Issues in the Measurement of Capital Services, Depreciation, Asset Price Changes and Interest Rates

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Abstract

The chapter considers the measurement of capital services aggregates under alternative assumptions about the form of depreciation, the opportunity cost of capital and the treatment of capital gains. Four different models of depreciation are considered: (1) one hoss shay or light bulb depreciation; (2) straight line depreciation; (3) declining balance or geometric depreciation and (4) linearly declining efficiency profiles. The chapter also considers the differences between cross section and time series depreciation and anticipated time series depreciation (which adds anticipated obsolescence of the asset to normal cross section depreciation of the asset). Finally, issues involving the measurement of certain intangible capital stocks are considered.

Key Words

Capital services, user costs, depreciation models, obsolescence, anticipated asset prices, intangible assets.

Journal of Economic Literature Classification Codes

C43, C82, D24, D92.

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1. Introduction[1]

In this chapter, we discuss some of the problems involved in constructing price and quantity series for both capital stocks and the associated flows of services when there are general and asset specific price changes in the economy.[2]

In section 2, we present the basic equations relating stocks and flows of capital assuming that data on the prices of vintages of a homogeneous capital good are available. This framework is not applicable under all circumstances but it is a framework that will allow us to disentangle the effects of general price change, asset specific price change and depreciation.

Section 3 continues the theoretical framework that was introduced in section 2. We show how information on vintage asset prices, vintage rental prices and vintage depreciation rates are all equivalent under certain assumptions; i.e., knowledge of any one of these three sequences or profiles is sufficient to determine the other two.

Section 4 discusses alternative sets of assumptions on nominal interest rates and anticipated asset price changes. We specify three different sets of assumptions that we will use in our empirical illustration of the suggested methods.

Section 5 discusses the significance of our assumptions made in the previous section and relates them to controversies in national income accounting. In particular, we discuss whether anticipated asset price decline should be an element of depreciation as understood by national income accountants.

Section 6 discusses the problems involved in aggregating over vintages of capital, both in forming capital stocks and capital services. Instead of the usual perpetual inventory method for aggregating over vintages, which assumes perfectly substitutable vintages of the same stock, we suggest the use of a superlative index number formula to do the aggregation.

Sections 7 to 10 show how the general algebra presented in sections 2 and 3 can be adapted to deal with four specific models of depreciation. The four models considered are the one hoss shay model, the straight line depreciation model, the geometric model of depreciation and the linear efficiency decline model. In section 11, we show how these models differ empirically by computing the corresponding stocks and flows using Canadian data on two asset classes. The details of the computations and the data used may be found in Diewert (2004).

Section 12 shows how our framework can be modified to model the treatment of some forms of intangible capital, such as investments in research and development.

Section 13 concludes with some observations on how statistical agencies might be able to use the material presented in this chapter.

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[2] We cover some of the same issues discussed in the recent paper by Hill and Hill (2003). However, Hill and Hill did not deal with the problems associated with adjusting nominal interest rates for general inflation.

2. The Fundamental Equations Relating Stocks and Flows of Capital

Before we begin with our algebra, it seems appropriate to explain why accounting for the contribution of capital to production is more difficult than accounting for the contributions of labour or materials. The main problem is that when a reproducible capital input is purchased for use by a production unit at the beginning of an accounting period, we cannot simply charge the entire purchase cost to the period of purchase. Since the benefits of using the capital asset extend over more than one period, the initial purchase cost must be distributed somehow over the useful life of the asset. This is *the fundamental problem of accounting*.

In a noninflationary environment, the value of an asset at the beginning of an accounting period is equal to the discounted stream of future rental payments that the asset is expected to yield. Thus the *stock value* of the asset is equal to the discounted future *service flows*[3] that the asset is expected to yield in future periods. Let the price of a new capital input purchased at the beginning of period t be P_0^t. In a noninflationary environment, it can be assumed that the (potentially observable) sequence of (cross sectional) vintage rental prices prevailing at the beginning of period t can be expected to prevail in future periods. Thus in this no general inflation case, there is no need to have a separate notation for future expected rental prices for a new asset as it ages. However, in an inflationary environment, it is necessary to distinguish between the observable rental prices for the asset at different ages at the beginning of period t and future *expected* rental prices for assets of various ages.[4] Thus let f_0^t be the (observable) rental price of a new asset at the beginning of period t, let f_1^t be the (observable) rental price of a one period old asset at the beginning of period t, let f_2^t be the (observable) rental price of a 2 period old asset at the beginning of period t, etc. Then *the fundamental equation* relating the *stock value of a new asset* at the beginning of period t, P_0^t, to the sequence *of cross sectional rental prices for assets of age n* prevailing at the beginning of period t, $\{f_n^t : n = 0,1,2,\ldots\}$ is[5]:

$$(1)\quad P_0^t = f_0^t + [(1+i_1^t)/(1+r_1^t)]\, f_1^t + [(1+i_1^t)(1+i_2^t)/(1+r_1^t)(1+r_2^t)]\, f_2^t + \ldots$$

In the above equation, $1+i_1^t$ is the *rental price escalation factor* that is *expected* to apply to a one period old asset going from the beginning of period t to the end of period t (or equivalently, to the beginning of period t+1), $(1+i_1^t)(1+i_2^t)$ is *the rental price escalation factor* that is *expected* to apply to a 2 period old asset going from the beginning of period t to the beginning of period t+2, etc. Thus the i_n^t *are expected rates of price change for used assets of varying ages n* that are formed at the beginning of period t. The term $1+r_1^t$ is the discount factor that makes a dollar received at the beginning of period t equivalent to a dollar received at the beginning of period t+1, the term $(1+r_1^t)(1+r_2^t)$ is the discount factor that makes a dollar received at the beginning of period t equivalent to a dollar received at the beginning of period t+2, etc. Thus the r_n^t are one period *nominal interest rates* that represent the *term structure of interest rates* at the beginning of period t.[6]

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[3] Walras (1954) (first edition published in 1874) was one of the earliest economists to state that capital stocks are demanded because of the future flow of services that they render. Although he was perhaps the first economist to formally derive a user cost formula as we shall see, he did not work out the explicit discounting formula that Böhm-Bawerk (1891; 342) was able to derive.

[4] Note that these future expected rental prices are not generally observable due to the lack of futures markets for these future period rentals of the assets of varying ages.

[5] The sequence of (cross sectional) vintage rental prices $\{f_n^t\}$ is called the *age-efficiency profile* of the asset.

[6] Peter Hill has noted a major problem with the use of equation (1) as the starting point of our discussion: namely, *unique assets* will by definition not have used versions of the same asset in the marketplace during

We now generalize equation (1) to relate the *stock value of an n period old asset* at the beginning of period t, P_n^t, to the sequence *of cross sectional vintage rental prices* prevailing at the beginning of period t, $\{f_n^t\}$; thus for n = 0,1,2,…, we assume:

(2) $P_n^t = f_n^t + [(1+i_1^t)/(1+r_1^t)]\, f_{n+1}^t + [(1+i_1^t)(1+i_2^t)/(1+r_1^t)(1+r_2^t)]\, f_{n+2}^t + \ldots$

Thus older assets discount fewer terms in the above sum; i.e., as n increases by one, we have one less term on the right hand side of (2). However, note that we are applying the same price escalation factors $(1+i_1^t)$, $(1+i_1^t)(1+i_2^t)$, …, to escalate the cross sectional rental prices prevailing at the beginning of period t, f_1^t, f_2^t,…, and to form estimates of future expected rental prices for each vintage of the capital stock that is in use at the beginning of period t.

The rental prices prevailing at the beginning of period t for assets of various ages, f_0^t, f_1^t, … are potentially observable.[7] These cross section rental prices reflect the relative efficiency of the various vintages of the capital good that are still in use at the beginning of period t. It is assumed that these rentals are paid (explicitly or implicitly) by the users at the beginning of period t. Note that the sequence of asset stock prices for various ages at the beginning of period t, P_0^t, P_1^t, … is not affected by general inflation provided that the general inflation affects the expected asset rates of price change i_n^t and the nominal interest rates r_n^t in a proportional manner. We will return to this point later.

The physical productivity characteristics of a unit of capital of each age are determined by the sequence of cross sectional rental prices. Thus a brand new asset is characterized by the vector of current rental prices for assets of various ages, f_0^t, f_1^t, f_2^t, … , which are interpreted as "physical" contributions to output that the new asset is expected to yield during the current period t (this is f_0^t), the next period (this is f_1^t), and so on. An asset which is one period old at the start of period t is characterized by the vector f_1^t, f_2^t, …, etc.[8]

We have not explained how the expected rental price rates of price change i_n^t are to be estimated. We shall deal with this problem in section 4 below. However, it should be noted that there is no guarantee that our expectations about the future course of rental prices are correct.

At this point, we make some simplifying assumptions about the expected rates of rental price change for future periods i_n^t and the interest rates r_n^t. We assume that these anticipated specific price change escalation factors at the beginning of each period t are all equal; i.e., we assume:

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the current period and so the cross sectional rental prices f_n^t for assets of age n in period t will not exist for these assets! In this case, the f_n^t should be interpreted as expected future rentals that the unique asset is expected to generate at today's prices. The $(1+i_n^t)$ terms then summarize expectations about the amount of asset specific price change that is expected to take place. This reinterpretation of equation (1) is more fundamental but we chose not to make it our starting point because it does not lead to a completely objective method for national statisticians to form reproducible estimates of these future rental payments. However, in many situations (e.g., the valuation of a new movie), the statistician will be forced to attempt to implement Hill's (2000) more general model. In section 12 below, we apply a variant of the expected rentals interpretation of our equations to value intangible capital.

[7] This is the main reason that we use this escalation of cross sectional rental prices approach to capital measurement rather than the more fundamental discounted future expected rentals approach advocated by Hill.

[8] Triplett (1996; 97) used this characterization for capital assets of various vintages.

(3) $i_n^t = i^t$; $\quad n = 1,2,\ldots$

We also assume that the term structure of (nominal) interest rates at the beginning of each period t is constant; i.e., we assume:

(4) $r_n^t = r^t$; $\quad n = 1,2,\ldots$

However, note that as the period t changes, r^t and i^t can change.

Using assumptions (3) and (4), we can rewrite the system of equations (2), which relate the sequence or profile of *stock prices* of age n at the beginning of period t $\{P_n^t\}$ to the sequence or profile of (cross sectional) *rental prices* for assets of age n at the beginning of period t $\{f_n^t\}$, as follows:

$$\begin{aligned}
(5)\ P_0^t &= f_0^t + [(1+i^t)/(1+r^t)]\, f_1^t + [(1+i^t)/(1+r^t)]^2\, f_2^t + [(1+i^t)/(1+r^t)]^3\, f_3^t + \ldots \\
P_1^t &= f_1^t + [(1+i^t)/(1+r^t)]\, f_2^t + [(1+i^t)/(1+r^t)]^2\, f_3^t + [(1+i^t)/(1+r^t)]^3\, f_4^t + \ldots \\
P_2^t &= f_2^t + [(1+i^t)/(1+r^t)]\, f_3^t + [(1+i^t)/(1+r^t)]^2\, f_4^t + [(1+i^t)/(1+r^t)]^3\, f_5^t + \ldots \\
&\ldots \\
P_n^t &= f_n^t + [(1+i^t)/(1+r^t)]\, f_{n+1}^t + [(1+i^t)/(1+r^t)]^2\, f_{n+2}^t + [(1+i^t)/(1+r^t)]^3\, f_{n+3}^t + \ldots
\end{aligned}$$

On the left hand side of equations (5), we have the sequence of period t asset prices by age starting with the price of a new asset, P_0^t, moving to the price of an asset that is one period old at the start of period t, P_1^t, then moving to the price of an asset that is 2 periods old at the start of period t, P_2^t, and so on. On the right hand side of equations (5), the first term in each equation is a member of the sequence of rental prices by age of asset that prevails in the market (if such markets exist) at the beginning of period t. Thus f_0^t is the rent for a new asset, f_1^t is the rent for an asset that is one period old at the beginning of period t, f_2^t is the rent for an asset that is 2 periods old, and so on. This sequence of current market rental prices for the assets of various vintages is then extrapolated out into the future using the anticipated price escalation rates $(1+i^t)$, $(1+i^t)^2$, $(1+i^t)^3$, etc. and then these future expected rentals are discounted back to the beginning of period t using the nominal discount factors $(1+r^t)$, $(1+r^t)^2$, $(1+r^t)^3$, etc. Note that given the period t expected asset inflation rate i^t and the period t nominal discount rate r^t, we can go from the (cross sectional) sequence of vintage rental prices $\{f_n^t\}$ to the (cross sectional) sequence of vintage asset prices $\{P_n^t\}$ using equations (5). We shall show below how this procedure can be reversed; i.e., we shall show how given the sequence of cross sectional asset prices, we can construct estimates for the sequence of cross sectional rental prices.

Böhm-Bawerk (1891; 342) considered a special case of (5) where all service flows f_n were equal to 100 for $n = 0,1,\ldots,6$ and equal to 0 thereafter, where the asset inflation rate was expected to be 0 and where the interest rate r was equal to .05 or 5 %.[9] This is a special case of what has come to be known as the *one hoss shay model* and we shall consider it in more detail in section 7.

Note that equations (5) can be rewritten as follows:[10]

[9] Böhm-Bawerk (1891; 343) went on and constructed the sequence of vintage asset prices using his special case of equations (5).

[10] Christensen and Jorgenson (1969; 302) do this for the geometric depreciation model except that they assume that the rental is paid at the end of the period rather than the beginning. Variants of the system of equations (6) were derived by Christensen and Jorgenson (1973), Jorgenson (1989; 10), Hulten (1990; 128) and Diewert and Lawrence (2000; 276). Irving Fisher (1908; 32-33) also derived these equations in words.

$$
\begin{aligned}
(6)\quad P_0^t &= f_0^t + [(1+i^t)/(1+r^t)]\, P_1^t \quad ; \\
P_1^t &= f_1^t + [(1+i^t)/(1+r^t)]\, P_2^t \quad ; \\
P_2^t &= f_2^t + [(1+i^t)/(1+r^t)]\, P_3^t \quad ; \\
&\ldots \\
P_n^t &= f_n^t + [(1+i^t)/(1+r^t)]\, P_{n+1}^t \; ; \ldots
\end{aligned}
$$

The first equation in (6) says that the value of a new asset at the start of period t, P_0^t, is equal to the rental that the asset can earn in period t, f_0^t,[11] plus the expected asset value of the capital good at the end of period t, $(1+i^t)\, P_1^t$, but this expected asset value must be divided by the discount factor, $(1+r^t)$, in order to convert this future value into an equivalent beginning of period t value.[12]

Now it is straightforward to solve equations (6) for the sequence of period t cross sectional rental prices, $\{f_n^t\}$, in terms of the cross sectional asset prices, $\{P_n^t\}$:

$$
\begin{aligned}
(7)\quad f_0^t &= P_0^t - [(1+i^t)/(1+r^t)]\, P_1^t &&= (1+r^t)^{-1}\, [P_0^t\,(1+r^t) - (1+i^t)\, P_1^t] \\
f_1^t &= P_1^t - [(1+i^t)/(1+r^t)]\, P_2^t &&= (1+r^t)^{-1}\, [P_1^t\,(1+r^t) - (1+i^t)\, P_2^t] \\
f_2^t &= P_2^t - [(1+i^t)/(1+r^t)]\, P_3^t &&= (1+r^t)^{-1}\, [P_2^t\,(1+r^t) - (1+i^t)\, P_3^t] \\
&\ldots \\
f_n^t &= P_n^t - [(1+i^t)/(1+r^t)]\, P_{n+1}^t &&= (1+r^t)^{-1}\, [P_n^t\,(1+r^t) - (1+i^t)\, P_{n+1}^t] \; ; \ldots
\end{aligned}
$$

Thus equations (5) allow us to go from the sequence of rental prices by age n $\{f_n^t\}$ to the sequence of asset prices by age n $\{P_n^t\}$ while equations (7) allow us to reverse the process.

Equations (7) can be derived from elementary economic considerations. Consider the first equation in (7). Think of a production unit as purchasing a unit of the new capital asset at the beginning of period t at a cost of P_0^t and then using the asset throughout period t. However, at the end of period t, the producer will have a depreciated asset that is expected to be worth $(1+i^t)\, P_1^t$. Since this offset to the initial cost of the asset will only be received at the end of period t, it must be divided by $(1+r^t)$ to express the benefit in terms of beginning of period t dollars. Thus the expected net cost of *using* the new asset for period t[13] is $P_0^t - [(1+i^t)/(1+r^t)]\, P_1^t$.

The above equations assume that the actual or implicit period t rental payments f_n^t for assets of different ages n are made at the *beginning* of period t. It is sometimes convenient to assume that the rental payments are made at the *end* of each accounting period. Thus we define the *end of period t rental price or user cost* for an asset that is n periods old at the beginning of period t, u_n^t, in terms of the corresponding *beginning of period t rental price* f_n^t as follows:

$$(8)\quad u_n^t \equiv (1+r^t)\, f_n^t \;\; ; \;\; n = 0,1,2,\ldots$$

[11] Note that we are implicitly assuming that the rental is paid to the owner at the beginning of period t.

[12] Another way of interpreting say the first equation in (6) runs as follows: the purchase cost of a new asset P_0^t less the rental f_0^t (which is paid immediately at the beginning of period t) can be regarded as an investment, which must earn the going rate of return r^t. Thus we must have $[P_0^t - f_0^t](1+r^t) = (1+i^t)P_1^t$ which is the (expected) value of the asset at the end of period t. This line of reasoning can be traced back to Walras (1954; 267).

[13] This explains why the rental prices f_n^t are sometimes called *user costs*. This derivation of a user cost was used by Diewert (1974; 504), (1980; 472-473), (1992a; 194) and by Hulten (1996; 155).

Thus if the rental payment is made at the end of the period instead of the beginning, then the beginning of the period rental f_n^t must be escalated by the interest rate factor $(1+r^t)$ in order to obtain the end of the period user cost u_n^t.

Using equations (8) and the second set of equations in (7), it can readily be shown that the sequence of end of period t user costs $\{u_n^t\}$ can be defined in terms of the period t sequence of asset prices by age $\{P_n^t\}$ as follows:

$$
\begin{aligned}
(9)\quad u_0^t &= P_0^t\,(1+r^t) - (1+i^t)\,P_1^t \\
u_1^t &= P_1^t\,(1+r^t) - (1+i^t)\,P_2^t \\
u_2^t &= P_2^t\,(1+r^t) - (1+i^t)\,P_3^t \\
&\ldots \\
u_n^t &= P_n^t\,(1+r^t) - (1+i^t)\,P_{n+1}^t\,;\ \ldots
\end{aligned}
$$

Equations (9) can also be given a direct economic interpretation. Consider the following explanation for the user cost for a new asset, u_0^t. At the end of period t, the business unit expects to have an asset worth $(1+i^t)\,P_1^t$. Offsetting this benefit is the beginning of the period asset purchase cost, P_0^t. However, in addition to this cost, the business must charge itself either the explicit interest cost that occurs if money is borrowed to purchase the asset or the implicit opportunity cost of the equity capital that is tied up in the purchase. Thus offsetting the end of the period benefit $(1+i^t)\,P_1^t$ is the initial purchase cost and opportunity interest cost of the asset purchase, $P_0^t\,(1+r^t)$, leading to a end of period t net cost of $P_0^t\,(1+r^t) - (1+i^t)\,P_1^t$ or u_0^t.

