State Dependent Pricing and Business Cycle Asymmetries

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STATE DEPENDENT PRICING AND BUSINESS CYCLE ASYMMETRIES

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Abstract. We present a tractable model of state-dependent pricing and study price adjustment in response to shocks. We find a distinct asymmetry in this response. Positive shocks to marginal cost generate greater price flexibility than negative shocks of the same magnitude. This arises from a strategic linkage between firms in price adjustment incentives. With a positive marginal cost shock, prices are strategic complements: firms have greater incentive to increase prices when other firms increase theirs. But for negative shocks, prices are strategic substitutes: firms have less incentive to lower prices when others lower theirs. We analyze this asymmetry in a dynamic general equilibrium model, and examine the response of aggregate prices and quantities to monetary policy shocks. For empirically relevant shocks there is a substantial difference between time- and state-dependent pricing models. In addition, our state-dependent pricing model can account for business cycle asymmetries in output found in empirical studies.

1. Introduction

A large literature in macroeconomics studies the role of nominal rigidities in dynamic general equilibrium settings. In these models, nominal prices adjust slowly in response to shocks. According to the usual argument, the presence of small fixed costs of changing prices makes it unprofitable for firms to adjust prices frequently. Firms choose to adjust prices when the benefits outweigh the fixed costs. This means that the degree of price rigidity – the

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fraction of price-adjusting firms – at a point in time depends on the state of the economy. That is, price adjustment is *state-dependent*.

In contrast, most macroeconomic models of price rigidity employ *time-dependent* rules for price adjustment. In these models, the frequency of a firm’s price adjustment does not depend on its current revenue or cost conditions. Classic papers in this literature are Taylor (1980) and Calvo (1983).

The usual argument for this approach is that for relatively small shocks, a firm’s gain from changing its price would be less than the explicit cost of price adjustment. Hence, variation in the degree of price rigidity is unlikely to be quantitatively important in response to business cycle shocks. In light of this, most of the analysis in this literature is confined to studying local properties of linearized model dynamics about a steady state. This means that one cannot distinguish between the effects of small and large business cycle shocks on output and inflation; nor can one distinguish between the effects of positive and negative shocks.

Many empirical studies have found evidence of non-linear and asymmetric effects of business cycle shocks on inflation and output (see DeLong and Summers, 1988; Cover, 1992; Macklem et al., 1996; Peltzman, 2000; Ravn and Sola, 2004; and the references therein). Positive monetary policy shocks have smaller expansionary effects on output relative to the contractions associated with negative shocks of the same magnitude; that is, the effect of monetary policy shocks are asymmetric. In addition, large monetary policy shocks (both positive and negative) lead to smaller output multipliers than shocks of smaller magnitude; the effect of monetary shocks are non-linear.

Models of state-dependent pricing represent a potential explanation for these non-linearities and asymmetries. However, the analysis of these models is technically challenging due to the induced heterogeneity in prices charged across firms. Study of state-dependent pricing models has usually made progress in specialized environments not amenable to quantitative business cycle analysis (see, for instance, Ball and Romer, 1990; Caplin and Leahy, 1991; Ball and Mankiw, 1994; and Conlon and Liu, 1997). By contrast, Dotsey et al. (1999) construct a model of state-dependent pricing within a more familiar dynamic general equilibrium environment. Due to the large state space in that model however, the analysis requires local linearization. This does not easily lend itself to the study of non-linear and asymmetric effects of monetary policy shocks.

In this paper, we study a simple ‘hybrid’ time- and state-dependent pricing model. We follow a suggestion of Ball and Mankiw (1994) and assume that firms specify prices in contracts of fixed duration. However, during the life of a given contract, a firm can ‘opt-out’ and revise its price by incurring a (fixed) recontracting cost. The model is made dynamic by specifying

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1 See also, Rotemberg and Woodford (1997), Chari et al. (2000), and Christiano et al. (2001). These papers represent an extremely small subset of the relevant research. For good surveys, see Goodfriend and King (1997), Taylor (1999), and Gali (2002).
that firms set price contracts in a staggered fashion, as in Taylor (1980).\footnote{Hence, our work can be seen as an extension of the work of Dotsey et al. (1999) in a more tractable framework. See also Ireland (1997) for an application of a hybrid price-setting model to the study of disinflation.} The fixed duration of contracts admits model solutions with a limited state space (this is the time-dependent aspect of the model). However, the state-dependent nature of price determination is preserved since a firm's incentive to opt-out will be determined by the state of nature. By allowing for this hybrid form of price determination, we are able to provide a complete analysis of the non-linear dynamics of our stochastic model.

A central result of the paper is the presence of a distinct asymmetry in a firm's state-dependent price adjustment decision in response to shocks. Positive shocks to marginal cost generate greater price flexibility than negative shocks of the same magnitude. This asymmetry arises due to a strategic linkage between firms in the incentive to change prices. With a positive marginal cost shock, prices are strategic complements: a firm has more incentive to increase its price when other firms increase theirs. But for a negative shock, prices are strategic substitutes: a firm has less incentive to lower its price when other firms lower theirs.

We integrate state-dependent pricing into a dynamic general equilibrium model, and examine the response of aggregate prices and quantities to monetary policy shocks. We stress two results. First, there is a substantial difference between time-dependent and state-dependent pricing models for empirically relevant shocks. In our state-dependent pricing model the responses of aggregate price and output to monetary policy shocks are highly non-linear. When we calibrate our model to postwar US data and empirically plausible estimates of the fixed costs of price adjustment, the predicted output response is smaller (and the price response is larger) than that suggested by time-dependent models.

Secondly, the aggregate response of the calibrated model inherits the asymmetry that we find in the incentives facing individual firms to adjust prices. At the aggregate level, the price level responds much more to a positive money shock than to a negative shock of the same magnitude. Consequently, the output response to positive shocks is much smaller than the response to negative shocks. This confirms the findings of the empirical literature mentioned above. Performing simulations with our model, we find that it can broadly account for the magnitude of the business cycle asymmetries that have been found empirically.

In addition, we explore the sensitivity of our results to the degree of ‘real rigidity,’ as defined by Ball and Romer (1990). To do this in our model, we dampen the responsiveness of real marginal costs to output fluctuations. When marginal costs are ‘flat,’ there is little incentive for price adjustment in response to either positive or negative monetary policy shocks. In this sense, one might argue that the consideration of state-dependent pricing is of little relevance.
However, we show that this argument is fragile. With real rigidity, there is a more traditional strategic complementarity present: a firm’s adjustment incentive is increasing in the number of price-adjusting firms. This is because with flat real marginal costs, nominal marginal costs are sensitive to the degree of price flexibility. Hence, state-dependent pricing models are likely to admit two locally isolated equilibria: one in which few firms adjust if other firms are not adjusting, and one in which all firms adjust if all other firms are adjusting.\(^3\) With real rigidity, multiple equilibria exist for relatively modest sized shocks. But the interesting finding is that this result interacts with our asymmetry result: the model is more likely to display multiple equilibria for positive shocks than for negative shocks.

Before proceeding, we note that Burstein (2002) also documents asymmetric responses in a model with state-dependent pricing. However, the tractability of our hybrid model allows us a more detailed discussion, and to isolate the source of the asymmetry. We find that the asymmetry is due to: (i) the strategic linkage between firms’ pricing decisions, and (ii) the positive covariance of aggregate price and marginal cost in equilibrium. We show that this asymmetry is operational for positive and negative shocks of all magnitudes; also, it is operational both with and without real rigidity.

In the next section, we present a simple static version of our state-dependent pricing model. We do this to highlight the key features generating asymmetries. In Section 3, we outline the dynamic version of our model, and show how the modelling of state-dependent pricing is made tractable when price contracts are of limited duration. Sections 4 and 5 present quantitative results from the dynamic model. Section 6 discusses potential extension to the analysis, and Section 7 concludes.

2. A Static Economy

In this section we introduce a simple model of a firm’s pricing decision when it can reset its price upon payment of a fixed cost. We then show, in a one-period general equilibrium model, how this determines equilibrium price flexibility. In Section 3, we consider a dynamic version of the model in which firms choose multi-period contract prices in a staggered fashion, and whether to adjust prices during the life of a contract.

