller’s auction. Formally,

**Definition 1** \( V_i \subseteq [0,1] \) is a credible veto set for \( i \) if there exists a continuation equilibrium \( b(V_i) \) and the corresponding equilibrium payoff \( U_i(v_i, b(V_i)) \) of \( v_i \) in the seller’s auction such that

1. \( V_i \neq \emptyset \),
2. \( U_i(v_i, b(V_i)) > U^m(v_i) \Rightarrow v_i \in V_i \),
3. \( U_i(v_i, b(V_i)) < U^m(v_i) \Rightarrow v_i \notin V_i \), and
4. in the continuation auction, other bidders’ beliefs about \( i \)’s value satisfy Bayes’ rule.

**Definition 2** The cartel mechanism is ratifiable against the seller’s auction if for all \( i \) either

1. there does not exist a credible veto set for \( i \), or
2. there exists a credible veto set \( V_i \) and a corresponding continuation equilibrium in the auction \( b(V_i) \) such that \( U_i(v_i, b(V_i)) = U^m(v_i) \) for all \( v_i \in V_i \).

The above definitions are slightly different from Cramton and Palfrey’s (1995), since we have already incorporated the fact that the cartel mechanism is incentive compatible and individually rational (with passive beliefs). Given these definitions, we now present the main result of our paper.

**Proposition 1** The efficient cartel mechanism \( m \) is not ratifiable.

The basic intuition behind the result is as follows. In the presence of a participation cost, low-value bidders have relatively more to gain by participating in the cartel mechanism than high-value bidders. By vetoing the cartel mechanism, a high-value bidder sends a signal that she has a high value, which would discourage other bidders from participating in the seller’s
auction and benefit the vetoer. On the other hand, a low-value bidder is not able to gain from vetoing. This makes vetoing by high-value bidders credible.

To present the above intuition formally, and as the first step for proving the result, we will construct a credible veto set. Suppose that when one of the bidders vetoes the cartel mechanism, others believe that her value exceeds a cutoff point \( v_N < 1 \).\(^{11}\) We will show that there is an equilibrium of the seller's auction with these revised beliefs such that the vetoer’s payoff in this equilibrium is larger (respectively, smaller) than her cartel payoff if her value is larger (respectively, smaller) than \( v_N \). In other words, it is precisely the types in \([v_N, 1]\) who would want to veto the cartel mechanism if a veto causes other bidders to believe that the vetoer’s type is in \([v_N, 1]\), so that \( V_i = [v_N, 1]\) is a credible veto set for any bidder \( i \).

Consider the second-price auction where the vetoer \( i \)'s value is distributed on \([v_N, 1]\) according to \( F_N(v) \equiv \frac{F(v) - F(v_N)}{1 - F(v_N)} \), which is derived from \( F(.) \) using the Bayes’ rule, and the ratifiers' values are distributed on \([0, 1]\) according to \( F(.) \). The equilibrium we consider in this auction is given by

\[
\begin{align*}
  b^*_i(v_i) &= v_i \text{ for all } v_i \in [v_N, 1], \\
  b^*_j(v_j) &= \begin{cases} 
    \text{No} & \text{if } v_j \leq v_Y \\
    v_j & \text{if } v_j > v_Y 
  \end{cases} \text{ for all } j \neq i,
\end{align*}
\]

where all the ratifiers (who are symmetric) use the same cutoff point \( v_Y \), and \( v_Y \) is determined by the indifference condition between participating and not participating in the auction.

For any given ratifier, the maximum of others' values is distributed on \([v_N, 1]\) according to \( \tilde{G}(y) \equiv F(y)^{n-2} F_N(y) \). Let \( \tilde{v}_Y \) be the solution to

\[
\int_{v_N}^{\tilde{v}_Y} (\tilde{v}_Y - y) d\tilde{G}(y) = c,
\]

where the left-hand side denotes the expected payoff of a \( \tilde{v}_Y \)-type ratifier whenever \( \tilde{v}_Y \leq 1 \). We have, \( v_Y = \min\{1, \tilde{v}_Y\} \) and \( v_N < v_Y \leq 1 \). Notice that \( v_Y \) is a strictly increasing function of \( v_N \) until it reaches 1 for some value of \( v_N \) and stays there for higher values of \( v_N \).