It is interesting to note that in both the accounting and financial management literature of the past century, there was a reluctance to treat the opportunity cost of *equity capital* tied up in capital inputs as a genuine cost of production.[14] However, more recently, there is an acceptance of an imputed interest charge for equity capital as a genuine cost of production.[15]

In the following section, we will relate the asset price profiles $\{P_n^t\}$ and the user cost profiles $\{u_n^t\}$ to *depreciation profiles*. However, before turning to the subject of depreciation, it is important to stress that the analysis presented in this section is based on a number of restrictive assumptions, particularly on future price expectations. Moreover, we have not explained how these asset price expectations are formed and we have not explained how the period t nominal interest rate is to be estimated (we will address these topics in section 7 below). We have not explained what should be done if the sequence of second hand asset prices $\{P_n^t\}$ is not available and the sequences of vintage rental prices or user costs, $\{f_n^t\}$ or $\{u_n^t\}$, are also not available (we will address this problem in later sections as well). We have also assumed that asset values and user costs are independent of how intensively the assets are used. Finally, we have not modeled uncertainty (about future prices and the useful lives of assets) and attitudes towards risk on the part of producers. Thus the analysis presented in this chapter is only a start on the difficult problems associated with measuring capital input.

3. Cross Section Depreciation Profiles

Recall that in the previous section, P_n^t was defined to be the price of an asset that was n periods old at the beginning of period t. Generally, the decline in asset value as we go

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[14] This literature is reviewed in Diewert and Fox (1999; 271-274).

[15] Stern Stewart & Co. has popularized the idea of charging for the opportunity cost of equity capital and has called the resulting income concept, EVA, Economic Value Added.

from one vintage to the next oldest is called *depreciation*. More precisely, we define the *cross section depreciation* D_n^t [16] of an asset that is n periods old at the beginning of period t as

(10) $D_n^t \equiv P_n^t - P_{n+1}^t$; $n = 0,1,2,\ldots$

Thus D_n^t is the value of an asset that is n periods old at the beginning of period t, P_n^t, minus the value of an asset that is n+1 periods old at the beginning of period t, P_{n+1}^t.

Obviously, given the sequence of period t cross section asset prices $\{P_n^t\}$, we can use equations (10) to determine the period t sequence of declines in asset values by age, $\{D_n^t\}$. Conversely, given the period t cross section depreciation sequence or *profile*, $\{D_n^t\}$, we can determine the period t asset prices by age n by adding up amounts of depreciation:

(11) $P_0^t = D_0^t + D_1^t + D_2^t + \ldots$
$P_1^t = D_1^t + D_2^t + D_3^t + \ldots$
$\ldots$
$P_n^t = D_n^t + D_{n+1}^t + D_{n+2}^t + \ldots$

Rather than working with first differences of asset prices by age, it is more convenient to reparameterize the pattern of cross section depreciation by defining the *period t depreciation rate* δ_n^t for an asset that is n periods old at the start of period t as follows:

(12) $\delta_n^t \equiv 1 - [P_{n+1}^t/P_n^t] = D_n^t / P_n^t$; $n = 0,1,2,\ldots$

In the above definitions, we require n to be such that P_n^t is positive.[17]

Obviously, given the sequence of period t asset prices by age n, $\{P_n^t\}$, we can use equations (12) to determine the period t sequence of *cross section depreciation rates*, $\{\delta_n^t\}$. Conversely, given the cross section sequence of period t depreciation rates, $\{\delta_n^t\}$, as well as the price of a new asset in period t, P_0^t, we can determine the period t asset prices by age as follows:

(13) $P_1^t = (1 - \delta_0^t)\, P_0^t$
$P_2^t = (1 - \delta_0^t)(1 - \delta_1^t)\, P_0^t$
$\ldots$
$P_n^t = (1 - \delta_0^t)(1 - \delta_1^t)\ldots(1 - \delta_{n-1}^t)\, P_0^t$; $\ldots$

The interpretation of equations (13) is straightforward. At the beginning of period t, a new capital good is worth P_0^t. An asset of the same type but which is one period older at the beginning of period t is less valuable by the amount of depreciation $\delta_0^t P_0^t$ and hence

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[16] This terminology is due to Hill (1999) who distinguished the decline in second hand asset values due to aging (cross section depreciation) from the decline in an asset value over a period of time (time series depreciation). Triplett (1996; 98-99) uses the cross section definition of depreciation (calling it deterioration) and shows that it is equal to the concept of capital consumption in the national accounts but he does this under the assumption of no expected real asset price change. We will examine the relationship of cross section to time series depreciation in section 5 below.

[17] This definition of depreciation dates back to Hicks (1939 ;176) at least and was used extensively by Edwards and Bell (1961; 175), Hulten and Wykoff (1981a) (1981b) (who call it deterioration), Diewert (1974; 504) and Hulten (1990; 128) (1996; 155).

is worth $(1 - \delta_0^t)\, P_0^t$, which is equal to P_1^t. An asset which is two periods old at the beginning of period t is less valuable than a one period old asset by the amount of depreciation $\delta_1^t\, P_1^t$ and hence is worth $P_2^t = (1 - \delta_1^t)\, P_1^t$ which is equal to $(1 - \delta_1^t)(1 - \delta_0^t)\, P_0^t$ using the first equation in (13) and so on. Suppose $L \geq 1$ is the first integer which is such that δ_{L-1}^t is equal to one. Then P_n^t equals zero for all $n \geq L$; i.e., at the end of L periods of use, the asset no longer has a positive rental value. If L = 1, then a new asset of this type delivers all of its services in the first period of use and the asset is in fact a nondurable asset.

Now substitute equations (12) into equations (9) in order to obtain the following formulae for the sequence of the *end of the period t user costs by age n*, $\{u_n^t\}$, in terms of the price of a new asset at the beginning of period t, P_0^t, and the sequence of cross section depreciation rates, $\{\delta_n^t\}$:

(14) $u_0^t = [(1+r^t) - (1+i^t)(1 - \delta_0^t)]\, P_0^t$

$u_1^t = (1 - \delta_0^t)[(1+r^t) - (1+i^t)(1 - \delta_1^t)]\, P_0^t$

…

$u_n^t = (1 - \delta_0^t)\ldots(1 - \delta_{n-1}^t)[(1+r^t) - (1+i^t)(1 - \delta_n^t)]\, P_0^t$; …

Thus given P_0^t (the beginning of period t price of a new asset), i^t (the nominal rate of new asset price change that is expected at the beginning of period t), r^t (the one period nominal interest rate that the business unit faces at the beginning of period t) and given the sequence of cross section vintage depreciation rates prevailing at the beginning of period t (the δ_n^t), then we can use equations (14) to calculate the sequence of the end of the period user costs for period t, the u_n^t. Of course, given the u_n^t, we can use equations (8) to calculate the beginning of the period user costs (the f_n^t) and then use the f_n^t to calculate the sequence of asset prices by age P_n^t using equations (5) and finally, given the P_n^t, we can use equations (12) in order to calculate the sequence of depreciation rates for assets of age n at the beginning of period t, the δ_n^t. Thus *given any one of these sequences or profiles, all of the other sequences are completely determined*. This means that assumptions about depreciation rates, the pattern of user costs by age of asset or the pattern of asset prices by age of asset *cannot be made independently of each other*.[18]

It is useful to look more closely at the first equation in (14), which expresses the user cost or rental price of a new asset at the end of period t, u_0^t, in terms of the depreciation rate δ_0^t, the one period nominal interest rate r^t, the new asset inflation rate i^t that is expected to prevail at the beginning of period t and the beginning of period t price for a new asset, P_0^t:

(15) $u_0^t = [(1+r^t) - (1+i^t)(1 - \delta_0^t)]\, P_0^t = [r^t - i^t + (1+ i^t)\delta_0^t]\, P_0^t$.

Thus the user cost of a new asset u_0^t that is purchased at the beginning of period t (and the actual or imputed rental payment is made at the end of the period) is equal to $r^t - i^t$ (a nominal interest rate minus an asset inflation rate which can be loosely interpreted[19] as a *real interest rate*) times the initial asset cost P_0^t plus $(1+ i^t)\delta_0^t P_0^t$ which is *depreciation* on

[18] This point was first made explicitly by Jorgenson and Griliches (1967; 257); see also Jorgenson and Griliches (1972; 81-87). Much of the above algebra for switching from one method of representing vintage capital inputs to another was first developed by Christensen and Jorgenson (1969; 302-305) (1973) for the geometrically declining depreciation model. The general framework for an internally consistent treatment of capital services and capital stocks in a set of vintage accounts was set out by Jorgenson (1989) and Hulten (1990; 127-129) (1996; 152-160).

[19] We will provide a more precise definition of a real interest rate later.

the asset at beginning of the period prices, $\delta_1^t P_0^t$, times *one plus the expected rate of asset price change*, $(1+ i^t)$.[20] If we further assume that the expected rate of asset price change i^t is 0, then (15) further simplifies to:

(16) $u_0^t = [r^t + \delta_0^t]\, P_0^t$.

Under these assumptions, the user cost of a new asset is equal to the interest rate plus the depreciation rate times the initial purchase price.[21] This is essentially the user cost formula that was obtained by Walras (1954; 268-269) in 1874.

However, the basic idea that a durable input should be charged a period price that is equal to a depreciation term plus a term that would cover the cost of financial capital goes back to Babbage (1835; 287) and others[22].

Babbage did not proceed further with the user cost idea. Walras seems to have been the first economist who formalized the idea of a user cost into a mathematical formula. However, the early industrial engineering literature also independently came up with the user cost idea; Church (1901; 734 and 907-908) in particular gave a very modern exposition of the ingredients needed to construct user costs or machine rents.

Church was well aware of the importance of determining the "right" rate to be charged for the use of a machine in a multiproduct enterprise. This information is required not only to price products appropriately but to determine whether an enterprise should make or purchase a particular commodity. Babbage (1835; 203) and Canning (1929; 259-260) were also aware of the importance of determining the right machine rate charge:[23]

The above equations relating asset prices by age n, P_n^t, beginning of the period user costs by age n, f_n^t, end of the period user costs, u_n^t, and the (cross section) depreciation rates δ_n^t are the fundamental ones that we will specialize in subsequent sections in order to measure both wealth capital stocks and capital services under conditions of inflation. In

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[20] This formula was obtained by Christensen and Jorgenson (1969; 302) for the geometric model of depreciation but it is valid for any depreciation model. Griliches (1963; 120) also came very close to deriving this formula in words: "In a perfectly competitive world the annual rent of a machine would equal the marginal product of its services. The rent itself would be determined by the interest costs on the investment, the deterioration in the future productivity of the machine due to current use, and the expected change in the price of the machine (obsolescence)."

[21] Using equations (13) and (14) and the assumption that the asset inflation rate $i^t = 0$, it can be shown that the user cost of an asset that is n periods old at the start of period t can be written as $u_n^t = (r^t + \delta_n^t)P_n^t$ where P_n^t is the beginning of period t second hand market price for the asset.

[22] Solomons (1968; 9-17) indicates that interest was regarded as a cost for a durable input in much of the nineteenth century accounting literature. The influential book by Garcke and Fells (1893) changed this.

[23] Under moderate inflation, the difficulties with traditional cost accounting based on historical cost and no proper allowance for the opportunity of capital, the proper pricing of products becomes very difficult. Diewert and Fox (1999; 271-274) argued that this factor contributed to the great productivity slowdown that started around 1973 and persisted to the early 1990's. The traditional method of cost accounting can be traced back to a book first published in 1887 by the English accountants, Garcke and Fells (1893; 70-71). Their rather crude approach to cost accounting should be compared to the masterful analysis of Church! Garcke and Fells (1893; 72-73) endorsed the idea that deprecation was an admissible item of cost that should be allocated in proportion to the prime cost (i.e., labour and materials cost) of manufacturing an article but they explicitly ruled out interest as a cost. The aversion of accountants to include interest as a cost can be traced back to the influence of Garcke and Fells.

the following section, we shall consider several options that could be used in order to determine empirically the interest rates r^t and the expected asset rates of price change i^t.

4. The Empirical Determination of Interest Rates and Rates of Asset Price Change

We consider initially three broad approaches[24] to the determination of the nominal interest rate r^t that is to be used to discount future period value flows by the business units in the aggregate under consideration:

- Use the ex post rate of return that will just make the sum of the user costs exhaust the gross operating surplus of the production sectors for the aggregate under consideration.
- Use an aggregate of nominal interest rates that the production sectors in the aggregate might be facing at the beginning of each period.
- Take a fixed real interest rate and add to it actual ex post consumer price inflation or anticipated consumer price inflation.

The first approach was used for the entire private production sector of the economy by Jorgenson and Griliches (1967; 267) and for various sectors of the economy by Christensen and Jorgenson (1969; 307). It is also widely used by statistical agencies. It has the advantage that the value of output for the sector will exactly equal the value of input in a consistent accounting framework. It has the disadvantages that it is subject to measurement error and it is an *ex post rate of return* which may not reflect the economic conditions facing producers at the beginning of the period. This approach (incorrectly in our view) transforms pure profits (or losses) into a change in the opportunity cost of financial capital.

The second approach suffers from aggregation problems. There are many interest rates in an economy at the beginning of an accounting period and the problem of finding the "right" aggregate of these rates is not a trivial one.

The third approach works as follows. Let the consumer price index for the economy at the beginning of period t be c^t say. Then the ex post general consumer inflation rate for period t is ρ^t defined as:

(17) $1 + \rho^t \equiv c^{t+1}/c^t$.

Let the production units under consideration face the real interest rate r^{*t}. Then by the Fisher (1896) effect, the relevant nominal interest rate that the producers face should be approximately equal to r^t defined as follows:

(18) $r^t \equiv (1+r^{*t})(1+\rho^t) - 1$.

The Australian Bureau of Statistics assumes that producers face a real interest rate of 4 per cent. This is consistent with long run observed economy wide real rates of return for most OECD countries which fall in the 3 to 5 per cent range. We shall choose this third method for defining nominal interest rates and choose the real rate of return to be 4 % per

[24] Other methods for determining the appropriate interest rates that should be inserted into user cost formulae are discussed by Harper, Berndt and Wood (1989) and in Chapter 5 of Schreyer (2001). Harper, Berndt and Wood (1989) evaluate empirically 5 alternative rental price formulae using geometric depreciation but making different assumptions about the interest rate and the treatment of asset price change. They show that the choice of formula matters (as we will later).

annum; i.e., we assume that the nominal rate r^t is defined by (18) with the real rate defined by

$$(19)\ r^{*t} \equiv .04$$

assuming that the accounting period chosen is a year.[25]

We turn now to the determination of the asset expected rates of price change[26], the i^t, which appear in most of the formulae derived in the preceding sections of this chapter. There are three broad approaches that can be used in this context:

- Use actual ex post rates of price change for a new asset over each period.
- Assume that each asset rate of price change is equal to the general inflation rate for each period.
- Estimate anticipated rates of asset price change for each period.

In what follows, we will compute cross sectional user costs using Canadian data on investments for two broad classes of assets (nonresidential construction and machinery and equipment) for 4 different sets of assumptions about depreciation or the relative efficiency of assets by age. We will undertake these computations in an inflationary environment and make each of the three sets of assumptions about the asset inflation rates listed above for each of the 4 depreciation models, giving 12 models in all that will be compared. If the various models give very different results, this indicates that the statistical agency computing capital stocks and service flows under inflation must choose its preferred model with some care.

When we assume that the rate of price change for each asset is equal to the general inflation rate ρ^t defined by (17), the equations presented earlier simplify. Thus if we replace $1+i^t$ by $1+\rho^t$ and $1+r^t$ by $(1+r^*)(1+\rho^t)$, equations (5), which relate the period t asset prices by age n P_n^t to the rental prices f_n^t, become:

$$
\begin{aligned}
(20)\ P_0^t &= f_0^t + [1/(1+r^*)]\, f_1^t + [1/(1+r^*)]^2\, f_2^t + [1/(1+r^*)]^3\, f_3^t + \ldots \\
P_1^t &= f_1^t + [1/(1+r^*)]\, f_2^t + [1/(1+r^*)]^2\, f_3^t + [1/(1+r^*)]^3\, f_4^t + \ldots \\
&\ldots \\
P_n^t &= f_n^t + [1/(1+r^*)]\, f_{n+1}^t + [1/(1+r^*)]^2\, f_{n+2}^t + [1/(1+r^*)]^3\, f_{n+3}^t + \ldots
\end{aligned}
$$

Note that only the constant real interest rate r* appears in these equations.

If we replace $1+i^t$ by $1+\rho^t$ and $1+r^t$ by $(1+r^*)(1+\rho^t)$, equations (14), which relate *the end of period user costs* u_n^t to the depreciation rates δ_n^t, become:

$$
\begin{aligned}
(21)\ u_0^t &= (1+\rho^t)[(1+r^*) - (1-\delta_0^t)]\, P_0^t = (1+\rho^t)[r^* + \delta_0^t]\, P_0^t \\
u_1^t &= (1+\rho^t)(1-\delta_0^t)[(1+r^*) - (1-\delta_1^t)]\, P_0^t = (1+\rho^t)(1-\delta_0^t)[r^* + \delta_1^t]\, P_0^t \\
&\ldots \\
u_n^t &= (1+\rho^t)(1-\delta_0^t)\ldots(1-\delta_{n-1}^t)[(1+r^*) - (1-\delta_n^t)]\, P_0^t \\
&= (1+\rho^t)(1-\delta_0^t)\ldots(1-\delta_{n-1}^t)\, [r^* + \delta_n^t]\, P_0^t.
\end{aligned}
$$

---

[25] If we are in a high inflation situation so that the accounting period becomes a quarter or a month, then r^{*t} must be chosen to be appropriately smaller.

[26] These are sometimes called *revaluation terms* in user cost formulae.

Now use equations (8) and $1+r^t = (1+r^*)(1+\rho^t)$ and substitute into (21) to obtain the following equations, which relate the *beginning of period user costs* f_n^t to the depreciation rates δ_n^t:

$$(22)\ f_0^t = (1+r^*)^{-1}[r^* + \delta_0^t]\, P_0^t$$
$$f_1^t = (1+r^*)^{-1}(1 - \delta_0^t)[r^* + \delta_1^t]\, P_0^t$$
$$\ldots$$
$$f_n^t = (1+r^*)^{-1}(1 - \delta_0^t)\ldots(1 - \delta_{n-1}^t)\, [r^* + \delta_n^t]\, P_0^t.$$

Note that only the constant real interest rate r* appears in equations (22) but equations (21) also have the general inflation rate $(1+\rho^t)$ as a multiplicative factor.

As mentioned above, in our third class of assumptions about rates of asset price change, we want to estimate *anticipated rates of asset price change* and use these estimates as our i^t in the various formulae we have exhibited. Unfortunately, there are any number of forecasting methods that could be used to estimate the anticipated asset rates of price change. We will take a somewhat different approach than a pure forecasting one: we will simply *smooth* the observed ex post new asset rates of price change and use these smoothed rates as our estimates of anticipated rates.[27] A similar forecasting problem arises when we use ex post actual consumer price index inflation rates (recall (17) and (18) above) in order to generate anticipated general inflation rates. Thus in our third set of models, we will use both smoothed asset inflation rates and smoothed general inflation rates as our estimates for anticipated rates. In our first class of models, we will use actual ex post rates in both cases.