2.1. Firms. Firms produce differentiated goods. Let nominal production costs be linear in output, and equal to $MC$. All firms set prices before the state of the world is known, i.e. before demand, marginal cost, and the firm-specific fixed cost of price-adjustment is realized. Any firm may adjust its price ex-post if it pays the firm-specific fixed cost.

\(^3\)For the initial exposition of this result in a static model, see Ball and Romer (1991). Our results indicate that this complementarity extends to a dynamic model with staggered price-setting.
Following Dotsey et al. (1999), let the fixed cost be stochastic, and its realization unknown to the firm when it sets its price. Hence, firms are ex-ante identical and set the same price. The (ex-post) fixed cost realization expressed in real units is given by $\varphi_i$ for firm $i$; the firm’s nominal cost of price adjustment is given by $MC \cdot \varphi_i$. We assume that the distribution of real fixed costs is uniform on $[0, \varphi_{\text{max}}]$. Ex-post, firms are randomly assigned to the unit interval, so that for firm $i$:

$$\varphi_i = \varphi_{\text{max}} \cdot i, \ i \in [0, 1].$$

At this point we can think of these either as ‘menu’ costs, or more broadly as decision-making or renegotiation costs associated with price change.

An equilibrium has the property that a measure, $z$, of firms will choose to adjust their prices ex-post, where $0 < z \leq 1$. Given this, a firm setting its price prior to the realization of the state of the world faces the maximization problem:

$$\max_{\bar{P}} E \left\{ \alpha \left[ (1 - z) d (\bar{P} - MC) + z \tilde{\Pi} \right] \right\},$$

where

$$d = \left( \frac{\bar{P}}{P} \right)^{-\lambda} X, \ \lambda > 1,$$

is demand for the firm’s good, $X$ is aggregate demand (or aggregate output), $P$ is the aggregate price level in each state of the world, and $\alpha$ is the firm’s state-contingent discount factor. The profit (gross of fixed cost) that the firm earns if it adjusts its price ex-post is denoted by $\tilde{\Pi}$. The price which solves this problem is written as:

$$\bar{P} = \hat{\lambda} E \left[ (1 - z) d MC \right] / E \left[ (1 - z) d \right],$$

where $\hat{\lambda} = \lambda / (\lambda - 1)$ is the mark-up factor.

After observing the realized state of the world, the firm decides whether to adjust its price. To characterize this decision, note that firms which adjust ex-post set the optimal price given by the mark-up rule, $\bar{P} = \hat{\lambda} MC$, and receive profits:

$$\tilde{\Pi} = \frac{\hat{\lambda} - \lambda}{\lambda - 1} MC^{1 - \lambda} P^\lambda X.$$

If the firm maintains its pre-set price, its profits are given by:

$$\bar{\Pi} = d (\bar{P} - MC).$$

Given this, we express the real gross gain from price adjustment as:

$$\Delta (MC, X, P) = \frac{P^\lambda \bar{P}^{1 - \lambda} X}{MC} \left[ \frac{\hat{\lambda} - \lambda}{\lambda - 1} \left( \frac{MC}{P} \right)^{1 - \lambda} + \frac{MC}{P} - 1 \right].$$

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4 The functional forms for the firm’s discount factor and demand function are the ones considered in our dynamic model of Section 3.
This is simply the difference in profits, $\Pi - \bar{\Pi}$, normalized by $MC$. Because the nominal fixed cost is expressed in the same units as marginal cost, this is the appropriate normalization. Hence, a firm will choose to re-set its price if $\Delta (MC, X, P)$ is greater than its fixed cost, $\varphi_i$.

2.2. Determinants of the Gain to Price Adjustment. To explore the factors which effect the firms adjustment decision, we take a third-order approximation of (2.3) around a non-stochastic steady-state equilibrium, in which $MC^* = M^*, \ P^* = \bar{P}^*$, and $\bar{P}^* = \hat{\lambda}M^*;^5$ here, $M^*$ is the non-stochastic value of the money stock. This approximation is given by:

\begin{equation}
\Delta (mc, x, p) \approx K \left[ \lambda mc^2 \frac{\lambda (\lambda + 1)}{3} mc^3 + \lambda^2 mc^2 p + \lambda mc^2 x \right],
\end{equation}

where $K = X^*/2$ is a constant, and lowercase letters denote log deviations from steady-state. The right-hand side of (2.4) measures the incentive that the firm has to alter its price in the face of shocks to marginal cost ($mc$), the prices of all other firms ($p$), and aggregate demand ($x$). Up to a first-order, this expression must be zero since the firm sets its price optimally to maximize expected (discounted) profits.

Up to a second-order, the incentive to adjust depends only on deviations in marginal cost from steady-state. Changes in the aggregate price level or aggregate demand can only matter to the extent that changes in marginal cost have first-order effects. Since these effects are zero, shocks to $x$ and $p$ are irrelevant up to a second-order. Hence, up to a second-order, the incentive for price adjustment is symmetric across positive and negative marginal cost shocks. Moreover, the critical parameter determining this incentive is the elasticity, $\lambda$. The higher is $\lambda$, the greater the gain to adjusting.

However, expression (2.4) also contains third-order terms. These terms differ from zero insofar as $mc^2 \neq 0$. Moreover, when $\lambda$ is large, these effects are non-negligible. These effects introduce asymmetries in the incentives for price adjustment across positive and negative shocks.

We characterize these asymmetries through the following example. Assume that $mc = m$, so that the marginal cost shock is driven solely by a shock to money (or nominal aggregate demand). In addition, assume that real aggregate demand is determined by the quantity equation, $x = m - p$. Both assumptions are obtained as special cases of the more general, dynamic model we consider in Section 3. Given this specification, (2.4) can be

We are making a simplification here by assuming that the pre-set price is equal to the expected value of marginal cost times the mark-up. The exact price is given by (2.2). It is easily shown that when no other firms adjust, $z = 0$ and $P$ is fixed, so that this simplification is exactly correct. Generally, whether the firm prices above or below expected marginal cost (adjusted by the mark-up) depends upon whether $\text{cov}((1 - z)\alpha d, MC)$ is positive or negative. In our quantitative analysis of the dynamic model – where we do not make any simplification with respect to $\bar{P}$ – we find that this covariance effect is negligible, so that there is only a trivial difference between the approximation here and the true solution.
rewritten as:

\[
(2.5) \quad \Delta (m, p) \approx K \left[ \lambda m^2 - \frac{\lambda(\lambda - 2)}{3} m^3 + \lambda (\lambda - 1) m^2 p \right].
\]

The second-order effect of marginal cost shocks is the same as before. Now there are two third-order terms. The first concerns changes in marginal costs that are of third-order magnitude. Ignoring \( p \), so long as \( \lambda > 2 \), the firm actually has a greater incentive to adjust its price in response to negative shocks than to positive shocks. The intuition comes from (2.3). Optimal profits (before scaling by \( MC \)), \( \bar{\Pi} \), are decreasing and convex in \( MC \), while fixed-price profits, \( \Pi \), are linearly decreasing in \( MC \). Hence, a fall in \( MC \) generates a greater increase in the gain to price adjustment than does a rise in \( MC \).

Now consider the term involving \( m^2 p \). This captures the effect of a change in the price of all other firms on the incentive for the firm to adjust its own price, in response to a shock to marginal cost. Suppose that \( p \) responds in the same direction as \( m \).\(^6\) Then when \( m > 0 \), the aggregate price will rise. This leads to a fall in the non-adjusting firm’s relative price, and thus an increase in demand for its output. This in turn increases the gain from adjusting its own price. The gain is captured in the term \( \lambda(\lambda - 1)m^2 p \), which is positive. Moreover, for a high value of \( \lambda \), this third order term may be large, as shown below.\(^7\)

On the other hand, when there is a negative marginal cost shock, a fall in aggregate prices leads to a fall in demand for the individual firm. This mitigates the incentive for it to adjust its own price, since the term \( \lambda(\lambda - 1)m^2 p \) is negative. Hence, this relative demand effect provides a greater incentive to price adjustment in response to positive shocks than to negative shocks.