\(^{11}\) Naturally, \( v_N \) depends on the participation cost \( c \), as well as other exogenous variables, this dependence is suppressed in the notation. Also, “\( N \)” is for saying “no” to the cartel mechanism (the vetoer), and “\( Y \)” is for saying “yes” (ratifiers).

\(^{12}\) Types in \([v_N, 1]\) strictly prefer to veto, whereas \( v_N \) is indifferent.
Since the best any player can do is to bid her valuation in the second-price auction (if she chooses to participate), the expected payoff of any type-$v_i$ bidder $i$ if she chooses to veto the cartel mechanism is (given the ratifiers’ belief that her type is in $[v_N, 1]$ and the equilibrium we are considering in the auction)

$$U_i(v_i, b^*([v_N, 1])) = \max \{ \int_0^{\max\{v_i,v_Y}\}} (v_i - p(y))dG(y) - c, 0 \},$$

where

$$p(y) = \begin{cases} 0 & \text{if } y \leq v_Y \\ y & \text{if } y > v_Y \end{cases}$$

is the price paid by the vetoer in case she wins the object in the auction.

Comparing this payoff with the one from the cartel mechanism leads to the following lemma, the proof of which is given in the Appendix.

**Lemma 1** There exists a unique $v_N \in (c, 1)$ such that $U_i(v_N, b^*([v_N, 1]) = U^m(v_N)$ and $U_i(v_i, b^*([v_N, 1])) > (<)U^m(v_i)$ for $v_i > (<)v_N$.

Lemma 1 shows that if after a veto of the cartel mechanism by bidder $i$ others believe that the $v_i$ exceeds $v_N$ (and the continuation equilibrium $b^*$ is played), then $i$ would benefit from such a veto iff her value exceeds $v_N$. Thus, $V_i = [v_N, 1]$ is a credible veto set for player $i$. Furthermore, there does not exist any credible veto set where all types in the set are indifferent between vetoing and ratifying. This is stated in the following lemma, with its proof to be found in the Appendix.

**Lemma 2** There does not exist a credible veto set $V_i$ and a corresponding continuation equilibrium in the auction $b(V_i)$ such that $U_i(v_i, b(V_i)) = U^m(v_i)$ for all $v_i \in V_i$.

Proposition 1 then follows from Definition 2 and Lemmas 1 and 2.

Note that $v_N$ in Lemma 1 generally depends on the distribution $F(.)$, the number of bidders $n$ and participation cost $c$. To see the impact of a positive participation cost, consider two extreme cases. Given $F(.)$ and $n$, when $c$ is large, the veto set is such that $v_Y = 1$ so that ratifiers do not participate in the continuation auction game. For instance, suppose $n = 2$, $F(v) = v$, and $c = 2/3$. Then it can be easily computed that $v_Y = 1$ and $v_N = 0.686$. In
the other extreme, as \( c \) goes to 0, both \( v_N \) and \( v_Y \) approach the same limit \( v_N^* \) satisfying

\[
\int_0^{v_N^*} y dG(y) = \int_0^1 [y - \frac{1 - F(y)}{f(y)}] G(y) dF(y).
\]

Notice that \( v_N^* > 0 \). This implies that even when \( c = 0 \), the efficient cartel mechanism is not ratifiable. We do not want to emphasize this however. The issue is, when \( c = 0 \), in the post-veto auction where the vetoer’s type is known to be in \( [v_N^*, 1] \), there is an equilibrium in weakly dominant strategies in which all types of ratifiers participate. If this is the post-veto equilibrium, then bidders clearly will not have an incentive to veto. Notice that, a positive \( c \), however small, eliminates this issue, ratifiers with values less than \( v_N^* \) will never participate in the post-veto auction, and the cutoff they use will approach to \( v_N^* \) from above as \( c \) goes to zero.