Before we proceed to consider our four specific depreciation models, we briefly consider in the next section a topic of some current interest: namely the interaction of (foreseen) obsolescence and depreciation. We also discuss cross section versus time series depreciation.

5. Obsolescence and Depreciation

We begin this section with a definition of the time series depreciation of an asset. Define the *ex post time series depreciation* of an asset that is n periods old at the beginning of period t, E_n^t, to be its second hand market price at the beginning of period t, P_n^t, less the price of an asset that is one period older at the beginning of period t+1, P_{n+1}^{t+1}; i.e.,

$$(23)\ E_n^t \equiv P_n^t - P_{n+1}^{t+1} \quad ; n = 0,1,2,\ldots$$

Definitions (23) should be contrasted with our earlier definitions (10), which defined the *cross section amounts of depreciation* for the same assets at the beginning of period t, $D_n^t \equiv P_n^t - P_{n+1}^t$.

We can now explain why we preferred to work with the cross section definition of depreciation, (10), over the time series definition, (23). The problem with (23) is that time series depreciation captures the effects of changes in *two* things: changes in *time*

[27] Unfortunately, different analysts may choose different smoothing methods so there may be a problem of a lack of reproducibility in our estimating procedures. Harper, Berndt and Wood (1989; 351) note that the use of time series techniques to smooth ex post asset inflation rates and the use of such estimates as anticipated price change dates back to Epstein (1977).

(this is the change in t to t+1)[28] and changes in the *age* of the asset (this is the change in n to n+1).[29] Thus time series depreciation aggregates together two effects: the asset specific price change that occurred between time t and time t+1 (asset revaluation due to general inflation and asset specific price change) *and* the effects of asset aging (depreciation). Thus the time series definition of depreciation combines together two distinct effects.

The above definition of ex post time series depreciation is the original definition of depreciation and it extends back to the very early beginnings of accounting theory.[30]

However, what has to be kept in mind that these early authors who used the concept of time series depreciation were implicitly or explicitly assuming *that prices were stable across time*, in which case, time series and cross section depreciation coincide.

P. Hill (2000; 6) and Hill and Hill (2003; 617)[31] recently argued that a form of time series depreciation that included expected obsolescence was to be preferred over cross section depreciation for national accounts purposes. Since the depreciation rates δ_n^t defined by (12) are cross section depreciation rates and they play a key role in the beginning and end of period t user costs f_n^t and u_n^t defined by (14), (21) and (22), it is necessary to clarify their use in the context of Hill's point that these depreciation rates should *not* be used to measure depreciation in the national accounts.

Our response to the Hill critique is twofold:

- Cross section depreciation rates as we have defined them *are* affected by anticipated obsolescence in principle but
- Hill is correct in arguing that cross section depreciation will not generally equal ex post time series depreciation or anticipated time series depreciation.

Before discussing the above two points in detail, it is necessary to discuss the concept of obsolescence in a bit more detail. Wykoff (2004), in his discussion of this chapter, takes a narrow "technological" definition of obsolescence. In his view, an asset can only become obsolete if a new model of the asset becomes available which can deliver at least the service flow of the old asset at a lower price. In his view, if there is no technological change embodied in the new asset, then by definition, there is no obsolescence. However, it is possible to define obsolescence more broadly and include the effects of changes in the economy that reduce the demand for the asset's services to such an extent that its real price falls.[32] In what follows, we will use the second broader concept of

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[28] This change could be captured by either $P_n^t - P_n^{t+1}$ or $P_{n+1}^t - P_{n+1}^{t+1}$.

[29] This change could be captured by either $P_n^t - P_{n+1}^t$ or $P_n^{t+1} - P_{n+1}^{t+1}$.

[30] See for example Matheson (1910; 35) and Hotelling (1925; 341).

[31] We agree in general with P. Hill (2000) and Hill and Hill (2003) that expected obsolescence should be added to cross sectional depreciation to form an overall depreciation charge. However, Hill and Hill assumed that there was no general inflation in their exposition so some clarification is needed to deal with this complication.

[32] This broader definition goes back to Church at least: "Even though a machine is used fairly and uniformly as contemplated when the rate of depreciation was fixed there is another influence that may shorten its period of usefulness in an unexpected way. The progress of the technical art in which it is employed may develop more efficient machines for doing the same work, so that it becomes advisable to scrap it long before it is worn out. The machine becomes obsolete and the loss of value from this cause is called 'obsolescence'. Again, unless the machine is of a very generalized type, such as an engineer's lathe, another type of misfortune may overtake it. If it is a machine that can only be used for certain definite

obsolescence. One more point must be considered at this point. If there is technological obsolescence due to a new and improved model of the asset being made available, then we assume that the price of the new model has been (somehow) quality adjusted so that the quality adjusted price is measured in quantity units that are comparable to the older models.

Now consider the first dot point above. Provisionally, we define *anticipated obsolescence* as a situation where the expected new asset rate of price change (adjusted for quality change) i^t is *negative*.[33] For example, everyone anticipates that the quality adjusted price for a new computer next quarter will be considerably lower than it is this quarter.[34] Now turn back to equations (5) above, which define the profile of vintage asset prices P_n^t at the start of period t. It is clear that the negative i^t plays a role in defining the sequence of vintage asset prices as does the sequence of vintage rental prices that is observed at the beginning of period t, the f_n^t. Thus in this sense, cross sectional depreciation rates certainly embody assumptions about anticipated obsolescence.

Thus for an asset that has a finite life, as we move down the rows of equations (5), the number of discounted rental terms decline and hence asset value declines, which is Griliches' (1963; 119) concept of *exhaustion*. If the cross sectional rental prices are monotonically declining (due to their declining efficiency), then as we move down the rows of equations (5), the higher rental terms are being dropped one by one so that the asset values will also decline from this factor, which is Griliches' (1963; 119) concept of *deterioration*. Finally, a negative anticipated asset inflation rate will cause all future period rentals to be discounted more heavily, which could be interpreted as Griliches' (1963; 119) concept of *obsolescence*.[35] Thus all of these explanatory factors are imbedded in equations (5).

Now consider the second dot point: that cross section depreciation is not really adequate to measure time series depreciation in some sense to be determined.

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kinds of work or some special article, as for example many of the machines used in automobile and bicycle manufacture, it may happen that changes in demand, or in style, make the manufacture of that special article no longer profitable. In this case, unless the machine can be transformed for another use, it is a dead loss." A.H. Church (1917; 192-193).

[33] Paul Schreyer and Peter Hill noted a problem with this provisional definition of anticipated obsolescence as a negative value of the expected asset inflation rate: it will not work in a high inflation environment. In a high inflation environment, the nominal asset inflation rate i^t will generally be positive but we will require this nominal rate to be less than general inflation in order to have anticipated obsolescence. Thus our final definition of *anticipated obsolescence* is that the real asset inflation rate i^{*t} defined later by (28) be negative; see the discussion just above equation (30) below.

[34] Our analysis assumes that the various vintages of capital are adjusted for quality change (if any occurs) as they come on the market. In terms of our Canadian empirical example to follow, we are assuming that Statistics Canada correctly adjusted the published investment price deflators for machinery and equipment and nonresidential construction for quality change. We also need to assume that the form of quality change affects all future efficiency factors (i.e., the f_n^t) in a proportional manner. This is obviously only a rough approximation to reality: technical change may increase the durability of a capital input or it may decrease the amount of maintenance or fuel that is required to operate the asset. These changes can lead to nonproportional changes in the f_n^t.

[35] However, it is more likely that what Griliches had in mind was Hill's second point; i.e., that time series depreciation will be larger than cross section depreciation in a situation where i^{*t} is negative.

Define the *ex ante time series depreciation* of an asset that is n periods old at the beginning of period t, ε_n^t, to be its second hand market price at the beginning of period t, P_n^t, less the *anticipated price* of an asset that is one period older at the beginning of period t+1, $(1+i^t)\,P_{n+1}^t$; i.e.,

$$(24)\ \varepsilon_n^t \equiv P_n^t - (1+i^t)\,P_{n+1}^t \quad ; n = 0,1,2,\ldots$$

Thus anticipated time series depreciation for an asset that is t periods old at the start of period t, ε_n^t, differs from the corresponding cross section depreciation defined by (10), $D_n^t \equiv P_n^t - P_{n+1}^t$, in that the anticipated new asset rate of price change, i^t, is missing from D_n^t. However, *note that the two forms of depreciation will coincide if the expected asset rate of price change* i^t *is zero.*

We can use equations (12) and (13) in order to define the ex ante depreciation amounts ε_n^t in terms of the cross section depreciation rates δ_n^t. Thus using definitions (24), we have:

$$\begin{aligned}
(25)\ \varepsilon_n^t &\equiv P_n^t - (1+i^t)\,P_{n+1}^t && n = 0,1,2,\ldots\\
&= P_n^t - (1+i^t)(1-\delta_n^t)\,P_n^t && \text{using (12)}\\
&= [1 - (1+i^t)(1-\delta_n^t)]\,P_n^t &&\\
&= (1-\delta_1^t)(1-\delta_2^t)\ldots(1-\delta_{n-1}^t)[1 - (1+i^t)(1-\delta_n^t)]\,P_0^t && \text{using (13)}\\
&= (1-\delta_1^t)(1-\delta_2^t)\ldots(1-\delta_{n-1}^t)[\delta_n^t - i^t(1-\delta_n^t)]\,P_0^t. &&
\end{aligned}$$

We can compare the above sequence of ex ante time series depreciation amounts ε_n^t with the corresponding sequence of cross section depreciation amounts:

$$\begin{aligned}
(26)\ D_n^t &\equiv P_n^t - P_{n+1}^t && n = 0,1,2,\ldots\\
&= P_n^t - (1-\delta_n^t)\,P_n^t && \text{using (12)}\\
&= [1 - (1-\delta_n^t)]\,P_n^t &&\\
&= (1-\delta_1^t)(1-\delta_2^t)\ldots(1-\delta_{n-1}^t)[\delta_n^t]\,P_0^t && \text{using (13).}
\end{aligned}$$

Of course, if the anticipated rate of asset price change i^t is zero, then (25) and (26) coincide and ex ante time series depreciation equals cross section depreciation. If we are in the provisional expected obsolescence case with i^t negative, then it can be seen comparing (25) and (26) that $\varepsilon_n^t > D_n^t$ for all n such that $D_n^t > 0$; i.e., if i^t is negative (and $0 < \delta_n^t < 1$), then ex ante time series depreciation exceeds cross section depreciation over all in use vintages of the asset. If i^t is *positive* so that the rental price of each vintage is expected to rise in the future , then ex ante time series depreciation is *less* than the corresponding cross section depreciation for all assets that have a positive price at the end of period t. This corresponds to the usual result in the vintage user cost literature where capital gains or an ex post price increase for a new asset leads to a negative term in the user cost formula (plus a revaluation of the cross section depreciation rate). Here we are restricting ourselves to *anticipated* capital gains rather than the *actual* ex post capital gains and we are focusing on depreciation concepts rather than the full user cost.

This is not quite the end of the story in the high inflation context. National income accountants often readjust asset values at either the beginning or end of the accounting period to take into account general price level change. At the same time, they also want to decompose nominal interest payments into a real interest component and another component that compensates lenders for general price change. So r^{*t}

Recall (17), which defined the general period t inflation rate ρ^t and (18), which related the period t nominal interest rate r^t to the real rate r^{*t} and the inflation rate ρ^t. We rewrite (18) as follows:

(27) $1 + r^{*t} \equiv (1 + r^t)/(1 + \rho^t)$.

In a similar manner, we define the period t anticipated rate of real asset price change i^{*t} as follows:

(28) $1 + i^{*t} \equiv (1 + i^t)/(1 + \rho^t)$.

Recall definition (24), which defined the *ex ante time series depreciation* of an asset that is n periods old at the beginning of period t, Δ_n^t. The first term in this definition reflects the price level at the beginning of period t while the second term in this definition reflects the price level at the end of period t. We now express the second term in terms of the beginning of period t price level. Thus we define the *ex ante real time series depreciation* of an asset that is n periods old at the beginning of period t, Γ_n^t, as follows:

(29) $\Gamma_n^t \equiv P_n^t - (1+i^t)\, P_{n+1}^t/(1+\rho^t)$ $\quad n = 0,1,2,\ldots$
$= P_n^t - (1+i^t)(1-\delta_n^t)\, P_n^t/(1+\rho^t)$ $\quad$ using (12)
$= [(1+\rho^t) - (1+i^{*t})(1+\rho^t)(1-\delta_n^t)]\, P_n^t/(1+\rho^t)$ $\quad$ using (28)
$= (1-\delta_0^t)(1-\delta_1^t) \ldots (1-\delta_{n-1}^t)[1 - (1+i^{*t})(1-\delta_n^t)]\, P_0^t$ $\quad$ using (13)
$= (1-\delta_0^t)(1-\delta_1^t) \ldots (1-\delta_{n-1}^t)[\delta_n^t - i^{*t}(1-\delta_n^t)]\, P_0^t$.

The ex ante real time series depreciation amount Γ_n^t defined by (29) can be compared to its cross section counterpart D_n^t, defined by (25) above. Of course, if the real anticipated asset inflation rate i^{*t} is zero, then (29) and (25) coincide and real ex ante time series depreciation equals cross section depreciation.

We are now in a position to provide a more satisfactory definition of expected obsolescence, particularly in the context of high inflation. We now define *expected obsolescence* to be the situation where the *real* rate of asset price change i^{*t} is negative. If this real rate is negative, then it can be seen comparing (29) and (26) that

(30) $\Gamma_n^t > D_n^t$ $\quad$ for all n such that $D_n^t > 0$;

i.e., real anticipated time series depreciation exceeds the corresponding cross section depreciation provided that i^{*t} is negative.

Thus the general user cost formulae that we have developed from the vintage accounts point of view can be reconciled to reflect the point of view of national income accountants. We agree with Hill's point of view that cross section depreciation is not really adequate to measure time series depreciation as national income accountants have defined it since Pigou (1935; 240-241).

Pigou (1924) in an earlier work had a more complete discussion of the obsolescence problem and the problems involved in defining time series depreciation in an inflationary environment. Pigou (1924; 34-35) first pointed out that the national dividend or net annual income (or in modern terms, real net output) should subtract depreciation or capital consumption. Pigou (1924; 39-41) then went on to discuss the roles of obsolescence and general price change in measuring depreciation. Pigou was responsible for many of the conventions of national income accounting that persist down to the present day. He essentially argued that (unanticipated) capital gains or losses be excluded from income and that the effects of general price level change be excluded from estimates of depreciation. He also argued for the inclusion of (foreseen) obsolescence in depreciation. Unfortunately, he did not spell out *exactly* how all of this could be done in the accounts. Our algebra above can be regarded as an attempt to formalize these Pigovian complications.

It should be noted that the early industrial engineering literature also stressed that the possibility of obsolescence meant that depreciation allowances should be larger than those implied by mere wear and tear; see Babbage (1835; 285), Matheson (1910; 39-40) and Church (1917; 192-193). Both Matheson and Church noted that obsolescence could arise not only from new inventions but also from shifts in demand.

We will end this section by pointing out another important use for the concept of real anticipated time series depreciation. However, before doing this, it is useful to rewrite equations (5), which define the beginning of period t asset prices by age n, P_n^t, in terms of the beginning of period t rental prices f_n^t, and equations (7), which define the user costs f_n^t in terms of the asset prices P_n^t, using definitions (27) and (28), which define the period t real interest rate r^{*t} and expected asset inflation rate i^{*t} respectively in terms of the corresponding nominal rates r^t and i^t and the general inflation rate ρ^t. Substituting (27) and (28) into (5) yields the following system of equations:

$$
\begin{aligned}
(31)\ P_0^t &= f_0^t + [(1+i^{*t})/(1+r^{*t})]\, f_1^t + [(1+i^{*t})/(1+r^{*t})]^2\, f_2^t + [(1+i^{*t})/(1+r^{*t})]^3\, f_3^t + \ldots \\
P_1^t &= f_1^t + [(1+i^{*t})/(1+r^{*t})]\, f_2^t + [(1+i^{*t})/(1+r^{*t})]^2\, f_3^t + [(1+i^{*t})/(1+r^{*t})]^3\, f_4^t + \ldots \\
&\ldots \\
P_n^t &= f_n^t + [(1+i^{*t})/(1+r^{*t})]\, f_{n+1}^t + [(1+i^{*t})/(1+r^{*t})]^2\, f_{n+2}^t + [(1+i^{*t})/(1+r^t)]^3\, f_{n+3}^t + \ldots
\end{aligned}
$$

Similarly, substituting (27) and (28) into (7) yields the following system of equations:

$$
\begin{aligned}
(32)\ f_0^t &= P_0^t - [(1+i^{*t})/(1+r^{*t})]\, P_1^t = (1+r^{*t})^{-1}\, [P_0^t\,(1+r^{*t}) - (1+i^{*t})\, P_1^t] \\
f_1^t &= P_1^t - [(1+i^{*t})/(1+r^{*t})]\, P_2^t = (1+r^{*t})^{-1}\, [P_1^t\,(1+r^{*t}) - (1+i^{*t})\, P_2^t] \\
&\ldots \\
f_n^t &= P_n^t - [(1+i^{*t})/(1+r^{*t})]\, P_{n+1}^t = (1+r^{*t})^{-1}\, [P_n^t\,(1+r^{*t}) - (1+i^{*t})\, P_{n+1}^t]\ ;\ \ldots
\end{aligned}
$$

Note that the nominal interest and inflation rates have entirely disappeared from the above equations. In particular, the beginning of the period user costs f_n^t can be defined in terms or real variables using equations (32) if this is desired. On the other hand, entirely equivalent formulae for the cross section user costs can be obtained using the initial set of equations (7), which used only nominal variables. Which set of equations is used in practice can be left up to the judgment of the statistical agency or the user.[36] The point is that the *careful and consistent use of discounting should eliminate the effects of general inflation from our price variables*; discounting makes comparable cash flows received or paid out at different points of time.