This term highlights the strategic interaction between firms’ pricing decisions. However, it is not the standard strategic complementarity that arises in general equilibrium models via the effect of pricing decisions on marginal cost. This is obvious here, since the marginal cost shock is taken as exogenous and equal to \( m \). Rather, the link is due to the impact of changes in other firms prices on a firm’s demand. Moreover, the nature of the strategic interaction depends on the sign of the shock. For a positive marginal cost shock, prices are strategic complements: a firm has a greater incentive to increase its price if other firms increase theirs. But for a negative shock, a firm has less incentive to reduce its price the more other firms reduce theirs, so that prices are strategic substitutes.

\(^6\)As will be made clear below, this positive covariance arises naturally in equilibrium, when the aggregate price level depends on the prices set by all firms.

\(^7\)Equation (2.5) also incorporates the impact of changes in \( p \) on aggregate demand, since the example assumes that \( x = m - p \). Thus, the positive effect of a rise in aggregate prices on the non-adjusting firm’s demand is dampened by the negative effect on aggregate demand. But since \( \lambda > 1 \), the net effect is always positive. A similar qualification holds for the term involving \( m^3 \).
To better understand this, consider Figure 1, which plots the gain to price adjustment as a function of the money shock. Here we consider the common calibration of a 10 percent steady-state mark-up so that $\lambda = 11$. The blue line corresponds to the case when other firms do not adjust, so that $p = 0$, and the red line when all other firms adjust, setting $p = m$. If we ignore price adjustment on the part of other firms, the asymmetry is due solely to the $m^3$ term described above. In this case the third-order effect is small, and implies a slightly greater adjustment incentive for negative shocks; this can be seen from the blue line.

But if all other firms do adjust prices, the asymmetry term becomes $\lambda(2\lambda - 1)m^3/3$. As is clear from the red line, positive shocks generate a greater incentive to adjust if all other firms are adjusting. For negative shocks there is less incentive to adjust when other firms adjust. Moreover, the asymmetry is quantitatively large: the gain to price adjustment for a $+2\%$ money shock is 28\% greater than that for a $-2\%$ shock. For money shocks of absolute value 5\% and 10\%, the gain is respectively, 73\% and 173\% greater for positive relative to negative shocks.

Note that the strength of the asymmetry is tied critically to the price elasticity of demand. When goods markets are highly competitive the incentives to adjust prices become highly asymmetric. To see this, and to provide further intuition, take the case where $\lambda \to \infty$, so that the market structure approaches perfect competition. In steady-state, optimized profits are (approximately) zero. In response to a fall in marginal cost, if other firms lower prices, demand for the individual non-adjusting firm falls to (essentially) zero since goods are near perfect substitutes. However, the incentive for the firm to adjust its price is small: if the firm does not adjust, it has zero sales and earns zero profits; but if it does lower its price, its optimized profits are again near zero. This is not true for a positive shock to marginal cost. With (essentially) perfect competition, a rise in marginal cost drives the non-adjusting firm’s mark-up of price over marginal cost negative. If other firms raise prices in response to this shock, the non-adjusting firm’s demand becomes the market demand (because of the very high $\lambda$), exacerbating the marginal cost shock. As a result, the individual firm faces a large incentive to adjust its price: this allows it to adjust from earning large negative profits to near zero profits.\footnote{Burstein (2002) makes a similar argument by considering the losses incurred due to non-adjustment in the face of shocks of sufficiently large magnitude.}

These examples make clear that the asymmetry in a firm’s adjustment incentive depends critically on the following two features: (i) the effect of other firms’ pricing decisions on an individual firm’s demand, and (ii) a positive covariance between prices and marginal cost.

2.3. Equilibrium in the Static Model. So far, we have discussed only the adjustment incentives for a single firm, given the pricing decisions of
all other firms. How does this translate into equilibrium price flexibility? Continue to focus on the special case where \( mc = m \) and \( x = m - p \), so that expression (2.5) holds.

The distribution of fixed adjustment costs is as described above. In equilibrium, a measure, \( z \), of firms adjust their price in response to monetary shocks. To a first-order approximation, the response of aggregate prices is given by \( p = zm \); since we are concerned only with the qualitative features of equilibrium in this subsection, this simplification is sufficiently accurate to make our point.\(^9\) The equilibrium measure \( z \) is determined by the condition that the gross gain, \( \Delta (m, p) \), equals the real fixed cost of price adjustment for the marginal firm:

\[
(2.6) \quad K \left[ \lambda m^2 - \frac{\lambda (\lambda - 2)}{3} m^3 + \lambda (\lambda - 1) m^3 z \right] - z \varphi_{\text{max}} = 0.
\]

This equation is illustrated in Figure 2. The \( GG^+ \) locus illustrates the net gain to adjustment for a positive money shock; \( GG^- \) illustrates the net gain for a negative shock. In both cases, the gain depends on the measure of adjusting firms.\(^10\)

To illustrate the nature of the solution, take the baseline calibration with \( \lambda = 11 \), and assume that the maximum fixed cost of price change, \( \varphi_{\text{max}} \), is 2 percent of the firm’s revenue. For sufficiently small shocks of either sign, the two schedules are effectively identical, and only a small fraction of firms choose to adjust prices. But as the size of the shock rises, the equilibrium value of \( z \) increases. In addition, as shown in the figure, it is substantially greater for positive than for negative shocks. For shocks that are large enough, both net gain schedules rise above the zero axis, so that \( z = 1 \) and the equilibrium displays full price flexibility. However, full price flexibility occurs for smaller positive shocks than for negative shocks. As \( \lambda \) rises, Figure 2 shows that both curves shift up; for a given sized shock, price flexibility is increased. In our dynamic model we find that these asymmetries are quantitatively significant for standard elasticities used in calibrated studies. This is true despite the fact that the asymmetry is a third-order effect.

**2.4. Implications of Real Rigidity.** The previous discussion assumed that \( mc = m \), so that marginal cost is independent of firms’ pricing decisions. But recent debate in the sticky price literature has emphasized the importance of marginal costs being unresponsive to aggregate demand. How

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\(^9\)The qualitative features of equilibrium are robust to considering higher-order approximations of the response of \( p \). Again, in solving the dynamic model of the next section, the exact expression for \( p \) is used. See also the next footnote.

\(^10\)In particular, the gross gain to price change is greater for positive relative to negative shocks as long as \( z \geq (\lambda - 2) / [3(\lambda - 1)] \). In our quantitative dynamic model, this cutoff value for \( z \) is much smaller. This is due principally to the fact that a fraction of firms will automatically being setting new, 2-period contract prices, so that the aggregate price level is more sensitive to money shocks.
would this affect our results? To study this question we now assume that:

(2.7) \( mc = p + \phi x = (1 - \phi) p + \phi m, \quad \phi \in (0, 1) \).

For low values of \( \phi \), we obtain significant real rigidity in marginal costs. Take the case with \( \phi = 0.1 \), and substitute this into equation (2.4). Using the same calibration as Figure 2, we find that both the \( GG^+ \) and \( GG^- \) curves shift down, and are effectively coincident at low values of \( z \). This is shown in Figure 3. In this sense, real rigidity gives rise to equilibria with low degrees of price adjustment. With small \( \phi \), there is little response in marginal cost to a monetary shock. As a result, there is little incentive for an individual firm to adjust its price.

However, note from (2.6) that the gain from adjustment rises sharply as \( z \) approaches unity. As first shown in Ball and Romer (1991), strategic complementarities arise across firms’ pricing decisions when \( \phi < 1 \). If all firms adjust, marginal cost rises by the full amount of the money shock, regardless of the degree of real rigidity. Hence, if for a given shock, full price flexibility is the unique equilibrium in the model without real rigidities (\( \phi = 1 \)), then this continues to be an equilibrium with real rigidities, for all values of \( \phi \in (0, 1) \).

The bottom panel of Figure 3 illustrates this phenomenon, using the same large shock as in Figure 2. For a negative shock, there is a unique equilibrium where only a tiny fraction of firms adjust their price. But for a positive shock of the same magnitude, there are three equilibria: one with a very low value of \( z \), one with an intermediate \( z \), and one with \( z = 1 \). Since the intermediate \( z \) equilibrium is ‘unstable’ in the normal sense, we focus on the other two equilibria. Hence, the key asymmetry in price adjustment highlighted above continues to play a role when marginal cost exhibits real rigidity. If, in the model without real rigidity, we obtain full price flexibility for smaller positive shocks than for negative shocks, it is more likely that there will be multiple equilibria for a given positive shock than for a negative shock.