3 Discussion

Since the seller’s mechanism is a second-price auction, our analysis and main finding apply to the case of heterogeneous bidders. Under certain assumptions the analysis also carries over to partial cartels. Suppose \( k \) bidders form a cartel, where \( 1 < k < n \). Assume that the cartel uses a mechanism that is ex post efficient, incentive compatible and balances the budget (see Mailath and Zemsky, 1991), and that it will send a representative bidder to bid in the seller’s auction, if bid at all. From non-members’ view point, the cartel’s (maximum) value has a distribution \( F^k(.) \). As before we assume that a unanimous voting is required to ratify a cartel. Suppose one member vetoes and others vote “yes”. Assume also that the ratifiers believe that the vetoer’s type is in \( [v_N^*, 1] \) and that non-cartel members still think that the cartel is in effect. We can compute the equilibrium payoffs in the second-price auction game and determine the cutoff \( v_N \) in the same way as we presented in Section 2.

How would the seller respond to the presence of collusive bidding arrangements? The literature has mostly focused on the seller’s response to possible collusion by appropriately choosing a reserve price. For instance, Graham and Marshall (1987) have shown that the seller’s optimal reserve price with a bidding cartel is always greater than the one without a cartel. Our analysis suggests that an entry fee, independent of its magnitude, may be a useful
tool in dealing with bidder collusion. Moreover, it is as good an instrument as a reserve price in terms of maximizing revenue in this setup.

There are several empirical studies that offer evidence of collusion in many auction markets.\textsuperscript{13} In these studies, the status quo game is typically a first-price sealed-bid auction and there are often multiple units sold in these markets. In this paper we have restricted our study to a single-unit second-price auction. If the status quo game is a first-price sealed-bid auction, vetoing any cartel mechanism will generate asymmetric beliefs in the continuation game which would make it difficult to compute equilibrium payoffs.

Finally, it should be noted that in our setting we have only shown that the well-known efficient cartel mechanism is not ratifiable. We leave for future research the question of whether there are other (inefficient) collusive arrangements which are immune to problems associated with information leakage from members’ participation decisions.

\textsuperscript{13} Examples include highway construction contracts (Porter and Zona, 1993), school milk delivery (Pesendorfer, 2000, Porter and Zona, 1999), timber auctions (Baldwin, Marshall and Richard, 1997), and federal offshore oil and gas lease auctions (Hendricks, Porter, and Tan, 2003).
Appendix

Proof of Lemma 1. In what follows we drop the subscript \( i \) to simplify notation. Note that we have

\[
U(v, b^*(\lbrack v_N, 1 \rbrack)) = \begin{cases} 
0 & \text{if } v < \frac{c}{G(v_Y)} \\
G(v_Y)v - c & \text{if } \frac{c}{G(v_Y)} \leq v \leq v_Y \\
G(v)v - \int_{v_Y}^{v} yg(y)dy - c & \text{if } v > v_Y
\end{cases}
\]

We will first find a \( v_N \) for which \( U(v_N, b^*(\lbrack v_N, 1 \rbrack)) = U^m(v_N) \), and then check the inequalities.

Since \( c < v_N \leq v_Y \), it suffices to show (making dependence of \( v_Y \) on \( v_N \) explicit)

\[
G(v_Y(v_N))v_N - c - \pi - \int_{c}^{v_N} G(y)dy = 0.
\]

Let

\[
\phi(v) = G(v_Y(v))v - \int_{c}^{v} G(y)dy - c - \pi.
\]

Notice that

\[
\phi'(v) = G(v_Y(v)) + vg(v_Y(v))v_Y'(v) - G(v) > 0
\]

for \( v < 1 \) since \( v_Y(v) \geq v \). Since \( \phi(v) \) is continuous, \( \phi(c) < 0 \) and \( \phi(1) > 0 \), a unique solution to \( \phi(v) = 0 \) exists, and is our candidate for \( v_N \). Next, we show

\[
U(v, b^*(\lbrack v_N, 1 \rbrack)) > (\leq)U^m(v) \text{ for } v > (\leq)v_N.
\]