Recall definition (29), which defined Δ_n^t as the *ex ante real time series depreciation* of an asset that is n periods old at the beginning of period t. It is convenient to convert this amount of depreciation into a *percentage* of the initial price of the asset at the beginning

[36] In particular, it is not necessary for the statistical agency to convert all nominal prices into real prices as a preliminary step before "real" user costs are calculated. The above algebra shows that our nominal user costs f_n^t can also be interpreted as "real" user costs that are expressed in terms of the value of money prevailing at the beginning of period t.

of period t, P_n^t. Thus we define the *ex ante time series depreciation rate for an asset that is n periods old at the start of period t*, ε_n^t, as follows:[37]

$$(33)\quad \varepsilon_n^t \equiv \Delta_n^t / P_n^t \qquad ; n = 0,1,2,\ldots$$
$$= [P_n^t - (1+i^t)\, P_{n+1}^t/(1+\rho^t)] / P_n^t \qquad \text{using (29)}$$
$$= [P_n^t - (1+i^t)(1-\delta_n^t)\, P_n^t /(1+\rho^t)] / P_n^t \qquad \text{using (12)}$$
$$= [1 - (1+i^{*t})(1-\delta_n^t)] \qquad \text{using (28).}$$

Now substitute definition (12) for the cross section depreciation rate δ_n^t into the nth equation of (32) and we obtain the following expression for the beginning of period t user cost of an asset that is n periods old at the start of period t:

$$(34)\quad f_n^t = (1+r^{*t})^{-1}\, [P_n^t\, (1+r^{*t}) - (1+i^{*t})\, P_{n+1}^t] \qquad n = 0,1,2,\ldots$$
$$= (1+r^{*t})^{-1}\, [P_n^t\, (1+r^{*t}) - (1+i^{*t})(1-\delta_n^t)\, P_n^t] \qquad \text{using (12)}$$
$$= (1+r^{*t})^{-1}\, [(1+r^{*t}) - (1+i^{*t})(1-\delta_n^t)]\, P_n^t$$
$$= (1+r^{*t})^{-1}\, [r^{*t} + \varepsilon_n^t]\, P_n^t \qquad \text{using (33).}$$

Thus the period t vintage user cost for an asset that is n periods old at the start of period t, f_n^t, can be decomposed into the sum of two terms. Ignoring the discount factor, $(1+r^{*t})^{-1}$, the first term is $r^{*t}\, P_n^t$, which represents the *real interest cost* of the financial capital that is tied up in the asset, and the second term is $\varepsilon_n^t\, P_n^t = \Delta_n^t$, which represents a concept of *national accounts depreciation*.

The last line of (34) is important if at some stage statistical agencies decide to switch from measures of gross domestic product to measures of net domestic product. If this change occurs, then the user cost for each age n of capital, f_n^t, must be split up into two terms as in (34). The first term, $(1+r^{*t})^{-1}\, r^{*t}\, P_n^t$ times the number of units of that type of capital in use, could remain as a primary input charge while the second term, $(1+r^{*t})^{-1}\, \varepsilon_n^t\, P_n^t$ times the number of units of that age of capital in use, (this is real national accounts depreciation) could be treated as an intermediate input charge (similar to the present treatment of imports). The second term would be an offset to gross investment.[38]

This completes our discussion of the obsolescence problem.[39] In the next section, we turn our attention to the problem of aggregating across ages of the same capital good.

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[37] To see that there can be a very large difference between the cross section depreciation rate δ_n^t and the corresponding ex ante time series depreciation rate ε_n^t, consider the case of an asset whose vintages yield exactly the same service for each period in perpetuity. In this case, all of the vintage asset prices P_n^t would be identical and the cross section depreciation rates δ_n^t would all be zero. Now suppose a marvelous new invention is scheduled to come on the market next period which would effectively drive the price of this class of assets down to zero. In this case, i^{*t} would be -1 and substituting this expected measure of price change into definitions (33) shows that the ex ante time series depreciation rates would all equal one; i.e., under these conditions, we would have $\varepsilon_n^t = 1$ and $\delta_n^t = 0$ for all vintages n.

[38] Using this methodology, we would say that capital is being *maintained intact* for the economy if the value of gross investments made during the period (discounted to the beginning of the period) is equal to or greater than the sum of the real national accounts depreciation terms over all assets. This is *a maintenance of financial capital concept* as opposed to Pigou's (1935; 235) *maintenance of physical capital concept*.

[39] It should be noted that our discussion of the obsolescence issue only provides an introduction to the many thorny issues that make this area of inquiry quite controversial. For further discussion, see Oulton (1995), Scott (1995) and Triplett (1996) and the references in these papers.

6. Aggregation over Vintages of a Capital Good

In previous sections, we have discussed the beginning of period t *stock price* P_n^t of an asset that is n periods old and the corresponding beginning and end of period *user costs*, f_n^t and u_n^t. The stock prices are relevant for the construction of *real wealth measures* of capital and the user costs are relevant for the construction of *capital services measures*. We now address the problems involved in obtaining quantity series that will match up with these prices.

Let the period $t-1$ investment in a homogeneous asset for the sector of the economy under consideration be I^{t-1}. We assume that the starting capital stock for a new unit of capital stock at the beginning of period t is K_0^t and this stock is equal to the new investment in the asset in the previous period; i.e., we assume:

(35) $K_0^t \equiv I^{t-1}$.

Essentially, we are assuming that the length of the period is short enough so that we can neglect any contribution of investment to current production; a new capital good becomes productive only in the period immediately following its construction. In a similar manner, we assume that the capital stock available of an asset that is n periods old at the start of period t is K_n^t and this stock is equal to the gross investment in this asset class during period $t-n-1$; i.e., we assume:

(36) $K_n^t \equiv I^{t-n-1}$; $\qquad$ n = 0,1,2,…

Given these definitions, the value of the capital stock in the given asset class for the sector of the economy under consideration (*the wealth capital stock*) at the start of period t is

(37) $W^t \equiv P_0^t K_0^t + P_1^t K_1^t + P_2^t K_2^t + \ldots$
$\qquad = P_0^t I^{t-1} + P_1^t I^{t-2} + P_2^t I^{t-3} + \ldots$ $\qquad$ using (36).

Turning now to the capital services quantity, we assume that the quantity of services that an asset of a particular age at a point in time is proportional (or more precisely, is equal) to the corresponding stock. Thus we assume that the quantity of services provided in period t by a unit of the capital stock that is n periods old at the start of period t is K_n^t defined by (36) above. Given these definitions, the value of capital services for all vintages of asset in the given asset class for the sector of the economy under consideration (*the productive services capital stock*) during period t using the end of period user costs u_n^t defined by equations (8) above is

(38) $S^t \equiv u_0^t K_0^t + u_1^t K_1^t + u_2^t K_2^t + \ldots$
$\qquad = u_0^t I^{t-1} + u_1^t I^{t-2} + u_2^t I^{t-3} + \ldots$ $\qquad$ using (36).

Now we are faced with the problem of decomposing the value aggregates W^t and S^t defined by (37) and (38) into separate price and quantity components. If we assume that each new unit of capital lasts only a finite number of periods, L say, then we can solve this value decomposition problem using normal index number theory. Thus define the period *t stock price and quantity vectors*, $\mathbf{P^t}$ and $\mathbf{K^t}$ respectively, as follows:

(39) $\mathbf{P^t} \equiv [P_0^t,P_1^t,\ldots,P_{L-1}^t]$; $\mathbf{K^t} \equiv [K_0^t,K_1^t,\ldots,K_{L-1}^t] = [I^{t-1},I^{t-2},\ldots,I^{t-L-1}]$; t = 0,1,…,T.

Fixed base or chain indexes may be used to decompose value ratios into price change and quantity change components. In the empirical work which follows, we have used the chain principle.[40] Thus the value of the capital stock in period t, W^t, relative to its value in the preceding period, W^{t-1}, has the following index number decomposition:

(40) $W^t / W^{t-1} = P(\mathbf{P}^{t-1},\mathbf{P}^t,\mathbf{K}^{t-1},\mathbf{K}^t)\, Q(\mathbf{P}^{t-1},\mathbf{P}^t,\mathbf{K}^{t-1},\mathbf{K}^t)$; $t = 1,2,\ldots,T$

where P and Q are *bilateral price and quantity indexes* respectively.

In a similar manner, we define the period *t end of the period user cost price and quantity vectors*, $\mathbf{u}^t$ and $\mathbf{K}^t$ respectively, as follows:

(41) $\mathbf{u}^t \equiv [u_0^t,u_1^t,\ldots,u_{L-1}^t]$; $\mathbf{K}^t \equiv [K_0^t,K_1^t,\ldots,K_{L-1}^t] = [I^{t-1},I^{t-2},\ldots,I^{t-L-1}]$; $t = 0,1,\ldots,T$.

We ask that the value of capital services in period t, S^t, relative to its value in the preceding period, S^{t-1}, has the following index number decomposition:

(42) $S^t / S^{t-1} = P(\mathbf{u}^{t-1},\mathbf{u}^t,\mathbf{K}^{t-1},\mathbf{K}^t)\, Q(\mathbf{u}^{t-1},\mathbf{u}^t,\mathbf{K}^{t-1},\mathbf{K}^t)$; $t = 1,2,\ldots,T$

where again P and Q are *bilateral price and quantity indexes* respectively.

There is now the problem of choosing the functional form for either the price index P or the quantity index Q.[41] In the empirical work that follows, we used the *Fisher* (1922) *ideal price and quantity indexes*. These indexes appear to be "best" from the axiomatic viewpoint[42] and can also be given strong economic justifications.[43]

It should be noted that our use of an index number formula to aggregate both stocks and services by age is more general than the usual aggregation procedures, which essentially assume that the different vintages of the same capital good are perfectly substitutable so that linear aggregation techniques can be used.[44] However, as we shall see in subsequent sections, the more general mode of aggregation suggested here frequently reduces to the traditional linear method of aggregation provided that the period prices by age all move in strict proportion over time.

Many researchers and statistical agencies relax the assumption that an asset lasts only a fixed number of periods, L say, and make assumptions about the distribution of retirements around the average service life, L. In our empirical work that follows, for simplicity, we will stick to the sudden death assumption; i.e., that all assets in the given asset class are retired at age L. However, this simultaneous retirement assumption can readily be relaxed (at the cost of much additional computational complexity) using a methodology developed by Hulten (1990; 125), where he subdivided a vintage into subcomponents, each of which had a different expected length of life.

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[40] Given smoothly trending price and quantity data, the use of chain indexes will tend to reduce the differences between Paasche and Laspeyres indexes compared to the corresponding fixed base indexes and so chain indexes are generally preferred; see Diewert (1978; 895) for a discussion.
[41] Obviously, given one of these functional forms, we may use (40) to determine the other.
[42] See Diewert (1992b; 214-223).
[43] See Diewert (1976; 129-134).
[44] This more general form of aggregation was first suggested by Diewert and Lawrence (2000). For descriptions of the more traditional linear method of aggregation, see Jorgenson (1989; 4) or Hulten (1990; 121-127) (1996; 152-165).

We now have all of the pieces that are required in order to decompose the capital stock of an asset class and the corresponding capital services into price and quantity components. However, in order to construct price and quantity components for capital services, we need information on the relative efficiencies f_n^t of the various vintages of the capital input or equivalently, we need information on cross sectional vintage depreciation rates δ_n^t in order to use (42) above. The problem is that we do not have accurate information on either of these series so in what follows, we will assume a standard asset life L and make additional *assumptions* on the either the pattern of vintage efficiencies or depreciation rates. Thus in a sense, we are following the same somewhat mechanical strategy that was used by the early cost accountants like Daniels (1933; 303).

However, our mechanical strategy is more complex than that used by early accountants in that we translate assumptions about the pattern of cross section depreciation rates into implications for the pattern of cross section rental prices and asset prices, taking into account the complications induced by discounting and expected future asset price changes.

In the following sections, we will consider 4 different sets of assumptions and calculate the resulting aggregate capital stocks and services using Canadian data. We *illustrate* how the various depreciation models differ from each other using annual Canadian data on two broad classes of asset:[45]

- machinery and equipment and
- nonresidential structures.

We use Canadian data on gross investment in these two asset classes (in current and in constant dollars) because it extends back to 1926 and hence capital stocks can be formed without making arbitrary starting value assumptions.

Our first problem is to decide on the average age of retirement for each of these asset classes. One source is the OECD (1993) where average service lives for various asset classes were reported for 14 OECD countries. For machinery and equipment (excluding vehicles) used in manufacturing activities, the average life ranged from 11 years for Japan to 26 years for the United Kingdom. For vehicles, the average service lives ranged from 2 years for passenger cars in Sweden to 14 years in Iceland and for road freight vehicles, the average life ranged from 3 years in Sweden to 14 years in Iceland. For buildings and structures, the average service lives ranged from 15 years (for petroleum and gas structures in the US) to 80 years for railway structures in Sweden. Faced with this wide range of possible lives, we decided to follow the example of Angus Madison (1993) and assume an average service life of 14 years for machinery and equipment and 39 years for nonresidential structures. The Canadian data that we used may be found in Diewert (2004).

We turn now to our first efficiency and depreciation model.

7. The One Hoss Shay Model of Efficiency and Depreciation

In section 2 above, we noted that Böhm-Bawerk (1891; 342) postulated that an asset would yield a constant level of services throughout its useful life of L years and then collapse in a heap to yield no services thereafter. This has come to be known as the one

[45] More accurate models would work with more disaggregated investment series.

hoss shay or light bulb model of depreciation. Hulten (1990; 124) noted that this pattern of relative efficiencies has considerable intuitive appeal for many assets.

The basic assumptions of this model are that the period t efficiencies and hence cross sectional rental prices f_n^t are all equal to say f^t for ages n that are less than L periods old and for older ages, the efficiencies fall to zero. Thus we have:

$$(43)\quad f_n^t = f^t \quad \text{for } n = 0,1,2,\ldots,L-1; \qquad = 0 \quad \text{for } n = L, L+1, L+2, \ldots$$

Now substitute (43) into the first equation in (5) and get the following formula[46] for the rental price f^t in terms of the price of a new asset at the beginning of year t, P_0^t:

$$(44)\quad f^t = P_0^t/[1 + (\gamma) + (\gamma)^2 + \ldots + (\gamma)^{L-1}]$$

where the period t discount factor γ is defined in terms of the period t nominal interest rate r^t and the period t expected asset rate of price change i^t as follows:

$$(45)\quad \gamma \equiv (1 + i^t)/(1 + r^t).$$

Now that the period t rental price f^t for an unretired asset has been determined, substitute equations (43) into equations (5) and determine the sequence of period t asset prices by age n, P_n^t:

$$(46)\quad P_n^t = f^t\,[1 + (\gamma) + (\gamma)^2 + \ldots + (\gamma)^{L-1-n}] \quad \text{for } n = 0,1,2,\ldots,L-1; \qquad = 0 \quad \text{for } n = L, L+1, L+2, \ldots$$

Finally, use equations (8) to determine the end of period t rental prices, u_n^t, in terms of the corresponding beginning of period t rental prices, f_n^t:

$$(47)\quad u_n^t = (1 + r^t)f_n^t\,; \qquad n = 0,1,2,\ldots$$

Given the asset prices defined by (46), we could use equations (12) above to determine the corresponding cross section depreciation rates δ_n^t. We will not table these depreciation rates since our focus is on constructing measures of the capital stock and of the flow of services that the stocks yield.

We have data in current and constant dollars for investment in nonresidential structures and for machinery and equipment in Canada for the years 1926 to 1999 inclusive; see Diewert (2004) for a description of these data. As was mentioned in the previous section, we follow the example set by Maddison (1993) and assume an average service life of 14 years for machinery and equipment and 39 years for nonresidential structures. Thus 1965 is the first year for which we will have data on all 39 types of nonresidential structures. Now it is a straightforward matter to use the asset prices by age defined by (46) above (where L equals 39) and apply (40) in the previous section to aggregate over the 39 types of nonresidential capital using the Fisher (1922) ideal index number formula and form aggregate price and quantity series for the nonresidential construction (wealth) capital stock, P_{NR}^t and K_{NR}^t, for the years 1965-1999. These series, along with their annual average (geometric) growth rates, can be found in Diewert (2004) at 5 year intervals.

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[46] This formula simplifies to $P_0^t[1-(\gamma)^L]/[1-\gamma]$ provided that γ is less than 1 in magnitude. This last restriction does not hold for our Canadian data, since for some years, i^t exceeds r^t. However, (44) is still valid under these conditions.

Similarly, we use (46) above (where L equals 14) and apply (40) in the previous section to aggregate over the 14 ages of machinery and equipment using the Fisher ideal index number formula and form aggregate price and quantity series for the machinery and equipment (wealth) capital stock, P_{ME}^t and K_{ME}^t, for the years 1965-1999. These series, along with their annual average (geometric) growth rates, can also be found in Diewert (2004) at 5 year intervals. In this *first model*, we assume that producers exactly anticipate the asset rates of price change, i_{NR}^t and i_{ME}^t, for nonresidential construction and for machinery and equipment respectively; these ex post rates of price change are listed in Diewert (2004). Having constructed the aggregate price and quantity of nonresidential capital, P_{NR}^t and K_{NR}^t respectively, and the aggregate price and quantity of machinery and equipment, P_{ME}^t and K_{ME}^t respectively, we may again use the Fisher ideal formula and aggregate these two series into a single aggregate price and quantity series for the wealth stock, which we denote by $P(1)^t$ and $K(1)^t$, where the 1 indicates that this is our first model in a grand total of 12 alternative aggregate capital stock models.

Using equations (43), (44) and (47) along with the data tabled in Diewert (2004), we can construct the end of the period user costs for each of our 39 types of nonresidential construction capital. Now use equation (38) to construct the service flow aggregate for nonresidential construction for each year. Then we use (42) in the previous section (where L equals 39) to aggregate over the 39 types of nonresidential capital using the Fisher (1922) ideal index number formula and form the aggregate rental price for nonresidential construction, u_{NR}^t, and the corresponding services aggregate, k_{NR}^t, for the years 1965-1999.[47] These series, along with their annual average (geometric) growth rates, can be found in Diewert (2004) at 5 year intervals. Similarly, we use (42) above (where L equals 14) and aggregate over the 14 ages of machinery and equipment using the Fisher ideal index number formula and form aggregate capital services price and quantity series, u_{ME}^t and k_{ME}^t, for the years 1965-1999. These series, along with their annual average (geometric) growth rates, can also be found in Diewert (2004) at 5 year intervals. Having constructed the aggregate price and quantity of nonresidential capital services, u_{NR}^t and k_{NR}^t respectively, and the aggregate price and quantity of machinery and equipment services, u_{ME}^t and k_{ME}^t respectively, we may again use the Fisher ideal formula and aggregate these two series into a single aggregate price and quantity series for capital services, which we denote by $u(1)^t$ and $k(1)^t$, where the 1 again indicates that this is our first model in a grand total of 12 alternative aggregate capital stock models. The various data series will be compared graphically in section 11 below.

We turn now to our *second one hoss shay depreciation model*. In this model, instead of assuming that producers correctly anticipate each year's ex post asset inflation rates, it is assumed that producers use the current CPI inflation rate as estimators of anticipated asset inflation rates. This model turns out to be equivalent to the constant real interest rate model that is frequently used by statistical agencies.[48] In terms of computations, we simply replace the two ex post asset inflation rates, i_{NR}^t and i_{ME}^t, by the CPI inflation rate $□^t$ listed in Diewert (2004) and then repeat all of the computations made to implement Model 1 above.

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[47] Since all of the vintage rental prices are equal, it turns out that the aggregate rental price is equal to this common vintage rental price and the service aggregate is equal to the simple sum over the vintages. This result is an application of Hicks' (1939; 312-313) aggregation theorem; i.e., if all prices in the aggregate move in strict proportion over time, then any one of these prices can be taken as the price of the aggregate.

[48] The nominal interest rate is still used in forming the end of the period user costs; otherwise, only real interest rates are used in this model.

When we compare the service prices and quantities in Model 1, the perfect foresight model, with the corresponding service prices and quantities in Model 2, the constant real interest rate model, a number of things stand out:

- The Model 2 user costs are much less volatile (as could be expected);
- The Model 1 user costs grow much more quickly;
- The Model 2 levels of capital services are much higher but
- The Model 1 and 2 average growth rates for capital services are very similar.