3. A Dynamic Economy

In this section, we extend the model to a dynamic general equilibrium environment. The price-setting description of the model is made tractable by specifying that firms follow a hybrid time- and state-dependent pricing rule.

3.1. Pricing. The starting point is a simple, two-period version of Taylor’s (1980) model. Half of the unit measure of firms set (two-period) contract prices in odd periods, half in even periods. We also allow for the possibility

\[\text{Conversely, if there does not exist an equilibrium with full price adjustment in the model with } \phi = 1, \text{ then this cannot be an equilibrium outcome with } \phi < 1.\]
that prices are adjusted automatically for trend inflation in the second period of the contract.

Following Ball and Mankiw (1994), we add to this the assumption that firms in the second half of their contracts can, by paying a fixed cost (in terms of labour hours), 'opt-out' of their price after observing the current state of the world. If they do so, they simply choose a price that maximizes ex-post profit.\(^{12}\) We think of these fixed costs as being of two types. The first are price recontracting costs: information-processing and decision-making costs associated with determining a new price, and communication and negotiation costs associated with implementing the new price. The second is the more standard, physical 'menu costs.' Zbaracki et al. (2000) argue that the former may be at least an order of magnitude greater than strict menu costs. Indeed, if we allow second period contract prices to be automatically updated for inflation, we are assuming that the physical costs of price change are zero; we return to this later.

Following Dotsey et al. (1999), the fixed cost of opting-out is a random draw which is i.i.d. across firms, from a common distribution. Any single firm faces a fixed cost that is i.i.d. over time. Thus, the firm’s current draw contains no information in predicting its future cost. In addition, we assume that a firm that chooses to opt-out in the current period just goes back to staggering next period, i.e. setting its two-period price.\(^{13}\)

When a firm chooses its two-period price, it takes into account the possibility that it may choose to opt-out next period. The likelihood of this happening is determined by the firm’s fixed cost, relative to its market demand and production cost. Since fixed costs are i.i.d. both across firms and over time, the environment is symmetric; all firms face the same probability of opting-out from their price contract in the second period. As a result, all firms set the same two-period price, allowing us to minimize heterogeneity across firms.

This, and the assumption of a constant elasticity of substitution in demand across all firms, allows us to write the aggregate price index at time \(t\) as:

\[
P_t = \left[ \frac{1}{2} P_{1t}^{1-\lambda} + \frac{(1 - z_t)}{2} P_{2t}^{1-\lambda} + z_t \bar{P}_{1t}^{1-\lambda} \right]^{\frac{1}{1-\lambda}},
\]

Note, of course, that this hybrid price-setting can be derived from an alternative interpretation in which all price changes involve a fixed cost. However, firms must incur the fixed cost every second period. This would result in the same model, except for a minor change to the labour market clearing condition displayed below.

We do this so that the firm’s opt-out decision is static, which makes a detailed characterization of the degree of price flexibility intuitive. It is straightforward, however, to modify the model so that opt-out firms set a new, two-period contract price. This would involve one additional state variable (the measure of firms with inherited contract prices), and using the terminology of Dotsey et al. (1999), would add to the 'history dependence' in the model’s dynamics.
where $\lambda > 1$. Here, $P_{1t}$ is the price set by firms who are entering into new contracts now for periods $t$ and $t + 1$, and $P_{2t}$ is the second period price inherited by firms who entered into new contracts at date $t - 1$. Also, $z_t$ is the fraction of firms in the second period of their contract who opt-out; these firms choose the ex-post profit maximizing price, denoted $\tilde{P}_t$.

Given this notation, we require that $P_{2t} = f \cdot P_{1t-1}$, where $f$ is a scaling factor that is built into the contract price. If $f = 1$ there is no automatic updating, and we get the traditional Taylor specification; if $f = \pi^*$, prices are automatically adjusted for trend inflation. Note that which specification we choose impacts upon the interpretation (and calibration) of the fixed cost. If prices are not updated for trend inflation ($f = 1$), then the cost of price adjustment includes both recontracting costs and menu costs, whereas if $f = \pi^*$ the cost should be interpreted as only the former.

3.2. Firms. Each firm is a monopolist and produces a differentiated good using labour alone. Labour is hired in a competitive economy-wide market. Firm $i$’s production function is $C_{it} = H_{it}$. It follows that the firm’s nominal marginal cost is given by the nominal wage, $W_t$.

After observing the current state, a firm that is setting a new contract price at time $t$ solves the problem:

$$\max_{P_{1t}} \Pi(P_{1t}, P_t, W_t) + E_t \left\{ \alpha_{t+1} \max \left[ \Pi(fP_{1t}, P_{t+1}, W_{t+1}), \Pi(P_{t+1}, P_{t+1}, W_{t+1}) - W_{t+1} \varphi_{it+1} \right] \right\}.$$ 

We define the static profit function for a firm as:

$$\Pi(P_t, P, W) = (P_t - W) \left( \frac{P_t}{F} \right)^{-\lambda} C,$$

where $C$ is composite (aggregate) consumption. Also, $\alpha_{t+1}$ represents the state-contingent nominal discount factor used by the firm in maximizing profits;\footnote{Since the firm is owned by the representative household, it discounts future nominal profits in the same way as the household.} $\varphi_{it}$ denotes firm $i$’s realized price adjustment cost at date $t$ in units of labour; again, draws of $\varphi$ are i.i.d. across firms and over time, and are distributed according to the CDF, $F(\varphi)$. The firm evaluates its future profits taking into account that it may choose to opt-out at period $t + 1$. The first order condition in choosing $P_{1t}$ in a symmetric outcome is given by:

$$\left( \frac{P_{1t}}{P_t} \right)^{-\lambda} C_t \left( 1 - \lambda + \lambda \frac{W_t}{P_{1t}} \right) + E_t \left\{ \alpha_{t+1} (1 - z_{t+1}) \left( \frac{fP_{1t}}{P_{t+1}} \right)^{-\lambda} C_{t+1} \left[ f (1 - \lambda) + \lambda \frac{W_{t+1}}{P_{1t}} \right] \right\} = 0.$$
At any date $t$, after observing the current state, a measure of firms will choose to opt-out of their price inherited from date $t-1$. Ranking these firms in order of their realized fixed cost, we can determine the threshold, $\bar{\varphi}_t$, such that all firms with $\varphi_i > \bar{\varphi}_t$ stay with their inherited price, while all firms $\varphi_i \leq \bar{\varphi}_t$ choose to opt-out. A firm that opts-out at time $t$ sets an ex-post profit maximizing price given by:

\begin{equation}
\tilde{P}_t = \hat{\lambda}W_t,
\end{equation}

where $\hat{\lambda} = \lambda/(\lambda - 1)$ is the mark-up factor. Given this, $z_t = F(\bar{\varphi}_t)$ is determined by the marginal condition:

\begin{equation}
(fP_{t-1} - W_t) \left( fP_{t-1} \right)^{-\lambda} C_t = \frac{\lambda - \lambda}{\lambda - 1} P_t \lambda C_t W_t^{1-\lambda} - W_t F^{-1} (z_t).
\end{equation}

This condition equates the profits from remaining with the contract price inherited from period $t-1$, with the profits from opting-out, net of the fixed cost, for the marginal firm, $z_t$. If the right-hand side of equation (3.4) is greater than the left at the maximal fixed cost, $z = 1$.

### 3.3. Households

The representative household has utility over consumption, real money balances, and labour supply given by:

\begin{equation}
E_0 \sum_{t=0}^{\infty} \beta^t U \left( C_t, \frac{M_t}{P_t}, H_t \right), \quad 0 < \beta < 1,
\end{equation}

where $U = \ln(C) + \chi \ln(M/P) - \eta H$. We define composite consumption as a CES aggregator over the unit measure of differentiated goods:

\begin{equation}
C = \left[ \int_0^1 C_i^{\frac{\lambda-1}{\lambda}} di \right]^{\frac{\lambda}{\lambda-1}},
\end{equation}

where the elasticity of substitution is $\lambda > 1$. Given this, the price index, $P_t$, in equation (3.1) is the natural measure of the aggregate price level. The household receives income from wages and the profits of firms. Hence, the budget constraint for the household is:

\begin{equation}
P_tC_t + M_t + B_t = W_t H_t + \int_0^1 \Pi_i di + M_{t-1} + (1 + r_{t-1}) B_{t-1} + T_t.
\end{equation}

Here, $B_t$ is the household’s purchases of nominal one-period bonds, $W_t$ is the nominal wage, and $\Pi_i$ represents the profits of firm $i$. We define $r_t$ as the nominal interest rate. Also, $T_t$ is a lump-sum transfer from the monetary authority.