Fix \( c \), and hence \( v_N \) and \( v_Y \). Notice that, \( c \leq \frac{c}{G(v_Y)} < v_N \leq v_Y \), \( U(v, b^*(\lbrack v_N, 1 \rbrack)) - U^m(v) \) is continuous and given by

\[
\begin{align*}
-\pi & \quad \text{if } v < \frac{c}{G(v_Y)} \\
G(v_Y)v - \int_{c}^{v} G(y)dy - \pi - c & \quad \text{if } \frac{c}{G(v_Y)} \leq v \leq v_Y \\
G(v)v - \int_{v_Y}^{v} yg(y)dy - \int_{c}^{v} G(y)dy - \pi - c & \quad \text{if } v > v_Y
\end{align*}
\]

Also notice that \( U(v, b^*(\lbrack v_N, 1 \rbrack)) - U^m(v) < 0 \) for \( v \leq \frac{c}{G(v_Y)} \). Let

\[
\varphi(v) = G(v_Y)v - \int_{c}^{v} G(y)dy - \pi - c.
\]
Note that \( \varphi(v_N) = \phi(v_N) = 0 \). Moreover, \( \varphi'(v) = G(v_Y) - G(v) > 0 \) for \( v < v_Y \) and \( \varphi'(v_Y) = 0 \). It follows that \( \varphi(v) < 0 \) for \( v < v_N \), \( \varphi(v_N) = 0 \), and \( \varphi(v) > 0 \) for \( v > v_N \), with \( \varphi(v) \) reaching its maximum, which is strictly positive, at \( v_Y \). Finally, notice that

\[
\frac{d}{dv} [U(v, b^*([v_N, 1])) - U^m(v)] = 0
\]

for \( v > v_Y \). Hence,

\[
U(v, b^*([v_N, 1])) > (\!<)U^m(v) \text{ for } v > (\!<)v_N.
\]

The claim follows. ■

**Proof of Lemma 2.** Suppose there exists a credible veto set \( V_i \) for bidder \( i \) and a corresponding continuation equilibrium in the auction \( b \) such that \( U_i(v_i, b(V_i)) = U^m(v_i) \) for all \( v_i \in V_i \). Since \( U^m(v_i) > 0 \) for all \( v_i \), it must be the case that \( U_i(v_i, b(V_i)) > 0 \) for all \( v_i \in V_i \), and at \( b \), the cutoff used by the vetoor \( a^* = \inf V_i \). Let the cutoffs used by others be \( a_1 < a_2 < ... < a_k \), where \( a_i \) is used by \( n_j \) bidders and \( \sum_{j=1}^k n_j = n - 1 \).

If \( a^* < a_1 \), then for \( v_i \in (a^*, a_1) \),

\[
U_i(v_i, b(V_i)) - U^m(v_i) = v_i \prod_{j=1}^k F(a_j)^k - c - \int_c^{v_i} F(y)^{n-1} dy - \pi
\]

which is strictly increasing in \( v_i \). Since \( a^* = \inf V_i \), this leads to a contradiction with \( U_i(v_i, b(V_i)) = U^m(v_i) \) for all \( v_i \in V_i \) and the requirement that every type who has a strict incentive to veto must be in the veto set, i.e., part ii) of Definition 1.

If \( a^* = a_1 \), then for \( v_i \in (a^*, a_2) \),

\[
U_i(v_i, b(V_i)) - U^m(v_i) = \prod_{j=2}^k F(a_j)^k [F(a^*)^n a^* + \int_{a^*}^{v_i} F^{n_1} dy] - c - \int_{a^*}^{v_i} F^{n-1} dy - \pi,
\]

which is again strictly increasing, as long as \( n_1 \neq n - 1 \), i.e., all the other bidders use the same cutoff as the vetoor. But this is not possible unless \( V_i = [0, 1] \), which in turn is not possible, since if \( V_i = [0, 1] \), then \( U_i(v_i, b(V_i)) - U^m(v_i) < 0 \) (the cartel mechanism is individually rational with passive beliefs), a contradiction.

The remaining case, \( a^* > a_1 \), is not possible either, since the bidders who use \( a_1 \) are sure to lose the auction, and thus have a negative payoff. ■
References