Thus the two models give very different results overall. The average rate of price increase for the Model 2 capital services aggregate was 3.29% per year, which is much lower than the Model 1 estimate of 4.85% per year. On the quantity side, the Model 2 flow of nonresidential construction capital services increased from $2727 million to $11,564 million (constant 1965) Canadian dollars, for an annual average (geometric) growth rate of 4.34% while the Model 2 flow of machinery and equipment capital services increased from $3588 million to $34,556 million (constant 1965) Canadian dollars, for an annual average growth rate of 6.89%. The Model 2 capital services aggregate grew at an annual average growth rate of 5.49% compared to the Model 1 5.61% capital services annual average growth rate.

We turn now to our *third one hoss shay depreciation model*. In this model (Model 3), instead of assuming that producers correctly anticipate each year's ex post asset inflation rates, we assume that they can anticipate the *trends* in asset inflation rates. In Diewert (2004), we describe in detail how these trends were determined. In terms of computations, we use exactly the same program that we used to implement Model 1 except that we replace the rather volatile nominal interest rates r^t by the smoothed nominal interest rates that are listed in Diewert (2004). We also replace the two ex post asset inflation rates, $i_{NR}{}^t$ and $i_{ME}{}^t$, by their smoothed counterparts listed in Diewert (2004).

Comparing the numbers across the three models, there are some small differences between the capital stocks generated by our three variants of the one hoss shay model of depreciation but the average growth rates are virtually identical. There is more variation across the three models in the movement of the stock prices with Model 1 giving the highest rate of price growth for the capital aggregate (4.35% per year), followed by Model 3 (4.17% per year) and then Model 2 (3.97% per year). The Model 1,2 and 3 aggregate prices, P(1)-P(3), and quantities of capital, K(1)-K(3) respectively, are graphed in Figures 1-6; see the Figures in section 11 below.

The tremendous volatility of the Model 1 rental prices, u(1), will become evident from viewing Figure 7. *Thus the use of ex post asset inflation rates as ex ante or anticipated inflation rates leads to user costs that are extremely volatile.* The Model 3 aggregate user costs, u(3), while still more volatile than the constant real interest rate user costs, u(2), are reasonable and smooth out the fluctuations in the u(1) series. The u(2) series lies below the other two user cost series because the constant real interest rate user costs make no allowance for the extra depreciation that arises from the anticipated price declines that are due to obsolescence; i.e., the u(2) series ignores the systematic real price declines in the price of machinery and equipment. Thus while Model 2 is acceptable, we prefer Model 3, since this model includes the effects of anticipated obsolescence, whereas Model 2 does not.

Examination of Figures 4-6 in section 11 shows that all three one hoss shay models give rise to much the same aggregate capital stocks. The constant real interest rate capital stocks K(2) are the biggest, followed by the smoothed anticipated inflation stocks K(3) and the fully anticipated inflation stocks K(1) are the smallest. The aggregate capital services graphed in Figures 10-12 show much the same pattern but with more dispersion. The constant real interest rate aggregated capital services k(2) are the biggest, followed by the smoothed anticipated inflation capital services k(3) and the fully anticipated inflation capital services k(1) are the smallest.

We turn now to our second model of depreciation and efficiency.

8. The Straight Line Depreciation Model

The straight line method of depreciation is very simple in a world without price change: one simply makes an estimate of the most probable length of life for a new asset, L periods say, and then the original purchase price P_0^t is divided by L to yield an estimate of period by period depreciation for the next L periods. In a way, this is the simplest possible model of depreciation, just as the one hoss shay model was the simplest possible model of efficiency decline.[49] The use of straight line depreciation dates back to the 1800's at least; see Matheson (1910; 55), Garcke and Fells (1893; 98) and Canning (1929; 265-266).

We now set out the equations which describe the straight line model of depreciation in the general case when the anticipated asset rate of price change i^t is nonzero. Assuming that the asset has a life of L periods and that the cross sectional amounts of depreciation $D_n^t \equiv P_n^t - P_{n+1}^t$ defined by (10) above are all equal for the assets in use, then it can be seen that the beginning of period t vintage asset prices P_n^t will decline linearly for L periods and then remain at zero; i.e., the P_n^t will satisfy the following restrictions:

$$(48)\ P_n^t = P_0^t\,[L - n]/L \qquad n = 0,1,2,...,L$$
$$= 0 \qquad n = L+1,L+2,...$$

Recall definition (12) above, which defined the cross sectional depreciation rate for an asset that is n periods old at the beginning of period t, δ_n^t. Using (48) and the nth equation in (13), we have:

$$(49)\ (1-\delta_0^t)(1-\delta_1^t)\ldots(1-\delta_{n-1}^t) = P_n^t / P_0^t = 1-(n/L) \qquad \text{for } n = 1,2,...,L.$$

Using (49) for n and n+1, it can be shown that

$$(50)\ (1-\delta_n^t) = [L-(n+1)]/[L-n] \qquad n = 0,1,2,...,L-1.$$

Now substitute (49) and (50) into the general user cost formula (14) in order to obtain the *period t end of the period straight line user costs*, u_n^t:[50]

$$(51)\ u_n^t = (1-\delta_0^t)\ldots(1-\delta_{n-1}^t)[(1+r^t)-(1+i^t)(1-\delta_n^t)]\,P_0^t \qquad n = 0,1,2,...,L-1$$
$$= [1-(n/L)][(1+r^t)-(1+i^t)([L-(n+1)]/[L-n])]\,P_0^t.$$

Equations (48) give us the sequence of asset prices by age that are required to calculate the wealth capital stock while equations (51) give us the user costs by age that are required to calculate capital services for the asset. It should be noted that if the anticipated asset inflation rate i^t is large enough compared to the nominal interest rate r^t,

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[49] In fact, it can be verified that if the nominal interest rate r^t and the nominal asset inflation rate i^t are both zero, then the one hoss shay efficiency model will be entirely equivalent to the straight line depreciation model.

[50] The user costs for n = L, L+1,L+2,... are all zero.

then the user cost u_n^t can be negative. This means that the corresponding asset becomes an output rather than an input for period t.[51]

At this point, we can proceed in much the same manner as in the previous section. We use the asset prices defined by (48) above (where L equals 39) and apply (40) in section 7 to aggregate over the 39 types of nonresidential capital using the Fisher (1922) ideal index number formula and we form aggregate price and quantity series for the nonresidential construction (wealth) capital stock, P_{NR}^t and K_{NR}^t, for the years 1965-1999. These series, along with their annual average (geometric) growth rates, can be found in Diewert (2004) at 5 year intervals. Similarly, we use (48) above (where L equals 14) and apply (40) to aggregate over the 14 types of machinery and equipment using the Fisher ideal index number formula and we form aggregate price and quantity series for the machinery and equipment (wealth) capital stock, P_{ME}^t and K_{ME}^t, for the years 1965-1999. These series, along with their annual average (geometric) growth rates, can also be found in Diewert (2004) at 5 year intervals. In this *fourth model*, we assume that producers exactly anticipate the ex post asset rates of price change, i_{NR}^t and i_{ME}^t, for nonresidential construction and for machinery and equipment respectively. Having constructed the aggregate price and quantity of nonresidential capital, P_{NR}^t and K_{NR}^t respectively, and the aggregate price and quantity of machinery and equipment, P_{ME}^t and K_{ME}^t respectively, we may again use the Fisher ideal formula and aggregate these two series into a single aggregate price and quantity series for the wealth stock, which we denote by $P(4)^t$ and $K(4)^t$.

Using equations (51) along with the data tabled in Diewert (2004), we can construct the end of the period user costs for each of our 39 types of nonresidential construction capital. Now use equation (38) to construct the service flow aggregate for nonresidential construction for each year. Then we use (42) in the previous section (where L equals 39) to aggregate over the 39 types of nonresidential capital using the Fisher (1922) ideal index number formula and form the aggregate rental price for nonresidential construction, u_{NR}^t, and the corresponding services aggregate, k_{NR}^t, for the years 1965-1999.[52] These series, along with their annual average (geometric) growth rates, can be found in Diewert (2004) at 5 year intervals. Similarly, we use (42) above (where L equals 14) and aggregate over the 14 types of machinery and equipment using the Fisher ideal index number formula and we form aggregate capital services price and quantity series, u_{ME}^t and k_{ME}^t, for the years 1965-1999. These series, along with their annual average (geometric) growth rates, can also be found in Diewert (2004) at 5 year intervals. Having constructed the aggregate price and quantity of nonresidential capital services, u_{NR}^t and k_{NR}^t respectively, and the aggregate price and quantity of machinery and equipment services, u_{ME}^t and k_{ME}^t respectively, we may again use the Fisher ideal formula and aggregate these two series into a single aggregate price and quantity series for capital services, which we denote by $u(4)^t$ and $k(4)^t$.

We turn now to our *second straight line depreciation model.* In this *Model 5*, instead of assuming that producers correctly anticipate each year's ex post asset inflation rates, it is

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[51] However, one is led to wonder if the model is reasonable if some vintages of the asset have negative user costs while other vintages have positive ones.

[52] It turned out that some of our rental prices were negative. This may not be a major theoretical problem since in this case, the corresponding capital input becomes a net output. However, the computations were carried out using the econometrics computer program SHAZAM and the index number option fails when any price is negative. In this case, it was necessary to write up a subroutine that would compute the Fisher indexes when some prices were negative. The four inner products that are building blocks into the Fisher indexes must all be positive in order to take the positive square root. This condition was satisfied by the data in all cases.

assumed that producers use the current CPI inflation rate as estimators of anticipated asset rates of price change. In terms of computations, we simply replace the two ex post asset rates of price change, i_{NR}^t and i_{ME}^t, by the CPI inflation rate ρ^t listed in Diewert (2004) and then repeat all of the computations made to implement Model 4 above.

It turns out that the Model 5 constant real interest rate capital stocks (and prices) are *exactly* equal to their Model 4 counterparts. This follows from equations (48), which describe the pattern of asset prices by age: in both Models 4 and 5 (and 6 to be considered shortly), these asset prices do not depend on r^t or i^t and hence the resulting asset prices and capital stocks will be identical. Hence there is no need to table the capital stocks and prices for Model 5. However, the Model 5 user costs and capital service flows by age (listed in Diewert (2004) at 5 year intervals) are very different from their Model 4 counterparts.

We turn now to our *third straight line deprecation model*, which we call *Model 6*. In this model, instead of assuming that producers correctly anticipate each year's ex post asset inflation rates, we assume that they can anticipate the *trends* in asset rates of price change. In terms of computations, we use exactly the same program that we used to implement Model 4 except that we replace the rather volatile nominal interest rate r^t that is listed in Diewert (2004) by the smoothed nominal interest rate that is listed in Diewert (2004). We also replace the two ex post asset inflation rates, i_{NR}^t and i_{ME}^t, by their smoothed counterparts also listed in Diewert (2004).

As mentioned earlier, the Model 6 constant real interest rate capital stocks (and prices) are *exactly* equal to their Model 4 counterparts in Table 7. Hence there is no need to table the capital stocks and prices for Model 6. However, the Model 6 vintage user costs and capital service flows are very different from their Model 4 and 5 counterparts.

On the quantity side, Model 6 gives much the same results as the other two straight line depreciation models, Models 4 and 5; see Figures 10-12 below for graphs of k(4)-k(6). In particular, the average annual (geometric) rate of growth of aggregate capital services for Models 4, 5 and 6 was 5.30 %, 5.08% and 5.24% per year respectively. However, on the user cost side, the three models give very different results. The perfect foresight model, Model 4, gave the highest annual average growth rate for the aggregate price of capital services, 4.96% per year, while the constant real interest rate model, Model 5, gave the lowest average growth rate, 3.61% per year. The smoothed anticipated prices model, Model 6, gave an intermediate growth rate for the price of capital services, 4.31% per year. As can be seen from Figures 7-9 below, the Model 5 and 6 aggregate user costs were much smoother than the volatile Model 4 user costs.

We turn now to our third class of depreciation and efficiency models.

9. The Declining Balance or Geometric Depreciation Model

The declining balance method of depreciation dates back to Matheson (1910; 55) at least.[53] In terms of the algebra presented in section 3 above, the method is very simple: all of the cross sectional vintage depreciation rates δ_n^t defined by (12) are assumed to be equal to the same rate δ, where δ a positive number less than one; i.e., we have for all time periods t:

[53] Matheson (1910; 91) used the term "diminishing value" to describe the method. Hotelling (1925; 350) used the term "the reducing balance method" while Canning (1929; 276) used the term the "declining balance formula".

(52) $\delta_n^t = \delta$; $\qquad$ n = 0,1,2,... .

Substitution of (52) into (14) leads to the following formula for the sequence of period t vintage user costs:

$$(53)\ u_n^t = (1-\delta)^{n-1}\,[(1+r^t) - (1+i^t)(1-\delta)]\,P_0^t\ ; \qquad n = 0,1,2,...$$
$$= (1-\delta)^{n-1}\,u_0^t\ ; \qquad n = 1,2,... .$$

The second set of equations in (53) says *that all of the vintage user costs are proportional to the user cost for a new asset*. This proportionality means that we do not have to use an index number formula to aggregate over vintages to form a capital services aggregate. To see this, using (53), the period t services aggregate S^t defined earlier by (38) can be rewritten as follows:

$$(54)\ S^t \equiv u_0^t K_0^t + u_1^t K_1^t + u_2^t K_2^t + \ldots$$
$$= u_0^t\,[K_0^t + (1-\delta)\,K_1^t + (1-\delta)^2 K_2^t + \ldots\,]$$
$$= u_0^t K_A^t$$

where the *period t capital aggregate* K_A^t is defined as

$$(55)\ K_A^t \equiv K_0^t + (1-\delta)\,K_1^t + (1-\delta)^2 K_2^t + \ldots$$

If the depreciation rate δ and the vintage capital stocks are known, then K_A^t can readily be calculated using (55). Then using the last line of (54), we see that the value of capital services (summed over all ages), S^t, decomposes into the price term u_0^t times the quantity term K_A^t. Hence, it is not necessary to use an index number formula to aggregate over ages of the asset using this depreciation model.

A similar simplification occurs when calculating the wealth stock using this depreciation model. Substitution of (52) into (13) leads to the following formula for the sequence of period t asset prices by age n:

$$(56)\ P_n^t = (1-\delta)^{n-1} P_0^t\ ; \qquad n = 1,2,... .$$

Equations (56) say *that all of the period t asset prices are proportional to the price of a new asset*. This proportionality means that again, we do not have to use an index number formula to aggregate over vintages to form a capital stock aggregate. To see this, using (56), the period t wealth aggregate W^t defined earlier by (37) can be rewritten as follows:

$$(57)\ W^t \equiv P_0^t K_0^t + P_1^t K_1^t + P_2^t K_2^t + \ldots$$
$$= P_0^t\,[K_0^t + (1-\delta)\,K_1^t + (1-\delta)^2 K_2^t + \ldots\,]$$
$$= P_0^t K_A^t$$

where K_A^t was defined by (55). Thus K_A^t can serve as both a capital stock aggregate or a flow of services aggregate, which is a major advantage of this model.[54]

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[54] This advantage of the model has been pointed out by Jorgenson (1989) (1996b) and his coworkers. Its early application dates back to Jorgenson and Griliches (1967) and Christensen and Jorgenson (1969) (1973).

There is a further simplification of the model which is useful in applications. If we compare equation (55) for period t+1 and period t, we see that the following formula results using definitions (39):

$$(58)\ K_A^{t+1} \equiv K_0^{t+1} + (1 - \delta)\, K_A^{t}\,.$$

Thus the period t+1 aggregate capital stock, K_A^{t+1}, is equal to the investment in new assets that took place in period t, which is K_0^{t+1}, plus $1 - \delta$ times the period t aggregate capital stock, K_A^{t}. This means that given a starting value for the capital stock, we can readily update it just using the depreciation rate δ and the new investment in the asset during the prior period.

We now need to address the problem of determining the depreciation rate δ for a particular asset class. Matheson (1910; 69-91) was perhaps the first engineer to address this problem. On the basis of his experience, he simply postulated some approximate rates that could be applied, ranging from 3 to 20 per cent.

The algebra corresponding to Matheson's method for determining δ was explicitly described by the accountant Canning (1929; 276). Let the initial value of the asset be V_0 and let its scrap value n years later be V_n. Then V_0, V_n and the depreciation rate δ are related by the following equation:

$$(59)\ \ V_n = (1 - \delta)^n\, V_0.$$

Canning goes on to explain that $1 - \delta$ may be determined by solving the following equation:

$$(60)\ \ \log (1 - \delta) = [\log V_n - \log V_0]/n.$$

It is clear that Matheson used this framework to determine depreciation rates even though he did not lay out formally the above straightforward algebra.

However, Canning (1929; 276 pointed out that the scrap value, V_n, which is not determined very accurately from an a priori point of view, is the tail that is wagging the dog; i.e., this poorly determined value plays a crucial role in the determination of the depreciation rate.

An effective response to Canning's criticism of the declining balance method of depreciation did not emerge until relatively recently when Hall (1971), Beidelman (1973) (1976) and Hulten and Wykoff (1981a) (1981b) used an entire array of used asset prices at point in time in order to determine the geometric depreciation rate which best matched up with the data.[55] Another theoretical possibility would be to use information on rental prices by age of asset in order to deduce the depreciation rate.[56]

[55] Jorgenson (1996a) has a nice review of most of the empirical studies of depreciation. It should be noted that Beidelman (1973) (1976) and Hulten and Wykoff (1981a) (1996; 22) showed that equation (59) must be adjusted to correct for the early retirement of assets. The accountant Schmalenbach (1959; 91) (the first German edition was published in 1919) also noticed this problem.

[56] This possibility is mentioned by Hulten and Wykoff (1996; 15).

This brings us to our next problem: how should we convert our estimated asset lives of 39 years for structures and 14 years for machinery and equipment into comparable geometric rates?

One possible method for converting an average asset life, L periods say, into a comparable geometric depreciation rate is to argue as follows. Suppose that we believe that the straight line model of depreciation is the correct one and the asset under consideration has a useful life of L periods. Suppose further that investment in this type of asset is constant over time at one unit per period and asset prices are constant over time. Under these conditions, the long run equilibrium capital stock for this asset would be[57]:

(61) $1 + [(L-1)/L] + [(L-2)/L] + ... + [2/L] + [1/L] = L(L+1)/2L = (L+1)/2$.

Under the same conditions, the long run equilibrium geometric depreciation capital stock would be equal to the following sum:

(62) $1 + (1-\delta) + (1-\delta)^2 + ... = 1/[1-(1-\delta)] = 1/\delta$.

Now find the depreciation rate δ which will make the two capital stocks equal; i.e., equate (61) to (62) and solve for δ. The resulting δ is:

(63) $\delta = 2/(L+1)$.

Obviously, there are a number of problematical assumptions that were made in order to derive the depreciation rate δ that corresponds to the length of life L[58] but (63) gives us at least a definite method of conversion from one model to the other.

Since we assumed that the average length of life for nonresidential construction was L equal to 39 years, applying the conversion formula (63) implies that δ_{NR} equals .05; i.e., we assume that the declining balance or geometric depreciation rate for nonresidential construction in Canada is 5%. Similarly, our assumed life of 14 years for machinery and equipment translates into a δ_{ME} equal to a 13 1/3% geometric depreciation rate for this asset class.