The household’s choices are characterized in the standard way. The implicit labour supply function and the money demand equation are, respectively:

\begin{equation}
W_t = \eta P_t C_t,
\end{equation}
(3.6) \[ \frac{M_t}{P_t} = \chi C_t \left( \frac{1 + r_t}{r_t} \right). \]

Demand for a particular good \( i \) is given by:

(3.7) \[ C_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\lambda} C_t. \]

Finally, the household’s intertemporal Euler equation is written as:

(3.8) \[ \frac{1}{1 + r_t} = \beta E_t \left\{ \frac{P_tC_t}{P_{t+1}C_{t+1}} \right\}, \]

so that \( \alpha_{t+1} = \beta P_tC_t/P_{t+1}C_{t+1} \). In Section 4, we also present results from a version of the model in which preferences are defined over consumption and leisure, and money demand is introduced via a cash-in-advance constraint. The description of the model is entirely standard, except that we introduce a subsidy to production income. This subsidy undoes the distortion on labour supply, as well as the intertemporal valuation of profits from the firm’s perspective, due to the non-zero interest rate. For the sake of space, we omit presentation of the details and make them available upon request.

In the case of i.i.d. monetary policy shocks, the implications for output, hours, inflation and price flexibility are identical across the money-in-utility and cash-in-advance versions. For autocorrelated shocks, the implications differ quantitatively, and this is taken up in Section 4.

3.4. **Monetary Policy Rule.** The monetary authority follows a money growth rule of the form:

\[ M_t = M_{t-1} \exp (\mu + v_t), \]

where \( v_t = \rho v_{t-1} + \varepsilon_t \) and \( \varepsilon_t \) is mean zero and i.i.d. Money growth finances transfers \( T_t \), so that in the aggregate we have \( M_t = M_{t-1} + T_t \).

3.5. **Equilibrium.** A symmetric equilibrium in this economy is defined in the usual way. In particular, equilibrium must satisfy equations (3.1) – (3.8), in addition to the labour market clearing condition:

(3.9) \[ H_t = \frac{1}{2} \left[ C_{1t} + (1 - z_t) C_{2t} + z_t \tilde{C}_t + \int_{0}^{\tilde{\varphi}_t} \varphi dF(\varphi) \right]. \]

Here we define \( C_{1t}, C_{2t}, \) and \( \tilde{C}_t \), respectively, as time \( t \) output of firms that set their contract price in period \( t \), in period \( t - 1 \), and firms that opt-out of their price contracts in period \( t \). Absent government bond issue, it must be that \( B_t = 0 \) in equilibrium.

To render the model’s equilibrium stationary, we scale all date \( t \) nominal variables by the date \( t \) money stock. Given this, the model’s natural state space is simply \( (p_{1t-1}, v_t) \) where \( p_{1t-1} \) is the scaled contract price inherited at date \( t \). Because of this small dimensionality, we are able to fully characterize the model’s non-linear dynamics. We solve the model by approximating the
expectation terms in (3.2) and (3.8) with linear combinations of Chebychev polynomials. This is implemented via an iterative algorithm which adapts the projection methods of Judd (1992). With these expectation functions in hand, it is straightforward to derive exact (non-linear) solutions for decision variables from the model’s equilibrium conditions.

3.6. **Introducing Real Rigidity.** We also consider an amended version of the model which allows for persistent real effects of monetary shocks. Recent work by Christiano et al. (2001) and Dotsey and King (2001) show how adding factors such as variable capacity utilization and produced (intermediate) inputs impact upon the ability of sticky price models to generate endogenous persistence. To keep our model simple, we take a more parsimonious, ‘reduced form’ approach in modelling these features. For this experiment we replace the labour supply function, (3.5), with the following condition:

\[ W_t = \eta P_t C_t^\phi. \]

For values of $\phi < 1$ this model displays an elasticity of the real wage with respect to consumption (output) that is less than unity. As discussed in Jeanne (1998), we view this modification as a stand-in for a more complete story of labour market institutions which allows for a small response of real marginal cost to output. A very similar labour supply condition can be attained when the household’s preferences are of the Greenwood-Hercowitz-Huffman (1988) form; this specification generates a labour supply function with no income effects and Frisch labour supply elasticities of arbitrary magnitude (see King and Wolman, 1999; and Burstein, 2002).

4. **Quantitative Results**

To investigate the quantitative importance of the asymmetries discussed above, we study a calibrated version of our model. The discount factor is set at 0.98 to correspond to a time period of 6-months; all firms adjust their prices fully at the end of one year. In line with many calibration studies, the elasticity of substitution is set at $\lambda = 11$ to correspond to a steady state price-cost mark-up of 10 percent. We experiment with lower mark-ups below. Steady-state hours are set at 30 percent of the household’s time endowment. To gain intuition for the model’s dynamics, the baseline monetary policy rule sets $\mu = 0$ and $\rho = 0$, implying that the money stock follows a random walk without drift. In the baseline case and all experiments

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15 For a discussion of the issues involved in solving models with occasionally binding constraints (here, $z \leq 1$), see Christiano and Fisher (2000). Details of our solution method are available upon request.

16 See, for instance, Chari et al. (2000). The results of Basu and Fernald (1997) indicate mark-ups of similar magnitude or slightly lower.
considered, the standard deviation of money growth is set to 1.7%, which corresponds to that of postwar US data on M2.\textsuperscript{17} We also consider a more realistic calibration with trend inflation and autocorrelated money growth shocks below. We also begin with a model without real rigidity, so that the labour supply condition is given by (3.5) (or alternatively (3.10) with \( \phi = 1 \)).

A key aspect of the model is the distribution of costs that firms incur in opting-out of their price contracts. There is very little current evidence to guide the calibration of this distribution. We use the most simple possible specification, namely that the distribution of fixed costs is uniform over \([0, \varphi_{\text{max}}]\), so that the CDF is \( F(\varphi) = \varphi/\varphi_{\text{max}} \). We set \( \varphi_{\text{max}} \) at 4% of steady-state hours.

The calibration of this last value is based, in part, on the study of Zbaracki et al. (2000), the most comprehensive to date. They study the price-setting process of a US manufacturing firm and classify the fixed costs associated with the annual issuance of the firm’s list prices. These fall into three categories: managerial (information-processing, decision-making), customer (communication, negotiation), and physical or ‘menu’ costs. Of these, the first two categories comprise about 1.2% of the firm’s annual revenue, while menu costs comprise only about 0.05%.\textsuperscript{18} Since the model frequency is 6-months, this adds up to approximately 2.5% of semi-annual revenue for a one-time price change. Further evidence is found in structural econometric studies. Slade (1998) provides dollar estimates of the total fixed cost per-price-change for saltine crackers. This value (adjusted for inflation) is smaller than the per-product, per-price-change value found in Zbaracki et al. Willis (2000) estimates the total fixed cost of price change from a sample of magazine prices; his preferred specification indicates that at the firm level, this is about 4% of semi-annual revenue. Bils and Klenow (2003) indicate that among consumer goods, magazines display some of the stickiest prices. As a result, we view our 4% value to be a reasonable upper-bound on the distribution.\textsuperscript{19}

\subsection*{4.1. The Baseline Model.} In Figure 4 we present for the baseline calibration, the impact responses of selected model variables to monetary policy shocks between +10\% and −10\%. In the panels, the red locus describes the response for the state-dependent pricing model; the blue locus describes the

\textsuperscript{17}This value is very close to that estimated by Boukez et al. (2003) for the standard deviation of monetary policy shocks, when money is measured as M2.

\textsuperscript{18}Further direct evidence on the size of physical costs of price change is provided by Levy et al. (1997). They find that for typical large US supermarket chains, per-store menu costs total about 0.7\% of annual revenue. Given that these stores change prices on a weekly basis, this figure corresponds closely to the 0.05\% figure above.