There is one remaining problem to deal with and then we can proceed to table the results for three geometric depreciation models for Canada. The problem is this: before 1926, we do not have reliable investment data but the effects of investments made prior to 1926

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[57] Recall equations (48), which imply that the vintage asset prices are proportional. Hence Hicks' Aggregation Theorem will imply that the capital aggregate will be the simple sum on the left hand side of (61).

[58] The two assumptions that are the least justified are: (1) the assumption that the straight line depreciation model is the correct model to do the conversion and (2) the assumption that investment has been constant back to minus infinity. Hulten and Wykoff (1996; 16) made the following suggestions for converting an L into a δ: "Information is available on the average service life, L, from several sources. The rate of depreciation for non-marketed assets can be estimated using a two step procedure based on the 'declining balance' formula $\delta = X/L$. Under the 'double declining balance' formula, X = 2. The value of X can be estimated using the formula $X = \delta L$ for those assets for which these estimates are available. In the Hulten-Wykoff studies, the average value for of X for producer's durable equipment was found to be 1.65 (later revised to 1.86). For nonresidential structures, X was found to be 0.91. Once X is fixed, δ follows for other assets whose average service life is available."

live on forever in the infinite lived geometric depreciation model that we considered in equations (54) to (58) above. In the case of machinery and equipment investments made before 1926, by the time we get to 1965, what is left of the original investments is negligible. However, in the case of a \$1000 investment in nonresidential structures made in 1925, \$128.50 of it would still be available as a productive input in 1965, assuming a 5% geometric depreciation rate. Hence we need a method for estimating the geometric capital stock that is available at the start of 1926 in order to not bias downward our estimates of the geometric capital stock for nonresidential construction for the period 1965-1999. We decided to assume that nonresidential investment for the period prior to 1926 grew at the same rate that it grew during the years 1926-1999.[59] Thus for the years 1927 to 1999, we took investment in nonresidential construction during the current year divided by the corresponding investment in the prior year (both in constant dollars) as our dependent variable and regressed this variable on a constant. The estimated constant turned out to be 1.0509. Hence, for the prior to 1926 period, we assumed that investments in nonresidential construction grew at the rate $g \equiv .05$; i.e., a 5% growth rate. Thus if I_{NR}^{1926} was the investment in 1926, we assumed that the investments in prior years were:

$$(64)\quad I_{NR}^{1926}/(1+g),\ \ I_{NR}^{1926}/(1+g)^2,\ \ I_{NR}^{1926}/(1+g)^3,\ \ldots\ .$$

Using assumption (64), we can calculate an estimate of the starting capital stock for nonresidential construction at the start of 1927 as

$$(65)\quad K_{NR}^{1927} \equiv I_{NR}^{1926}\,\{1 + [(1-\delta)/(1+g)] + [(1-\delta)/(1+g)]^2 + [(1-\delta)/(1+g)]^3 + \ldots\}$$
$$= I_{NR}^{1926}\,\{1/(1 - [(1-\delta)/(1+g)])\}$$
$$= I_{NR}^{1926}\,(1 + g)/(g + \delta)$$

where $g = .05$ and $\delta = .05$. Now we can use formula (58) above, starting at the year $t = 1927$, to build up the capital stock for each of our two asset classes. For nonresidential construction, our starting 1927 capital stock was defined by (65) and for machinery and equipment, it was simply the 1926 investment in machinery and equipment, I_{ME}^{1926} say.

At this point, we can proceed in much the same manner as in the previous section. We have already explained how we can use equations (58) to form the aggregate capital stocks for nonresidential construction and machinery and equipment. From (57), it can be seen that the corresponding capital stock price is P_0^t, the price of a new vintage at the beginning of year t. These series, along with their annual average (geometric) growth rates, can be found in Table 11 of Diewert (2004) at 5 year intervals. In this *seventh model*, having constructed the aggregate price and quantity of nonresidential capital, P_{NR}^t and K_{NR}^t respectively, and the aggregate price and quantity of machinery and equipment, P_{ME}^t and K_{ME}^t respectively, we may again use the Fisher ideal formula and aggregate these two series into a single aggregate price and quantity series for the wealth stock, which we denote by $P(7)^t$ and $K(7)^t$.

Comparing the capital stock prices for Model 7 with those of Model 4, we find that these numbers are exactly the same. This is because in both the straight line depreciation model and the geometric model, the price of a new asset acts as the aggregate stock price over all vintages. However, when we use the Fisher formula to aggregate the two types of capital prices together to get either P(4) or P(7), we get slightly different numbers because the aggregate quantities of the two types of asset differ in the two models. The

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[59] This method for obtaining a starting value for the geometric capital stock is due to Kohli (1982); see also Fox and Kohli (1998).

Fisher ideal aggregate price for these two capital stock components increased from 1 to 3.6243 over this period. The price of a unit of nonresidential construction capital increased by 5.08% per year and the price of a unit of machinery and equipment capital increased by only 1.37% per year on average for Model 7. The average rate of price increase for the Model 7 capital aggregate was 3.86% per year. This should be compared to the average rate of price increase for the one hoss shay capital aggregate which was much higher at 4.35% per year. On the quantity side, the stock of nonresidential construction capital increased from \$32.8 billion to \$115.9 billion (constant 1965) Canadian dollars, for an annual average (geometric) growth rate of 3.78% (3.85% for the straight line model) while the stock of machinery and equipment capital increased from \$19.1 billion to \$199.7 billion (constant 1965) Canadian dollars, for an annual average growth rate of 7.15% (7.19% for the straight line model). The Model 7 declining balance capital aggregate grew at an annual average growth rate of 4.85%. The corresponding aggregate growth rates for the one hoss shay and straight line models were 4.95% and 4.88% per year respectively.

We turn now to the service flow part of our seventh model, where we assume that producers exactly anticipate the asset rates of price change, i_{NR}^t and i_{ME}^t, for nonresidential construction and for machinery and equipment respectively; these ex post rates are listed in Table A2 of Diewert (2004). The user cost for a new asset at the start of period t, u_0^t, is defined in equations (53). Equation (54) shows that this user cost matches up with the corresponding aggregated over ages capital stock so the computations are simplified in this model. Denote these user costs by u_{NR}^t and u_{ME}^t for our two assets and denote the corresponding service aggregates by k_{NR}^t and k_{ME}^t respectively. We renormalize these series so that both user costs are unity in 1965.[60] These series, along with their annual average (geometric) growth rates, can be found in Table 12 of Diewert (2004) at 5 year intervals. Having constructed the aggregate price and quantity of nonresidential capital services, u_{NR}^t and k_{NR}^t respectively, and the aggregate price and quantity of machinery and equipment services, u_{ME}^t and k_{ME}^t respectively, we may again use the Fisher ideal formula and aggregate these two series into a single aggregate price and quantity series for capital services, which we denote by $u(7)^t$ and $k(7)^t$.[61]

Comparison of the declining balance growth rates with the corresponding straight line growth rates shows that there are some substantial differences. For example, the average annual geometric rate of growth for the user cost of machinery and equipment was 3.40% per year for the straight line model versus 2.75% per year for the geometric model. The geometric model rate of capital services price growth of 4.51% per year should be compared to the straight line model rate of capital services price growth of 4.96% per year which in turn can be compared to the average rate of price increase for the one hoss shay capital services aggregate which was somewhat higher at 4.85% per year. The use of ex post asset inflation rates again leads to user costs that are extremely volatile; see Figure 7 below. On the quantity side, the Model 7 flow of nonresidential construction capital services increased from \$1916 million to \$6764 million (constant 1965) Canadian dollars, for an annual average (geometric) growth rate of 3.78% while the flow of machinery and equipment capital services increased from \$3069 million to \$32,069 million (constant 1965) Canadian dollars, for an annual average growth rate of 7.15%. The capital services aggregate grew at an annual average growth rate of 5.55% compared to the 4.85% annual average growth rate for the aggregate capital stock. The geometric model average rate of capital services growth rate of 5.55% per year can be compared to

[60] Before normalization, the service flow aggregates k_{NR}^t and k_{ME}^t are exactly equal to the corresponding stock aggregates. Thus the rates of growth of the corresponding stock and flow variables will be the same.
[61] These series are plotted in Figures 7 and 10 below.

the straight line growth rate of capital services of 5.30% per year and to the average rate of growth for the one hoss shay capital services aggregate of 5.61% per year.

We turn now to our *second geometric depreciation model*, which will eliminate the volatility problem mentioned in the last paragraph. In this *Model 8*, instead of assuming that producers correctly anticipate each year's ex post asset rates of price change, it is assumed that producers use the current CPI inflation rate as estimators of anticipated asset price change. In terms of computations, we simply replace the two ex post asset rates of price change, i_{NR}^t and i_{ME}^t, by the CPI inflation rate ρ^t listed in Table A2 of Diewert (2004) and then repeat all of the computations made to implement Model 7 above.

It turns out that the Model 8 constant real interest rate capital stocks (and prices) are *exactly* equal to their Model 7 counterparts in Table 11. This follows from equations (57), which show that the aggregate (over ages) stock price is equal to the price of a new asset, which in turn does not depend on our assumptions about interest rates or expected asset inflation rates. Hence there is no need to table the capital stocks and prices for Model 8 (or Model 9 below). However, the Model 8 vintage user costs and capital service flows are very different from their Model 2 counterparts and slightly different from their Model 5 counterparts. Table 13 in Diewert (2004) lists the Model 8 rental prices and flows of capital services for the geometric depreciation (constant real interest rate) Canadian capital stocks at 5 year intervals over the period 1965-1999.

The overall annual rate of growth for capital services for the straight line model was 5.08% per year compared to 5.37% per year for the geometric model where both models assumed constant real interest rates. This is not a large difference. In Figures 7 and 8 below, it can be seen that the user costs that correspond to the geometric model with constant real interest rates, u(8), is much less volatile than the corresponding geometric model that assumes perfect foresight, u(7).

We turn now to our *third geometric deprecation model*, which we call *Model 9*. In this model, instead of assuming that producers correctly anticipate each year's ex post asset rates of price change, we assume that they can anticipate the *trends* in these rates. In terms of computations, we use exactly the same program that we used to implement Model 7 except that we replace the rather volatile nominal interest rate r^t that was listed in Table A2 of Diewert (2004) by the smoothed nominal interest rate that is listed in Table A3 of Diewert (2004).. We also replace the two ex post asset rates of price change, i_{NR}^t and i_{ME}^t, by their smoothed counterparts listed in Table A3 of Diewert (2004).

As mentioned earlier, the Model 9 constant real interest rate capital stocks (and prices) are *exactly* equal to their Model 7 counterparts in Table 7. Hence there is no need to table the capital stocks and prices for Model 9. However, the Model 9 vintage user costs are somewhat different from their Model 7 and 8 counterparts. Table 14 in Diewert (2004) lists the Model 9 rental prices and flows of capital services for the Canadian capital stock at 5 year intervals over the period 1965-1999.

When we compare the two capital services, k_{NR}^t and k_{ME}^t across the 3 declining balance models, they turn out to be identical and hence so are their growth rates. Hence when we aggregate across these two assets to form the Model 7,8 and 9 capital services aggregates, we find that the average annual geometric growth rates are quite similar: 5.55%, 5.37% and 5.52% respectively. However, the corresponding rental price series for each type of asset, u_{NR}^t and u_{ME}^t, are no longer identical across the two models. The geometric aggregate rental price grew at an annual geometric rate of 3.88% per year while the straight line aggregate rental price grew at a 4.31% per year rate. In Figures 7 and 9 below, it can be seen that the user cost that corresponds to the geometric model with smoothed asset inflation rates, u(9), is much less volatile than the corresponding geometric model that assumes perfect foresight, u(7), but the trend in each series is similar.

We turn now to our fourth and final class of depreciation and relative efficiency models.

10. The Linear Efficiency Decline Model

Recall that our first class of models (the one hoss shay models) assumed that the efficiency (or cross section user cost) of the asset remained constant over the useful life of the asset. In our second class of models (the straight line depreciation models), we assumed that the cross section depreciation of the asset declined at a linear rate. In our third class of models (the geometric depreciation models), we assumed that cross section depreciation declined at a geometric rate. Comparing the third class with the second class of models, it can be seen that geometric depreciation is more *accelerated* than straight line depreciation; i.e., depreciation is relatively large for new vintages compared to older ones. In this section, we will consider another class of models that gives rise to an accelerated pattern of depreciation: the class of models that exhibit *a linear decline in efficiency*.[62]

It is relatively easy to develop the mathematics of this model. Let f_0^t be the period t rental price for an asset that is new at the beginning of period t. If the useful life of the asset is L years and the efficiency decline is linear, then the sequence of period t cross sectional user costs f_n^t is defined as follows:

$$(66)\quad f_n^t \equiv f_0^t\,[L - n]/L\;; \qquad n = 0,1,2,\ldots,L-1\;;$$
$$\equiv 0\;; \qquad n = L,L+1,L+2,\ldots\,.$$

Now substitute (66) into the first equation in (5) and get the following formula for the rental price f_0^t in terms of the price of a new asset at the beginning of year t, P_0^t:

$$(67)\quad f_0^t = LP_0^t/[L + (L-1)(\gamma) + (L-2)(\gamma)^2 + \ldots + 1(\gamma)^{L-1}]$$

where the period t discount factor γ is defined in terms of the period t nominal interest rate r^t and the period t expected asset rate of price change i^t in the usual way:

$$(68)\quad \gamma \equiv (1 + i^t)/(1 + r^t).$$

Now that f_0^t has been determined, substitute (67) into (66) and substitute the resulting equations into equations (5) and determine the sequence of period t asset prices by age n, P_n^t:

$$(69)\quad P_n^t = P_0^t\,[(L-n) + (L-n-1)(\gamma) + \ldots + 1(\gamma)^{L-1-n}]/[L + (L-1)(\gamma) + \ldots + 1(\gamma)^{L-1}] \quad \text{for } n = 0,1,2,\ldots,L-1$$
$$= 0 \quad \text{for } n = L, L+1,L+2,\ldots\,.$$

Finally, use equations (8) to determine the end of period t rental prices, u_n^t, in terms of the corresponding beginning of period t rental prices, f_n^t:

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[62] Diewert (2004) showed how linear efficiency decline models can be derived from one hoss shay models where maintenance expenditures are expected to increase linearly over time.

(70) $u_n^t = (1 + r^t)f_n^t$; $n = 0,1,2,...$

Given the asset prices by age n defined by (69), we could use equations (12) above to determine the corresponding cross section depreciation rates δ_n^t. We will not table these depreciation rates since our focus is on constructing measures of the capital stock and of the flow of services that the stocks yield. However, we will note that if we recall definition (10) for the period t cross section depreciation of an asset of vintage n, $D_n^t \equiv P_n^t - P_{n+1}^t$, and assume that the nominal interest rate r^t and the nominal asset rate of price change i^t are both zero, then using (69), it can be shown that

(71) $D_n^t \equiv P_n^t - P_{n+1}^t = P_0^t\, [L - n]/[L(L+1)/2]$ for $n = 0,1,2,...,L$;

i.e., when $r^t = i^t = 0$, *depreciation declines at a linear rate* for the linear efficiency decline model. When depreciation declines at a linear rate, the resulting formula for depreciation is called *the sum of the year digits formula*.[63] Thus just as the one hoss shay and straight line depreciation models coincide when $r^t = i^t = 0$, so too do the linear efficiency decline and sum of the digits depreciation models coincide.

In our *tenth Model*, we assume that producers exactly anticipate the asset rates of price change, i_{NR}^t and i_{ME}^t, for nonresidential construction and for machinery and equipment respectively. We use the Fisher ideal index to aggregate over ages using formula (69) above for the asset prices by age. Having constructed the aggregate price and quantity of nonresidential capital, P_{NR}^t and K_{NR}^t respectively, and the aggregate price and quantity of machinery and equipment, P_{ME}^t and K_{ME}^t respectively, we may again use the Fisher ideal formula and aggregate these two series into a single aggregate price and quantity series for the wealth stock, which we denote by $P(10)^t$ and $K(10)^t$. The average rate of price increase for the linear efficiency decline capital stock aggregate was 4.13% per year, which is lower than the corresponding rate of aggregate price increase for the one hoss shay aggregate of 4.35% per year; see Table 15 in Diewert (2004). On the quantity side, the stock of nonresidential construction capital increased from \$29.6 billion to \$98.5 billion (constant 1965) Canadian dollars, for an annual average (geometric) growth rate of 3.60% while the stock of machinery and equipment capital increased from \$15.0 billion to \$166.6 billion (constant 1965) Canadian dollars, for an annual average growth rate of 7.33%. Of course the levels of the capital aggregate are only about 2/3 to 3/4 of the corresponding one hoss shay levels due to the accelerated form of depreciation for the former model. The linearly declining efficiency capital aggregate grew at an annual average growth rate of 4.74%, which is lower than the corresponding rate of growth for the one hoss shay aggregate of 4.95%.

Using equations (66), (67) and (70) along with the data tabled in Tables A1 and A2 of Diewert (2004), we can construct the end of the period user costs for each of our 39 types of nonresidential construction capital. As usual, use equation (38) to construct the service flow aggregate for nonresidential construction for each year. Then we use (42) (where L equals 39) to aggregate over the 39 types of nonresidential capital using the Fisher (1922) ideal index number formula and form the aggregate rental price for nonresidential construction, u_{NR}^t, and the corresponding services aggregate, k_{NR}^t, for the years 1965-1999.[64] These series, along with their annual average (geometric) growth rates, can be found in Table 16 of Diewert (2004) at 5 year intervals. Similarly, we use

[63] Canning (1929; 277) describes the method in some detail so it was already in common use by that time.

[64] Since all of the rental prices by age of asset are proportional to each other, again Hicks' (1939; 312-313) aggregation theorem implies that all of the usual indexes are equal to each other.

(42) above (where L equals 14) and aggregate over the 14 types of machinery and equipment using the Fisher ideal index number formula and form aggregate capital services price and quantity series, u_{ME}^t and k_{ME}^t, for the years 1965-1999. These series, along with their annual average (geometric) growth rates, can also be found in Table 16 of Diewert (2004) at 5 year intervals. Having constructed the aggregate price and quantity of nonresidential capital services, u_{NR}^t and k_{NR}^t respectively, and the aggregate price and quantity of machinery and equipment services, u_{ME}^t and k_{ME}^t respectively, we may again use the Fisher ideal formula and aggregate these two series into a single aggregate price and quantity series for capital services, which we denote by $u(10)^t$ and $k(10)^t$.[65]

Table 16 in Diewert (2004) shows that the price of a unit of nonresidential construction capital services increased by 6.32% per year and the price of a unit of machinery and equipment capital services increased by 2.54% per year on average. The average rate of price increase for the linearly declining efficiency capital services aggregate was 4.32% per year, which is much less than the corresponding rate of price increase for the one hoss shay aggregate capital services price, which was 4.85% per year. On the quantity side, the flow of nonresidential construction capital services increased from $2066 million to $7467 million (constant 1965) Canadian dollars, for an annual average (geometric) growth rate of 3.85% while the flow of machinery and equipment capital services increased from $3162 million to $33,554 million (constant 1965) Canadian dollars, for an annual average growth rate of 7.19%. The capital services aggregate grew at an annual average growth rate of 5.56% compared to the 5.61% annual average growth rate for the corresponding one hoss shay capital services. As usual, the linear efficiency decline user costs u(10) that are based on the assumption of perfect foresight are very volatile; see Figure 7.

We turn now to our *second linear efficiency decline model*, which will eliminate the volatility problem mentioned in the last paragraph. In this *Model 11*, instead of assuming that producers correctly anticipate each year's ex post asset rates of price change, it is assumed that producers use the current CPI inflation rate as estimators of these rates. This model turns out to be equivalent to the constant real interest rate model. As usual, in terms of computations, we simply replace the two ex post asset rates of price change, i_{NR}^t and i_{ME}^t, by the CPI inflation rate $\square^t$ listed in Table A2 of Diewert (2004) and then repeat all of the computations made to implement Model 10 above.

The Model 11 capital stock quantities are very similar to the Model 10 quantities. The overall average growth rate for the price of the aggregate stock is a bit higher for Model 10 (4.13% per year) than for Model 11 (3.94% per year).

The one hoss shay capital services aggregate that assumes constant real interest rates, k(2), is quite close to the linear efficiency decline capital services aggregate that assumes constant real interest rates, k(11), and their average annual geometric growth rates are also close: 5.49% for k(2) versus 5.43% for k(11). However, k(11) is 15 to 20% bigger in levels than the first linear efficiency decline capital services aggregate k(10), which assumed that anticipated asset inflation rates were equal to ex post rates. The average annual geometric growth rate for k(10) was somewhat higher at 5.56% per year.

We turn now to our *third linear efficiency decline model*. In this model (*Model 12*), instead of assuming that producers correctly anticipate each year's ex post asset rates of price change, we assume that they can anticipate the *trends* in these rates. In terms of computations, we use exactly the same program that we used to implement Model 10

[65] These series are plotted in Figures 7 and 10 in section 11.

except that we replace the rather volatile nominal interest rates r^t that are listed in Table A2 of Diewert (2004) by the smoothed nominal interest rates that are listed in Table A3 of Diewert (2004). We also replace the two ex post asset inflation rates, i_{NR}^t and i_{ME}^t, by their smoothed counterparts listed in Table A3 of Diewert (2004). It turns out that there are some small differences between the capital stocks generated by our three variants of the linear efficiency decline model but the average growth rates are virtually identical. There is more variation across the three models in the movement of the stock prices with Model 10 giving the highest rate of price growth for the capital aggregate (4.13% per year), followed by Model 12 (4.04% per year) and then Model 11 (3.94% per year). However, there are large differences in the levels and small differences in the growth rates for capital services generated by the 3 models: the average annual geometric growth rates for k(10), k(11) and k(12) are 5.56%, 5.43% and 5.55% per year. The average annual geometric growth rates for K(10), K(11) and K(12) are 4.74%, 4.72% and 4.74% per year respectively. However, there is much more variation across the three models in the movement of the service prices with Model 10 giving the highest rate of price growth for the capital services aggregate (4.32% per year), followed by Model 12 (3.78% per year) and then Model 11 (3.27% per year).

Viewing Figures 7-9, the aggregate linear efficiency decline user cost series u(10), which assumes that anticipated asset inflation rates are equal to the actual ex post rates, is the highest very volatile curve. Smoothing these volatile asset inflation rates leads to the u(12) curve, which is much smoother and captures the trend in u(10). The constant real interest rate user cost series, u(11), lies far below the other two aggregate user cost series for much of the sample period.

Figures 10-12 plot the three linear efficiency decline aggregate capital services series, k(10)-k(12). Each of these series is reasonably smooth but note that they are spread out much more than the corresponding aggregate capital stock series, K(10)-K(12), that are plotted in Figures 4-6. *Thus the different assumptions on anticipated asset price movements generate substantially different measures of capital services for these linear efficiency decline models*. The constant real interest rate series, k(11), is the top curve, followed by the smoothed asset inflation rates model, k(12), and the ex post asset inflation rates model, k(10), is the lowest curve.

In the following section, we make some graphical comparisons across our 12 models.

11. A Comparison of the Twelve Models

In this section, we will compare stock prices and user costs across our four types of model that are based on alternative assumptions about the structure of depreciation or asset efficiency, holding constant our assumptions about nominal interest rates and anticipated asset price movements. We will also compare capital stocks and service flows across depreciation and relative efficiency models, holding constant our assumptions about nominal interest rates and anticipated asset price movements.

Figure 1 plots the aggregate capital stock prices generated by our four depreciation and efficiency models, assuming that ex post asset price movements are perfectly anticipated. Note the volatility of these series. The one hoss shay stock prices P(1) are the highest, followed by the linear efficiency decline prices P(10). The straight line and geometric depreciation prices, P(4) and P(7), are the lowest and are very close to each other.

Figure 2 plots the aggregate capital stock prices generated by our four depreciation and efficiency models, assuming that ex post asset price changes are equal to changes in the consumer price index. This model assumes a constant real interest rate of 4 per cent.

These stock prices are much smoother than those exhibited in Figure 1 and they are also much closer to each other. The one hoss shay and linear efficiency decline prices, P(2) and P(11), are virtually indistinguishable on the top, followed by the straight line depreciation prices P(5) and then followed very closely by the geometric stock prices P(8).

Figure 3 plots the aggregate capital stock prices generated by our four depreciation and efficiency models, assuming that anticipated asset price changes are equal to smoothed ex post asset price changes. These stock price series smooth out considerably the much rougher series exhibited in Figure 1. The one hoss shay stock prices P(3) are the highest, followed by the linear efficiency decline prices P(12). The straight line and geometric depreciation prices, P(6) and P(9), are the lowest and are very close to each other.

Figure 4 plots the aggregate capital stocks that correspond to the perfectly anticipated asset prices assumption for the four depreciation models. The one hoss shay capital stock curve K(1) is the highest, followed by the straight line depreciation curve K(4), which in turn is followed by the geometric depreciation curve K(7). The linear efficiency decline stock K(10) is the lowest curve. These results are intuitively plausible: the one hoss shay model has the least accelerated form of depreciation, followed by the straight line model, followed by the geometric depreciation model and the linear efficiency decline model generates the most accelerated form of depreciation. In an economy where investment is growing over time, the capital stocks corresponding to the least accelerated form of depreciation will grow the quickest, followed by the more accelerated forms and the capital stock corresponding to the most accelerated form of depreciation will grow the slowest. Figures 5 and 6 plot the aggregate capital stocks that correspond to the constant real interest rate and the smoothed asset price models: the results are much the same as those exhibited in Figure 4.

Figure 7 plots the aggregate user costs generated by our four classes of depreciation and efficiency models, assuming that ex post asset price movements are perfectly anticipated. Note that the user cost series in Figure 7 are even more volatile than the capital stock prices charted in Figure 1. The one hoss shay and straight line depreciation user costs, u(1) and u(4), are the highest, followed by the geometric depreciation and linear efficiency decline user costs, u(7) and u(10).

Figure 8 plots the aggregate user costs generated by our four classes of depreciation and efficiency models, assuming that ex post asset price changes are equal to changes in the consumer price index. This model assumes a constant real interest rate of 4 per cent. These user costs are much smoother than those exhibited in Figure 7 and they are also much closer to each other. The straight line depreciation user costs u(5) are on top, followed by the one hoss shay, geometric and linear efficiency decline user costs, u(2), u(8) and u(11), which are too close to each other to be distinguished visually.

Figure 9 plots the aggregate user costs generated by our four classes of depreciation and efficiency models, assuming that anticipated asset price changes are equal to smoothed ex post asset price changes. These user cost series smooth out considerably the much rougher series exhibited in Figure 7. The straight line and one hoss shay user costs, u(6) and u(3), are very close to each other on top but near the end of our sample period, the one hoss shay user costs u(3) dip below the straight line depreciation user costs u(6). The geometric depreciation and linear efficiency decline user costs, u(9) and u(12), are fairly close to each other on the bottom. These two models represent the most accelerated forms of depreciation.

Figures 10 and 11 plot the aggregate capital services that correspond to the perfectly anticipated asset price change and the constant real interest rate models. The aggregate services using ex post asset price changes plotted in Figure 10 are more volatile and more widely dispersed than the aggregate services plotted in Figures 11 and 12, as one might expect. The linear efficiency decline services are the top curve k(10), followed by the geometric depreciation services k(7), followed by the one hoss shay services k(1) and the straight line depreciation capital services k(4) are the bottom curve. The aggregate services using constant real interest rates plotted in Figure 11 are fairly similar to the smoothed capital services exhibited in Figure 12. For the constant real interest rate services in Figure 11, the one hoss shay and linear efficiency decline services, k(2) and k(11), are at the top followed very closely by the geometric depreciation services k(8) and the straight line depreciation capital services k(5) are the bottom curve. Figure 12 plots the aggregate capital services that correspond to the smoothed asset price change model; i.e., Figure 12 is the quantity counterpart to Figure 9. The linear efficiency decline capital services curve k(12) is the highest, followed closely by the geometric depreciation and one hoss shay curves, k(9) and k(3), which are very close to each other. The straight line depreciation curve k(6) is the lowest curve and is well below the other three curves. Thus overall, three of our four depreciation and efficiency models give rise to much the same measures of capital services, holding constant the assumptions about asset price changes and the reference interest rate. However, the straight line depreciation capital services seem to be consistently below the corresponding services generated by the other three classes of models.

Our conclusion at this point is that both the form of depreciation that is assumed (light bulb, straight line, geometric or linear efficiency decline) and the assumptions on interest rates and price expectations (perfect foresight, constant real rate or anticipated capital gains) matter. This means it will be necessary for statistical agencies to introduce surveys to determine when assets are retired or sold and it will be necessary for economists to decide what is the “best” set of assumptions concerning the nominal opportunity cost of capital and anticipated asset price changes.

12. The Treatment of Intangible Assets

Since this volume is primarily concerned with the treatment of intangible assets, we devote this section to indicating how the above treatment of tangible assets can be modified to deal with intangible assets.

Examples of expenditures on intangible assets are *advertising and marketing expenses* and *research and development expenditures*. Both of these categories of expenditures have the character that the present period outlays will create incremental revenues in the future for the firm that undertakes them. These current period expenditures on intangible assets have a different character than expenditures on tangible durable inputs, which can be used for a number of periods and then sold to other users.[66] The problem in this section is to determine how to *allocate* the cost outlays on intangible investments over future periods. Thus the accounting problems in the present section have a different character than in the previous sections, where a straightforward opportunity cost

[66] In many cases, the stream of future revenues created by an intangible investment can be sold on the marketplace (e.g., patents, trademarks and franchises), but this still does not solve the problem of how to distribute the intangible investment costs over future periods.

approach was used. In the present section, the approach taken is one of *matching* current costs with future expected revenues.[67]

To fix ideas, suppose that in period t, a firm has made expenditures on creating an intangible asset, which are equal to C^t:

$$(72)\ C^t \equiv \sum_{m=1}^{M} P_m^t Q_m^t$$

where P_m^t is the period t price for the mth type of input that is used to create the intangible asset and Q_m^t is the corresponding quantity purchased. These expenditures in period t are expected to generate a future stream of incremental revenues for the firm. Let R_0^t denote the immediate period t incremental revenues (which could be zero) and let R_n^t denote the incremental revenues that the period t expenditures C^t are expected to generate n periods from the present period t, for n = 1,2,… Let r^t be the (nominal) period t opportunity cost of financial capital.[68] Then the discounted value of these expected incremental revenues is:

$$(73)\ R^t \equiv R_0^t + R_1^t/(1+r^t) + R_2^t/(1+r^t)^2 + R_3^t/(1+r^t)^3 + \ldots$$

The problem is to allocate the current period cost C^t over future periods. Thus let C_n^t be the allocation of C^t to the accounting period that is n periods after period t for n = 0,1,2,… At first sight, it seems reasonable that these future cost allocations C_n^t should sum to C^t. However, this turns out not to be so reasonable: costs that are postponed to future periods must be escalated by the (nominal) interest rate r^t, so that the present value of discounted future costs is equal to the actual period t costs C^t. Thus the intertemporal cost allocations C_n^t should satisfy the following equation:

$$(74)\ C^t = C_0^t + C_1^t/(1+r^t) + C_2^t/(1+r^t)^2 + C_3^t/(1+r^t)^3 + \ldots$$

To see why discounting is necessary, consider the following simple example where we invest C^t during the present period and we anticipate the revenue R_2^t two periods from now. The expected discounted profits that this investment will generate are:

$$(75)\ \Pi \equiv -C^t + R_2^t/(1+r^t)^2.$$

The period by period cash flows for this project are $-C^t$, 0, R_2^t. We want to match the period t cost C^t with the period t+2 revenue flows. Thus we want to convert the cash flow stream $-C^t$, 0, R_2^t into an equivalent cash flow stream 0, 0, $-C_2^t + R_2^t$. If we choose

$$(76)\ C_2^t \equiv C^t(1+r^t)^2,$$

then it can be seen that these two cash flow streams have the same present value and C_2^t is the "right" period t+2 cost allocation. Put another way, if we simply carried forward the period t costs C^t and set C_2^t equal to C^t, we would be neglecting the fact that the costs took place in period t while the return on the investment was deferred until period t+2 and hence, we need to charge the opportunity cost of financial capital for two periods on the initial investment (for two periods) until it is expensed in period t+2.

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[67] Paton and Littleton (1940; 123) argued that the primary purpose of accounting is to match costs and revenues. For an excellent early discussion on the importance of matching costs to future revenues, see Church (1917; 193).

[68] Thus for simplicity, we are making assumption (4) in section 2.

How should the intertemporal cost allocations C_n^t be chosen? It is natural to make these cost allocations proportional to the corresponding period anticipated revenues. Thus choose the number □ so that the following equation is satisfied:

$$(77)\ C^t = \square R^t.$$

Thus we set the observed period t cost associated with the intangible investment C^t equal to the constant □ times the discounted value of the anticipated incremental revenue stream R^t that the investment is expected to yield.[69]

Typically, □ will be equal to or less than one, since otherwise, the period t intangible investment expenditures C^t should not be undertaken. If □ is less than one, then there will be an expected profit above the opportunity cost of capital, which could be some form of monopoly profit or a reward for risk taking.

Once □ has been determined by solving (77), then the intertemporal cost allocations C_n^t can be defined to be proportional to the corresponding anticipated incremental revenues R_n^t for future periods:

$$(78)\ C_n^t \equiv \square R_n^t\,; \qquad n = 0,1,2,\ldots$$

At this point, it is possible to use the algebra developed in sections 2 and 3 above with some slight modifications. We can convert the nominal cost allocation factors C_n^t into constant (period t) dollar cost allocations f_n^t as follows:

$$(79)\ f_n^t \equiv C_n^t/(1+\square^t)^n\,; \qquad n = 0,1,2,\ldots$$
$$= \square R_n^t/(1+\square^t)^n$$

where $\square^t$ is the period t consumer price inflation rate, which is expected to persist into the future.[70] The f_n^t defined by (79) are the counterparts to the period t cross sectional rental prices that were defined in section 2. Once these intertemporal constant dollar cost allocation factors f_n^t have been defined by (79), we can use equations (5) in section 2 to define the sequence of constant dollar asset values[71], P_0^t, P_1^t, P_2^t, …, except that the period t expected rate of asset price change i^t in equations (5) is replaced by the consumer price index inflation rate $\square^t$. If we then make use of (18), which expresses the nominal interest rate r^t in terms of the real rate r^{*t} and the CPI inflation rate $\square^t$, so that $1+r^t = (1+\square^t)(1+r^{*t})$, then equations (5) simplify to the following equations:

$$(80)\ C^t = P_0^t = f_0^t + f_1^t/(1+r^{*t}) + f_2^t/(1+r^{*t})]^2 + f_3^t/(1+r^{*t})]^3 + \ldots$$
$$P_1^t = f_1^t + f_2^t/(1+r^{*t}) + f_3^t/(1+r^{*t})]^2 + f_4^t/(1+r^{*t})]^3 + \ldots$$
$$P_2^t = f_2^t + f_3^t/(1+r^{*t}) + f_4^t/(1+r^{*t})]^2 + f_5^t/(1+r^{*t})]^3 + \ldots$$

The sequence of constant dollar "asset" values $C^t = P_0^t$, P_1^t, P_2^t, … shows how the period t intangible investment can be written down over time in constant period t dollars and equations (10) and (12) in section 3 show how a sequence of constant dollar depreciation rates $\square_n^t$ for the intangible investment can be obtained from the sequence of constant

[69] Of course, the practical problem that the national income accountant will face is: how can the future stream of incremental revenues be estimated?

[70] This expectational assumption could be relaxed at the cost of more notational complexity.

[71] Note that P_0^t is equal to C^t.

dollar “asset” values, P_n^t.[72] These depreciation rates δ_n^t can also be applied to the investment components Q_m^t to form estimated constant dollar input stocks for the intangible investments.[73] Thus the assumptions made about the shape of the anticipated future period incremental revenues generated by the intangible investment,[74] along with the matching of costs to revenues methodology, determine the pattern of depreciation that can be used to write down these costs associated with the intangible investment over time.[75]

The period t beginning of the period and end of period user cost charges, f_0^t and u_0^t respectively, for the intangible investment have the following forms:

$$(81)\ f_0^t \equiv P_0^t - [(1+\rho^t)/(1+r^t)]P_1^t$$
$$= P_0^t - P_1^t/(1+r^{*t})$$
$$= [P_0^t r^{*t} + D_0^t]/(1+r^{*t});$$

$$(82)\ u_0^t \equiv P_0^t(1+r^t) - (1+\rho^t)P_1^t$$
$$= (1+\rho^t)[P_0^t r^{*t} + D_0^t]$$

The above two formulae show that the period t “user costs” for the intangible investment does not consist solely of a depreciation charge, D_0^t: there are also real interest rate charges that must be added to the depreciation term.

It should be noted that the cost allocation model outlined above can be applied to other forms of “assets”; namely, deferred charges, prepaid expenses[76] and transfer fees when a reproducible asset is acquired. The one hoss shay form of revenue matching is probably the preferred method for dealing with this type of “asset”.

13. Conclusion

We have considered the problems involved in constructing price and quantity measures for both the capital stock and the flow of services yielded by the stock in an inflationary environment. In order to accomplish these tasks, the statistician will have to make decisions in a number of dimensions:

- What length of life L best describes the asset?
- What form of depreciation or asset efficiency is appropriate?

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[72] If the assumptions on the anticipated (real) incremental revenues are such that the f_n^t decline at the geometric depreciation rate δ, then this rate will carry over to P_n^t; i.e., we will have $P_n^t = (1-\delta)^n C^t$ for n = 0,1,2,… if $f_n^t = (1-\delta)^n f_0^t$ for n = 1,2,…

[73] It is not necessary for the statistical agency to do this but some users will be interested in the resulting M asset stocks that form capital stock aggregates of the Q_m^t. Normal index number theory can be used to aggregate these M stock components into an overall capital stock aggregate using the period t flow prices P_m^t as price weights.

[74] Thus the specific depreciation models presented in sections 7-10 can be adapted to the present context.

[75] It should be noted that the obsolescence problems discussed in section 5 do not occur in the present context because the asset inflation rate and the CPI inflation rate coincide. However, obsolescence problems can still occur when technical progress causes expectations about future incremental revenues to be revised downwards.