\textsuperscript{19}In terms of calibration studies, Dotsey et al. (1999) consider an upper-bound corresponding to 1.9\% of semi-annual revenue. Golosov and Lucas (2003) consider a degenerate distribution, with a fixed cost of 0.5\% of semi-annual revenue.
equivalent response with time-dependent pricing, that is, for the standard two-period Taylor model.\textsuperscript{20}

There are two key issues that can be addressed with Figure 4. First, when does the time-dependent model cease to be an accurate representation of the response to a shock, in the presence of endogenous adjustment by firms in the second period of their price contract? Secondly, to what extent are the responses of prices and real activity asymmetric across positive and negative shocks? The first question can be assessed by the degree to which the red locus in each panel departs from the blue one, or equivalently, by the total measure of firms who choose to opt-out in the first panel. The second question is assessed by the degree to which the response of the red locus is different for positive and negative shocks of the same size.

For the baseline calibration, the time-dependent model is a reasonably accurate characterization of the response of aggregate prices and consumption/output (the bottom panels) generated by the state-dependent model, within the range of $\pm 4$ percent shocks. Within this range, no more than 20 percent of firms choose to opt-out of their second period prices. Outside this range, however, deviations between the time-dependent and state-dependent model become increasingly obvious. For shocks greater than $\pm 4$ percent, prices respond by much more, and consumption by much less than implied by the time-dependent pricing model. The reason for the gap between the two pricing models is clear from the first panel: for large shocks a substantial measure of firms choose to reset their prices. Hence, the state-dependent pricing model displays distinct non-linearities in response to monetary shocks.

The state-dependent adjustment displayed in Figure 4 is also asymmetric. For a 6.5\% or higher positive shock, all firms adjust their prices so there is full neutrality of money (apart from the adjustment costs). But even for a negative 10\% money shock, only 80 percent of second period prices will have adjusted. The source of this asymmetry is the same as that discussed in Section 2.

4.2. Extensions to the Baseline Case. To highlight the role of the elasticity of demand and the strategic link across firms’ pricing decisions, Figure 5 repeats the exercise of Figure 4, but assuming that the steady state mark-up is only 2 percent (so $\lambda = 51$). In this case, there is a dramatic asymmetry in the adjustment process. For positive shocks of +3 percent, all firms have adjusted their prices; but even for a $-10$ percent shock, just over half of firms in the second period of their contract have adjusted their prices.\textsuperscript{21}

\textsuperscript{20}The solution for time-dependent pricing is obtained by setting the range of the price adjustment cost very high, so that effectively no firms choose to opt-out.

\textsuperscript{21}Note that with $\lambda = 51$, the responses for the time-dependent model are also highly asymmetric. In fact, positive money shocks generate smaller price responses relative to negative shocks. This is due to the fact that the aggregate price index (3.1) at date $t$ is a
We now add to the model a positive rate of trend money growth/inflation. We set \( \mu = 0.03 \) which matches the semi-annual growth rate of M2 in the postwar US data. The presence of trend inflation adds to the asymmetric response of price adjustment as highlighted by Ball and Mankiw (1994). Intuitively, with trend inflation, if firms cannot update their within-contract prices for inflation, firms will on average have second-period prices below their static profit maximizing levels. A small positive shock to marginal cost exacerbates this difference, further encouraging them to adjust prices. A small negative shock reduces the difference between price and marginal cost, and in this respect, diminishes the incentive to adjust. However, if contract prices are automatically updated for trend inflation, the response of the economy is virtually equivalent to that of Figure 4.

Figures 6 and 7 illustrate this. Figure 6 contrasts the impact of a money shock when firms do and do not update their second-period prices for inflation.\(^{22}\) Without updating, the asymmetry in price adjustment is exacerbated – the price adjustment schedule is essentially shifted uniformly to the left. In terms of the output response, this ‘trend inflation’ asymmetry reinforces the profit function asymmetry that we document. The peak positive response of output is at a +3% shock, while the peak negative response is at a −7% shock. Figure 7 compares the time-dependent (blue lines) and the state-dependent (red lines) model without trend updating (i.e., the red lines in Figures 6 and 7 coincide). The range of monetary policy shocks for which the time-dependent model is a close approximation to the state-dependent model is clearly asymmetric, as seen in the output and inflation responses. At least 20 percent of firms opt-out for positive shocks about +2.5% and greater; the equivalent cut-off for negative shocks is about −5.5%.

The experiments displayed in Figures 4 through 7 assume an i.i.d. distribution for shocks. But empirically, monetary policy shocks are persistent. We now consider a value of \( \rho = 0.6 \), which corresponds to the autocorrelation in postwar US M2 growth.\(^{23}\) In Figure 8, we examine the case with autocorrelated shocks, trend money growth of 3 percent (\( \mu = 0.03 \)), and without automatic updating of second-period prices (\( f = 1 \)). The key effect of persistent shocks is to further diminish the overlap between the time- and state-dependent models. At least 20 percent of firms opt-out of their second period prices for shocks greater (in absolute value) than +1% and −2.5%.

\(^{22}\)This corresponds to the case with \( f = \exp (\mu) \) and \( f = 1 \), respectively. If we allow for updating we are implicitly assuming that menu costs are zero, so that the fixed cost of price change refers only to managerial and customer costs. Given the evidence of Zbaracki et al. (2000), this is not an unreasonable assumption, and we leave the calibration of the fixed cost distribution unchanged.

\(^{23}\)This is very close to the estimate of \( \rho = 0.5 \) found by Christiano et al. (1998) as the persistence of identified monetary policy shocks in quarterly M2 data. See also Boukez et al. (2003) for estimates of persistence.
The state-dependent pricing model delivers full price flexibility at a positive shock of 2 percent, and a negative shock of 6 percent.

The importance of autocorrelated shocks in our model is due principally to the way in which money enters the model. As noted by Dotsey et al. (1999), with interest elastic money demand (as in equation (3.6)), persistent money shocks have greater effects on aggregate demand than i.i.d. shocks, through their impact on nominal interest rates. Because this increases the sensitivity of real marginal cost, it further diminishes the range for which the time-dependent model is relevant.

As an alternative to the model with interest elastic money demand, Figure 9 illustrates the same case as Figure 8, but for the cash-in-advance model. In this set-up, the interest elasticity of money demand is zero and marginal cost moves one-for-one with the current money shock. The effect of persistent money growth is less extreme relative to the money-in-utility specification. However, allowing for autocorrelated shocks still contributes to our asymmetry results, relative to the i.i.d. case. For instance, with \( \rho = 0.6 \), the output response peaks at a +2.5% money shock; this is compared to peaking at a +3.5% shock when \( \rho = 0 \) (in which case, the implications of the money-in-utility and cash-in-advance models are identical).

Note that in Figures 7 – 9, even when all firms adjust their price, the output effect is non-zero. In fact, for a positive shock and full price adjustment, the output response is negative. This is due to the ‘front-loading’ behaviour on the part of firms setting prices in the first period of their contracts, based on current and expected future monetary policy. These firms increase their price by more than in proportion to current marginal cost since they expect future money supply to be higher than today. Firms who are choosing to opt-out of their second period contracts adjust prices in proportion to the current marginal cost. Hence, in total, the aggregate price level is rising by more than the current money shock, and output is lower, relative to the case of a zero shock.

4.3. Empirical Estimates of Asymmetry. There is a sizeable empirical literature which has identified the presence of asymmetries in the response of output to monetary policy shocks. Though they differ in their econometric specifications, the procedure followed in these papers can be roughly summarized as follows. First, identify monetary policy shocks from a linear monetary policy rule; then run a linear regression of output growth on the identified shocks (and other control variables), allowing for different coefficients on positive relative to negative shocks. Though the quantitative results differ across papers, all of these studies find that positive monetary policy shocks generate small estimated output effects which are never statistically different from zero. On the other hand, negative shocks have large contractionary effects which are almost always statistically different from zero.
Table 1. Asymmetric Effects of Monetary Policy Shocks on Output: Regression Results. Note: Rows 2 and 3 give the coefficient estimates on positive and negative shocks in an output growth regression. Columns 2 – 5 present results from the cash-in-advance (CIA) and money-in-utility (MIU) models; in all cases, $\mu = 0.03$, $f = 1$, $\rho = 0.60$. Columns 6 through 9 present empirical results from: Macklem et al. (1996), Table 2a, column 2; Ravn and Sola (2004), Table 5, column 4; Cover (1992), Table 3, column 4; and Cover (1992), Table 5, column 4.