[76] Hatfield (1927; 16) gives several examples of this type of asset, including insurance payments which apply to multiple accounting periods, the stripping away of surface rock for a strip mine and prepaid expenses. Hatfield (1927; 18) notes that this type of asset is different from the usual sort of tangible asset since this type of asset cannot readily be converted into cash; i.e., it has no opportunity cost value.

- What assumptions should be made about the reference interest rate and the treatment of anticipated asset price change?

In this paper, we focused on the last two questions. We considered four classes of depreciation or efficiency and three types of assumption on the nominal interest rate r^t and on the anticipated asset rate of price change, i^t, giving 12 models in all. We evaluated these 12 models using aggregate Canadian data on two asset classes over the period 1926-1999. We found that the assumptions on the form of depreciation or asset efficiency by age were less important than the assumptions made about the reference interest rate and the treatment of anticipated asset price changes.[77]

We consider the third question above first. In order to answer this question, it is necessary to ask about the purpose for which the capital data will be used. For some purposes, it may be useful to use ex post asset price changes as anticipated price changes. For example, this approach may be useful in constructing estimates of taxable business income if capital gains are taxable. It may also be useful if we want to evaluate the ex post efficiency of a firm, industry or economy. However, for most other uses, assuming that anticipated price changes are equal to actual ex post price changes is very unsatisfactory since it is unlikely that producers could anticipate all of the random noise that seems to be inherent in series of actual ex post asset price changes. Moreover, this approach generates tremendous volatility in user costs and statistical agencies would face credibility questions if this approach were used.

Thus we restrict our attention to the choice between assuming a constant real interest rate or using smoothed ex post asset price changes as estimates of anticipated asset price changes. The assumption of constant real interest rates has a number of advantages:

- The resulting price and quantity series tend to be very smooth.
- The estimates are *reproducible*; i.e., any statistician given the same basic price and quantity data along with an assumed real interest rate will be able to come up with the same aggregate price and quantity measures.

However, the use of smoothed ex post asset price changes as measures of anticipated asset price changes has some advantages as well:

- Longer run trends in relative asset prices can be accommodated.
- The anticipated obsolescence phenomenon can be captured.

Each individual statistical agency will have to weigh the costs and benefits of the two approaches in order to decide which approach to use. I think that for most assets, it would be quite acceptable to use the constant real interest rate model and this would maximize reproducibility. However, with assets that have experienced rapid technical progress, I would prefer to use the smoothed expectations model, since this model will better capture obsolescence effects. I would also use the smoothed expectations model for land, since over long periods, land prices tend to appreciate faster than the general price level.

We now discuss which of our four sets of assumptions on the form of depreciation or vintage asset efficiency decline is "best".

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[77] Harper, Berndt and Wood (1989) also found that differing assumptions on r^t and i^t made a big difference empirically using U.S. data. However, they considered only geometric depreciation. Our paper can be viewed as an extension of their work to consider also variations in the form of depreciation.

The one hoss shay model of efficiency decline, while seemingly a priori attractive, does not seem to work well empirically; i.e., vintage depreciation rates tend to be much more accelerated than the rates implied by the one hoss shay model. If maintenance costs are linearly rising over time, a "gross" one hoss shay model gives rise to a linearly declining efficiency model,[78] which of course, is a model that exhibits very accelerated depreciation.

The straight line depreciation model, while not as inconsistent with the data as the one hoss shay model, also does not generate the pattern of accelerated depreciation that seems to characterize many used asset markets. However, given the simplicity of this model (to explain to the public), it could be used by statistical agencies.

The geometric depreciation model seems to be most consistent with the empirical studies on used assets of the four simple classes of model that we considered.[79] Of course, geometric depreciation has the disadvantage that it will never exhaust the full value of the asset.[80]

Finally, a good alternative to the geometric depreciation model is the linear efficiency decline model. However, this model may have a pattern of "over-accelerated" depreciation relative to the geometric model. What is required is more empirical work so that the actual pattern of depreciation can be determined. In particular, statistical agencies need to consider establishing capital asset surveys, which would ask firms not only what assets they purchased during the reference period, but also what assets they sold or scrapped during the reference period.[81]

We conclude by noting some limitations of the analysis presented in this paper:

- We have not dealt in great detail with the problems posed by *unique* assets, although the model presented in the previous section could be used.
- We have not dealt with the problems posed by assets that depreciate by *use* rather than by *age*.[82]
- We have neglected property taxes, income taxes and insurance premiums as additional components of user costs.
- We have neglected the problems posed by indirect commodity taxes on investment goods; this complication can lead to differences between investment prices and asset stock prices.
- We have neglected many forms of capital in our empirical work including inventories, land, knowledge capital (except for our brief discussion in the previous section), resource stocks and infrastructure capital.
- We have not discussed the many complexities involved in making quality adjustments for new types of capital.
- We have not discussed the problems posed by establishment deaths on asset lives and depreciation rates. We would expect asset lives to decrease during recessions but we have not spelled out exactly how to adjust for this factor.

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[78] See Diewert (2004).

[79] See Hulten and Wykoff (1981a) (1981b) and Jorgenson (1996a).

[80] Some statistical agencies solve this problem by "scrapping" the depreciated value of the asset when it reaches a certain age. This solves one problem but it introduces two additional problems: (i) the truncation age has to be decided upon and (ii) the theoretical simplicity of the model is lost.

[81] The survey should also ask for information on what the age and initial purchase prices of the sold or scrapped assets was.

[82] Our reason for neglecting use is simple: usually, the national statistician will not have data on the use of machines available.

However, we have provided a fairly comprehensive review of most of the issues surrounding the measurement of capital, including a method for forming intangible capital stocks.

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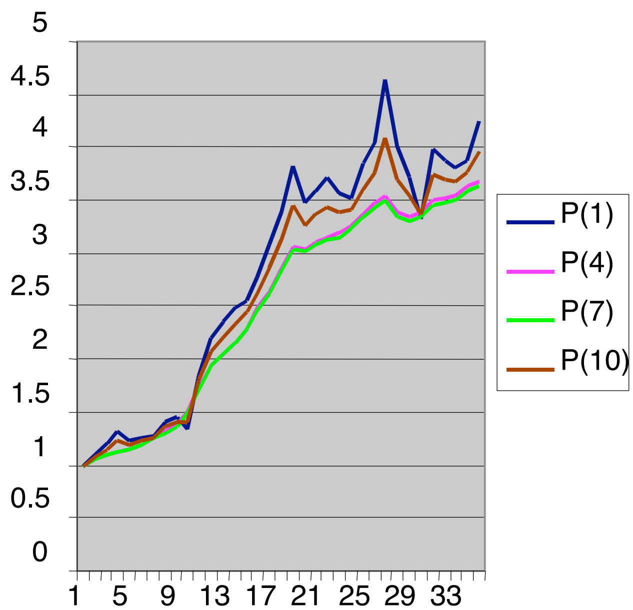


Figure 1: Capital Stock Prices using Ex Post Price Changes

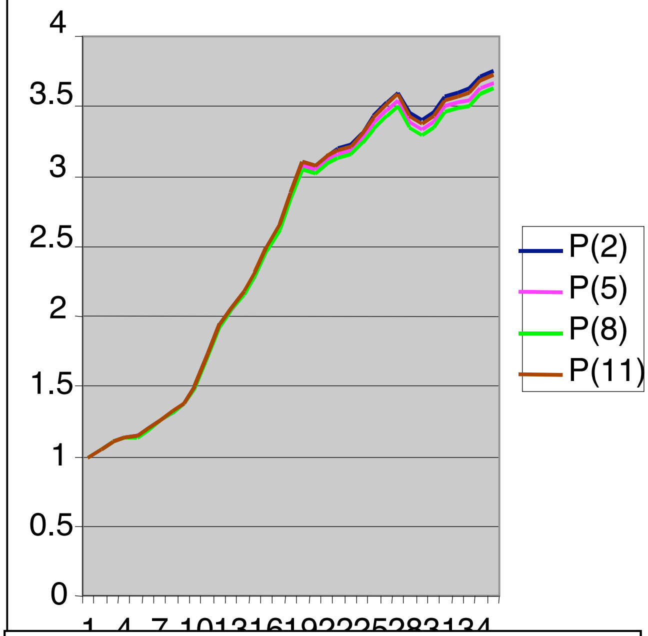
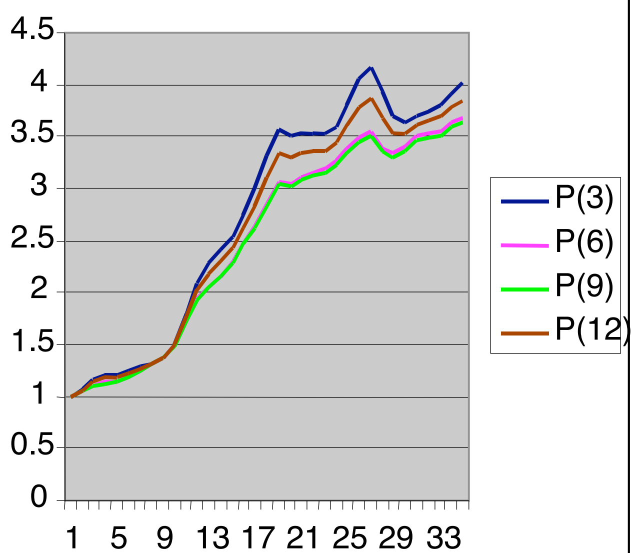


Figure 2: Alternative Stock Prices with Constant Real Interest Rates



Using Smoothed Asset Price Changes

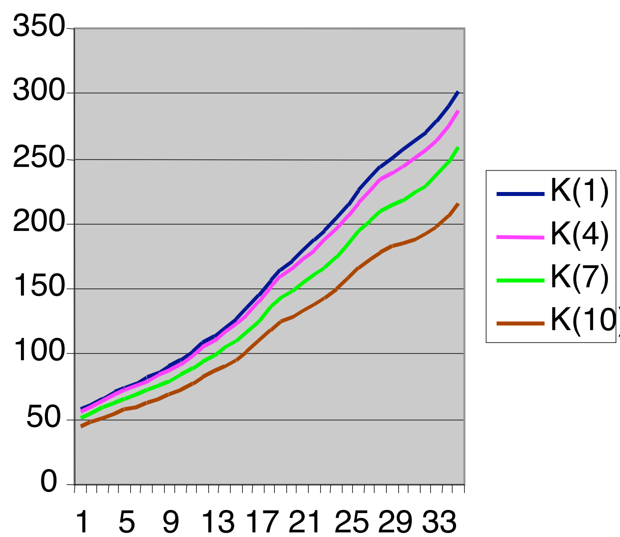


Figure 4: Alternative Capital Stocks Using Ex Post Price Changes

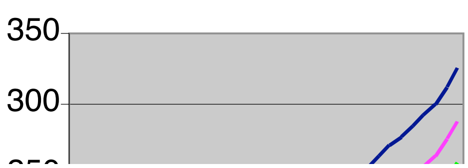


Figure 5: Alternative Capital Stocks Using Constant Real Rates

Figure 6: Alternative Capital Stocks Using Smoothed Asset Price Changes

Figure 7: Alternative User Costs Using Ex Post Asset Price Changes

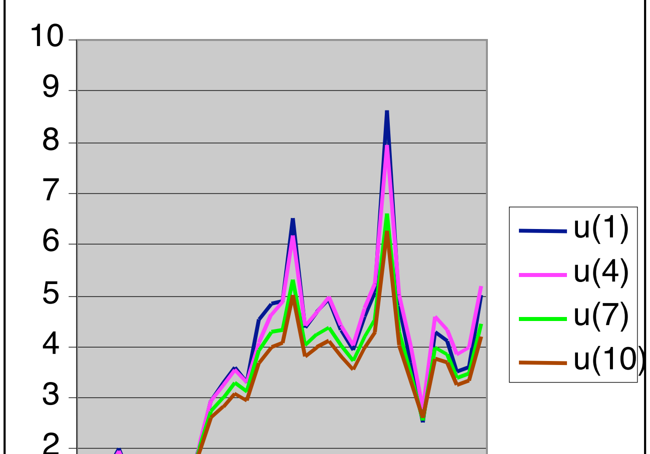


Figure 8:Alternative User Costs Using Constant Real Interest Rates

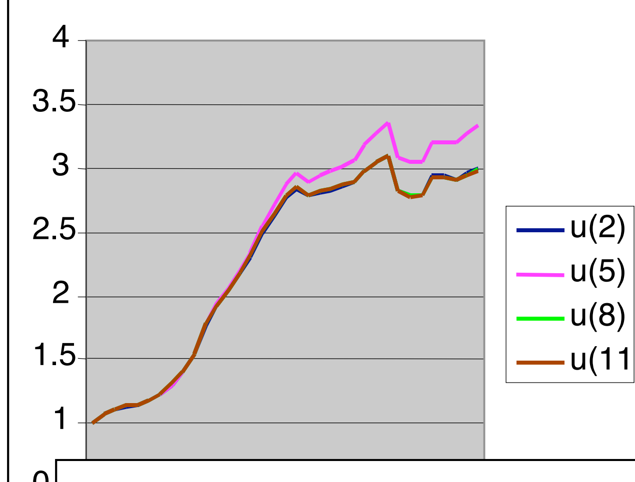


Figure 9: Alternative User Costs Using Smoothed Asset Price Changes

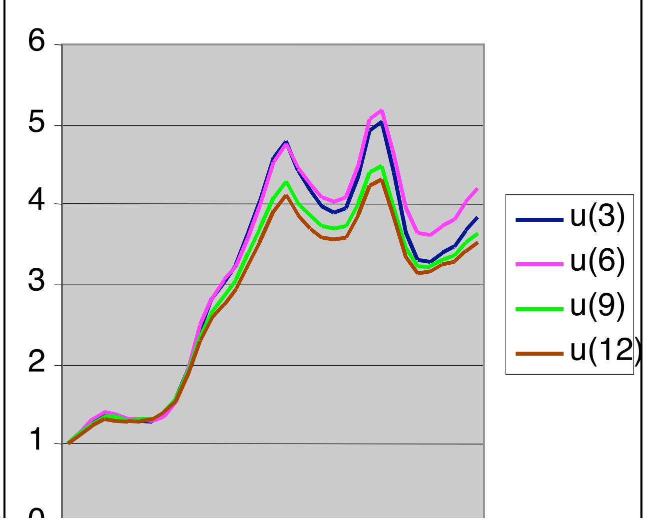
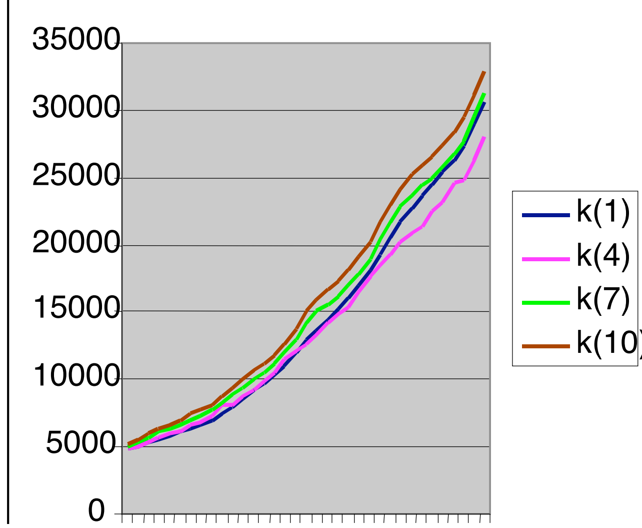


Figure 10: Alternative Capital Services Using Ex Post Price Changes



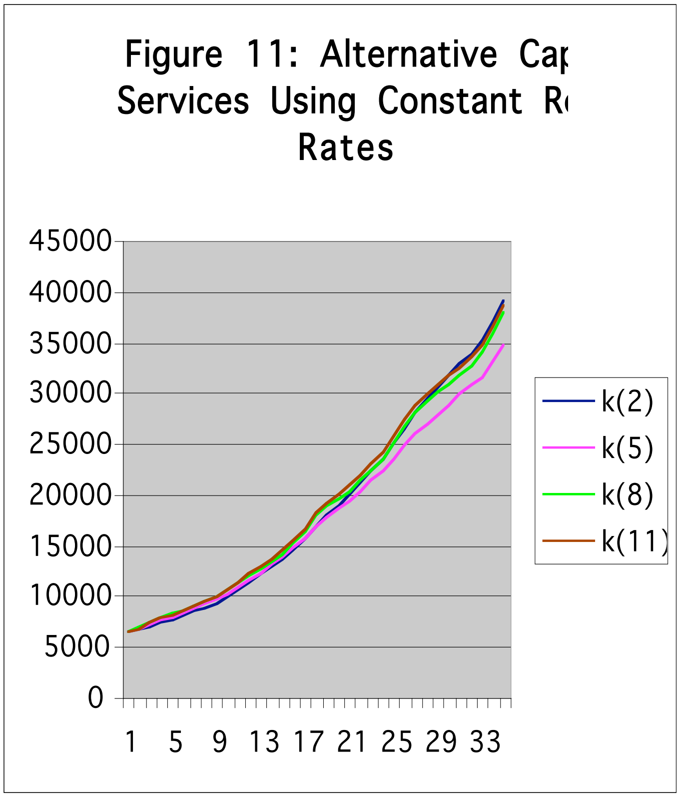


Figure 11: Alternative Cap
Services Using Constant R
Rates
45000
40000
35000
30000
25000
20000
15000
10000
5000
0
1 5 9 13 17 21 25 29 33
k(2)
k(5)
k(8)
k(11)

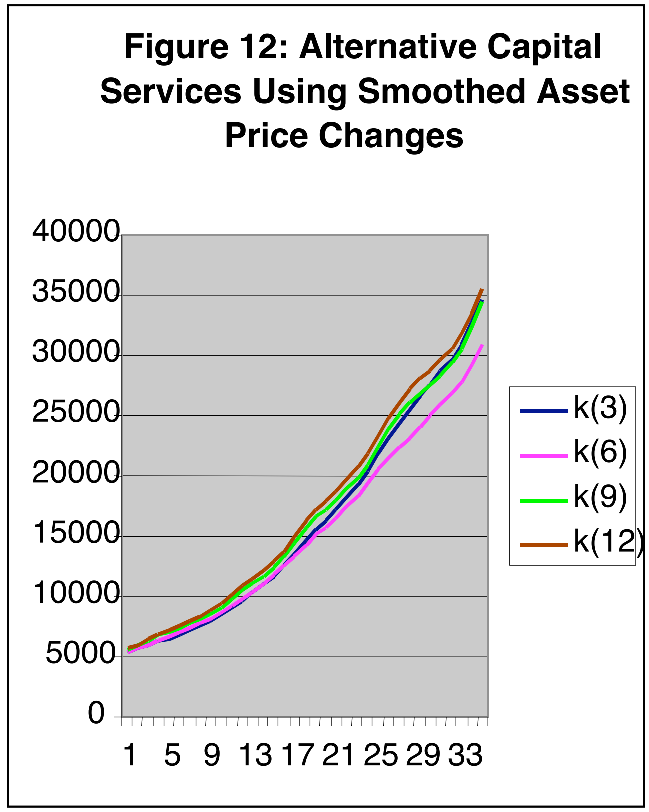


Figure 12: Alternative Capital Services Using Smoothed Asset Price Changes
40000
35000
30000
25000
20000
15000
10000
5000
0
1 5 9 13 17 21 25 29 33
k(3)
k(6)
k(9)
k(12)