<table>
<thead>
<tr>
<th></th>
<th>CIA $\lambda = 11$</th>
<th>CIA $\lambda = 16$</th>
<th>CIA $\lambda = 21$</th>
<th>MIU $\lambda = 11$</th>
<th>Macklem et al.</th>
<th>Ravn-Sola</th>
<th>Cover Table 3</th>
<th>Cover Table 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>positive</td>
<td>0.264</td>
<td>0.175</td>
<td>0.053</td>
<td>-0.088</td>
<td>0.54</td>
<td>0.000</td>
<td>-0.180</td>
<td>-0.093</td>
</tr>
<tr>
<td>negative</td>
<td>0.367</td>
<td>0.376</td>
<td>0.409</td>
<td>1.145</td>
<td>0.90</td>
<td>0.243</td>
<td>0.873</td>
<td>1.238</td>
</tr>
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</table>

In Table 1, we present a representative sample of these results. In Cover (1992), the monetary policy instrument is taken to be US M1. Ravn and Sola (2004) use the US Federal Funds Rate, and in Macklem et al. (1996) it is the term spread on Canadian interest rates.\(^24\)

From the table we see that there is quite a range in the magnitude of the coefficient estimates. Macklem et al. find the estimates on both positive and negative monetary policy shocks to be positive (i.e., a positive shock causes a rise in output, while a negative shock causes a fall in output), with the negative coefficient 1.6 times larger than the positive coefficient. Ravn and Sola estimate a zero coefficient on positive shocks, and a value of 0.243 for negative shocks.\(^25\) Cover finds a weak contractionary effect of positive shocks, and a strong effect of negative shocks.

We now investigate the ability of our model to match this range of estimates. We focus on the specification with trend inflation ($\mu = 0.03$), no automatic price updating ($f = 1$), and autocorrelated shocks ($\rho = 0.6$). We simulate the model and run a regression of output growth on a constant, lagged output growth, and positive and negative monetary policy shocks.

\(^{24}\)In Table 1, the results of Cover involve an output regression which includes a constant, lagged output growth, and in the column labelled ‘Table 3,’ only contemporaneous money shocks. In the column labelled ‘Table 5,’ the regression allows shocks to enter with up to a 4 period lag, and we present the sum of coefficients on positive and negative shocks. For the Ravn and Sola results, the regression includes a constant, lagged output growth, and contemporaneous shocks. For the Macklem et al. results, we present the sum of coefficients on the regression which includes a constant, 4 lags of output growth, 4 lags of the shock, and other domestic and foreign controls.

\(^{25}\)Ravn and Sola actually consider a wide range of regressions and test a series of restrictions. They ‘prefer’ a specification in which only negative shocks during low volatility regimes have a significant output effect; positive shocks during low volatility regimes and all shocks during high volatility regimes are estimated to have no effect.
Because there is no other shock in the model, we do not include other regressors since this would obviously cause collinearity problems.

We first discuss results for the cash-in-advance model with $\lambda = 11$. Here, the interest elasticity of money demand is zero and marginal cost moves one-for-one with money shocks. When this model is simulated, we obtain coefficient estimates on positive and negative innovations of 0.264 and 0.367, respectively. The ratio of these is 1.4, close to the 1.6 value found in Macklem et al. (1996). Moreover, if we consider the cash-in-advance model with larger elasticities we find greater asymmetry. These latter results are similar to those found in Ravn and Sola (2004). Hence, as discussed above, the quantitative magnitude of the asymmetries depends sensitively on the model calibration. Note, however, that across these experiments, the values of $\lambda$ lie within the range of mark-ups implied by Basu and Fernald (1997).

For the baseline money-in-utility model, the interest elasticity of money demand is unity, and marginal cost moves more than one-for-one with money shocks. For the same case ($\mu = 0.03, f = 1, \rho = 0.6$) and the baseline value of $\lambda = 11$, we estimate a coefficient on the positive shock equal to -0.088 and a coefficient on the negative shock equal to 1.145. Hence, for this representation of the model, we are able to match the extreme asymmetries obtained in Cover (1992).

5. Quantitative Implications of Real Rigidity

The overall result from our analysis is that once we allow for endogenous price adjustment, we find a substantial difference between pure time-dependent pricing models and our hybrid time-and-state-dependent model, in the response of the economy to monetary policy shocks. Prices adjust endogenously for moderate valued shocks, in such a way so as to make the time-dependent model wholly inaccurate for shocks of this magnitude. The second key discrepancy between the models is in the asymmetric response to positive and negative shocks.

One objection to our analysis is that we use a specification in which the elasticity of marginal cost to output is relatively high (although approximately the same as used in Chari et al., 2000). If, on the other hand, we assume substantial real rigidity, as in equation (3.10), the difference between models with endogenous and exogenous price adjustment rules might be considerably reduced.

Figure 10 shows that at one level, this criticism is correct. Figure 10 repeats the baseline experiment of Figure 4, but now sets $\phi = 0.05$ in equation (3.10), so that the real wage is insensitive to fluctuations in output. This implies that movements in nominal marginal costs are determined largely by those of aggregate prices. This in turn implies complementarity across firms in their pricing decisions.
The top two panels of Figure 10 display the impact responses of the switching fraction and output from the baseline model for ±10% monetary shocks. For shocks within this range, there always exists an equilibrium which displays very little price adjustment. In this low switching equilibrium, the impact responses from the time-dependent and state-dependent models are very similar; as the bottom panel reveals, this means that even in response to large shocks, both models exhibit persistent real fluctuations.

However, simply focusing on this equilibrium outcome with state-dependent pricing is misleading, since the strategic complementarity also operates in the opposite direction. If all firms do adjust their prices in response to money shocks (positive or negative), then the behavior of marginal cost is identical to that in the setting with \( \phi = 1 \). This has the following implication. If, for a given sized money shock, an equilibrium is such that all firms adjust in the economy without real rigidity, then this will remain an equilibrium in the economy with real rigidity; full adjustment of prices by all firms eradicates the presence of real rigidity.

Figure 11 illustrates this point. The loci show the net gain to price adjustment for a firm in the second period of its contract, as a function of the measure of adjusting firms. This is done for both a positive (blue line) and negative (red line) shock. This is the dynamic analog to Figure 3 from the static example. The calibration is again the same as in the baseline case, and the size of the shock is ±8 percent. In the case of the +8% shock, we see that there are three equilibria, where a small range of firms adjust, an intermediate range adjust, and all firms adjust. The intermediate equilibrium is unstable by standard reasoning and can be ignored. Nevertheless, the key result is that implications about the real effects of monetary shocks in models with real rigidity may be fragile. While there may be equilibria in which money has large, persistent real impacts and few firms choose to adjust prices ex-post, there may also be expectationally confirming equilibria in which all firms adjust, so that the real impacts of money shocks are negligible.

A novel feature of this argument is the presence of asymmetries. In Figure 11 there exist two stable equilibria for a positive 8% money shock. But for a negative 8% shock, there is a unique equilibrium in which very few firms adjust ex-post. Since in the baseline model without real rigidity not all firms adjust to a −8% shock, it cannot be an equilibrium that all firms adjust in the \( \phi = 0.05 \) case. Of course, because we do get full adjustment for +8% shocks in the baseline model (see Figure 4), this admits the possibility of multiple equilibria in the case with real rigidity. Finally, we note that in

\[ ^{26}\text{To solve the model, we add to the set of natural state variables an exogenous, } \{0, 1\} \text{i.i.d. sunspot shock. For points in the natural state space which admit multiple equilibria, the value of the sunspot determines which equilibrium firms coordinate upon. As a result, a firm’s contract pricing decision (in addition to other decision variables) depend not only on } \{p_{t-1}, v_t\}, \text{ but also on the distribution and realizations of the i.i.d. fixed cost and sunspot shocks.} \]
the case of trend inflation and autocorrelated shocks, full switching occurs for +2% and −6.5% shocks. Accordingly, we find that multiple equilibria are present for shocks greater (in absolute value) than these magnitudes. Hence, the presence of multiplicity is asymmetric across positive and negative shocks. Moreover, at least in the case of positive shocks, the possibility of multiple equilibria exists for very moderate sized shocks.

6. Discussion of Potential Extensions

We have employed a relatively parsimonious model of the monetary business cycle. In this section, we comment on the robustness of our results to several possible modifications. First, one could consider a more ‘neoclassical’ version allowing for physical capital accumulation. This would enrich the dynamic response of the model to monetary shocks. However, because of the short-run inelasticity of capital supply, (under standard assumptions) the primary effect would be greater diminishing returns to variable inputs, and greater sensitivity of real marginal cost to aggregate demand shocks. This would result in an even greater deviation of time-dependent pricing from state-dependent pricing, in a similar fashion to the experiments presented in Figures 8 and 9. An obvious further modification to counteract this would be the inclusion of variable capacity utilization, as in Christiano et al. (2001) and Dotsey and King (2001).

Many macro models are designed to generate persistent responses in output and inflation to monetary shocks. A number of studies have incorporated features that generate ‘flatter’ marginal costs curves. For instance, Christiano et al. (2001) include sticky wage-setting on the part of households, so that a key component of nominal marginal cost responds sluggishly to shocks. Dotsey and King (2001) include produced intermediate goods as a factor of production. Hence, if intermediate good prices are sticky, this introduces nominal rigidity into the costs of other price-setting firms using those goods as inputs. However, as long as these wages and input prices are set in a state-dependent manner, we retain the key strategic complementarity that is present in our more reduced-form analysis of real rigidities. In addition, a number of papers introduce features which dampen the responsiveness of price change to nominal marginal cost. We have in mind models with variable elasticity of demand as in Kimball (1995), so that a firm’s demand is highly sensitive to changes in relative price (see for instance, Chari et al., 2000; and Burstein et al., 2003). But again, this modification displays the type of strategic complementarity that we stress above. If few firms adjust their price, there is little incentive for an individual firm to do so, while

27 We view as an interesting open question the extent to which nominal wages are in fact sticky, and the magnitude of fixed costs associated with wage change. Note that in a model with state-dependent wage-setting, it is unclear how the incentives facing households would affect asymmetry in wage change decisions.
if many firms adjust prices, there is a large incentive for price change. As a result, we view the existence of multiple equilibria due to such complementarities to be a salient feature of state-dependent pricing models. Hence, it is unclear that models designed to generate realistic, persistent fluctuations are immune from displaying potentially large degrees of endogenous price adjustment.

Finally, we note that a number of researchers have suggested that hybrid time-and-state-dependent models might provide for the most realistic approach to the study of price rigidity (see for example, Ball and Mankiw, 1994; and Taylor, 1999). However, we view as an interesting open question the quantitative difference in the degree of non-linearity and asymmetry derived from our hybrid model, and a pure state-dependent pricing model. Answering this question would entail extending our simple two-period model, first, by having opt-out firms set new two-period prices, and second, by increasing the exogenous length of price contracts. Our conjecture is that as long as the frequency of price change is sufficiently high, as is true in current calibration studies, our results will not be significantly altered. Moreover, a model with multi-period price contracts and shorter time periods might shed light on evidence for asymmetric price adjustment presented in Peltzman (2000); in particular, the result that over a horizon of several quarters, output prices respond more strongly to positive cost shocks than they do to negative shocks.

7. Conclusions

In this paper we study the business cycle properties of a model with endogenous price adjustment; in particular, we study the impact of monetary policy shocks on inflation and output when firms face fixed costs of price adjustment. We make this analysis tractable by considering a hybrid model of time- and state-dependent price setting. State-dependency is modelled by allowing firms to opt-out of price contracts in response to the state of nature. However, because these contracts are of fixed duration, the state space maintains a manageable size. This allows us to characterize the complete non-linear dynamics generated by the model’s endogenous degree of price fixity.

We find that in calibrated version of our model, monetary policy shocks of moderate size generate a significant degree of endogenous price change. Hence in these cases, time-dependent pricing provides a poor approximation to the response of output and inflation generated by a model with fixed costs of price change. Also, we find that the model’s output and inflation responses are asymmetric across positive and negative monetary policy shocks. For positive shocks, prices are strategic complements so that firms have greater incentive to increase their price when other firms increase theirs. For negative shocks, prices are strategic substitutes, and firms have less incentive
to lower prices when other firms lower theirs. In equilibrium, this results in smaller output expansions associated with positive money shocks relative to output contractions due to negative shocks of the same magnitude. When we simulate our state-dependent pricing model, we find that it accounts for the magnitude of output asymmetries that have been found in empirical studies.

We find that this asymmetry is due to two crucial features: (i) the effect of other firms’ pricing decisions on an individual firm’s demand, and (ii) the positive equilibrium covariance of aggregate prices and marginal cost. These elements combine to generate the strategic complementarity and substitutability in state-dependent pricing behavior. This asymmetry is operational for shocks of all magnitudes, and also in the model both with and without real rigidities. Finally, in the model with real rigidities, this asymmetry leads to asymmetries in the presence of multiple equilibria across positive and negative shocks.

Finally, we note that the extent and importance of these asymmetries depends on the calibration of several key parameters. Hence, we believe that it is particularly important that more evidence be brought to bear on the size and distribution of fixed costs of price adjustment. Such evidence would allow researchers to better distinguish between the predictions and plausibility of competing sticky price models.

REFERENCES


Figure 1: Price Adjustment Incentives for the Individual Firm

- When other firms adjust, the normalized gross gain is shown by the red curve.
- When other firms don't adjust, the normalized gross gain is shown by the blue dashed curve.

The x-axis represents the shock, ranging from 0 to 1.1, while the y-axis represents the normalized gross gain, ranging from -0.01 to 0.1.
Figure 2: Net Gain to Price Adjustment for Positive and Negative Shocks

A: 2 percent money shock, $\lambda = 11$

B: 5 percent money shock, $\lambda = 11$

C: 5 percent money shock, $\lambda = 16$

$GG^{-}$ (negative shock, red)
$GG^{+}$ (positive shock, blue)

normalized net gain
measure of adjusting firms
Figure 3: Net Gain to Price Adjustment for Positive and Negative Shocks with Real Rigidity

A: 5 percent money shock, $\lambda = 11, \phi = 0.1$

B: 5 percent money shock, $\lambda = 16, \phi = 0.1$
Figure 4: Impact Response to x% Money Shock: Baseline

- **Switching Fraction**

- **1st Period Price, Marginal Cost**

- **Price Level**

- **Consumption**
Figure 5: Impact Response to x% Money Shock: $\lambda = 51$

- **Switching Fraction**
  - % response vs. % money shock
  - Graph shows a sharp increase at certain % money shock

- **1st Period Price, Marginal Cost**
  - Graph shows a linear increase with % money shock
  - Marked as 'time, state'

- **Price Level**
  - % response vs. % money shock
  - Graph shows a steady increase with % money shock
  - Marked as 'state'

- **Consumption**
  - % response vs. % money shock
  - Graph shows a steady increase with % money shock
  - Marked as 'time' and 'state'
Figure 6: Impact Response with Trend Inflation: Assessing the Ball-Mankiw Asymmetry

Switching Fraction

1st Period Price (Inflation)

Price Level (Inflation)

Consumption

$ f = 1 $

$ f = e^{\mu} $
Figure 7: Impact Response to x% Money Growth Shock: $\mu = 0.03$

Switching Fraction

1st Period Price (Inflation)

Price Level (Inflation)

Consumption

% response

% money growth rate shock

% money growth rate shock
Figure 8: Impact Response with Trend Inflation and AR Shock: $\mu = 0.03, \rho = 0.60$
Figure 9: Impact Response with Trend Inflation and AR Shock: $\mu = 0.03$, $\rho = 0.60$, CIA version

Switching Fraction

1st Period Price (Inflation)

Price Level (Inflation)

Consumption

% response vs % money growth rate shock
Figure 10: Real Rigidity, iid Money Shocks

Impact Response: Switching Fraction

Impact Response: Consumption

Impulse Response: Consumption, +10% shock
Figure 11: Net gain, 8% shocks; $\phi = 0.05$, $\rho = 0$