On Measuring Japan's Productivity, 1955-2003

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Abstract

The paper looks at the contribution of the market sector to changes in Japan's living standards over the years 1955-2003. Initially, a conventional Total Factor Productivity growth approach is taken where TFP growth is measured as year to year Fisher gross output growth divided by Fisher primary input growth. The slowdown in Japan's TFP performance in the post bubble period is documented. The paper also shows what happens when inventories and land are omitted from the list of primary inputs. The remainder of the paper looks at the market sector contribution to the growth in Japanese real income and decomposes this growth into three components: components due to changes in productivity, in real output prices (including changes in the terms of trade) and in primary input growth. The exact index number approach developed by Diewert and Morrison and Kohli is adapted to this real income context. Finally, the paper switches from a gross output concept to a theoretically preferred net output concept. In the net output context, it turns out that the role of capital deepening as a contributor to higher living standards diminishes and the role of productivity and labour growth becomes more important.

Key Words

Total factor productivity growth, Japanese growth in living standards, exact index numbers, capital deepening, real income growth, gross versus net output, terms of trade.

Journal of Economic Literature Classification System Numbers

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1. Introduction

The Japanese economy, the second largest in the world, has experienced tremendous growth in the past half century but there has been a pronounced slowdown in the past 15 years. The primary purpose of this paper is to look at the contribution of Japan's market oriented production sector to improvements in Japanese living standards over the period 1955-2003. In particular, we will focus on the relative contributions of Total Factor Productivity (TFP) growth, on the contribution of primary input growth and on the effects of changes in international prices to real income growth in the Japanese economy.

In section 2 of the paper, we present a conventional Fisher (1922) TFP growth accounting for the Japanese economy for the years 1955-2003. There have been several recent studies that do more or less the same thing so one might question the value of yet another study of Japanese TFP growth.² However, all of these alternative studies cover a much shorter period (with the exception of Nomura (2004)) and there are other significant differences. In the present study, we consider only the market sector of the Japanese economy where productivity improvements are possible under current national income accounting conventions. This means that we eliminated the general government sector from our growth accounting (since output is measured by input in this sector) and we also eliminated all residential housing services (and the services of consumer durables) from our output concept.³ We also followed the conventions introduced by Jorgenson and Griliches on the treatment of taxes; i.e., we adjusted prices for tax wedges whenever possible so that the adjusted prices reflect the prices that producers face.⁴ We also included the services of inventories and land as capital inputs. Our treatment of inventory change is also not conventional.⁵ We drew extensively on the web based resources of the Historical Statistics of Japan, the Cabinet Office and the Statistics Bureau in constructing our data set.⁶ We also made extensive use of the investment and

² Some of the recent studies are Hayashi and Prescott (2003), Miyagawa, Ito and Harada (2004), Jorgenson and Mitohashi (2005) and Jorgenson and Nomura (2005).

³ Owner occupied housing and consumer durables are also zero productivity sectors; i.e., output is proportional to input in these sectors. We should include the services of rental housing in our productivity data base but it proved to be too difficult to determine what portion of land and structures were used by owner occupied housing versus rental housing so we eliminated the services of all residential housing services from our market sector outputs. However, new investments in residential housing were included as outputs of the market sector.

⁴ Thus our suggested treatment of indirect commodity taxes in an accounting framework that is suitable for productivity analysis follows the example set by Jorgenson and Griliches who advocated the following treatment of indirect taxes: "In our original estimates, we used gross product at market prices; we now employ gross product from the producers' point of view, which includes indirect taxes levied on factor outlay, but excludes indirect taxes levied on output." Dale W. Jorgenson and Zvi Griliches (1972; 85).

⁵ The current SNA treatment of inventory change does not give rise to meaningful deflators and so we use the methodological approach suggested by Diewert (2005a), which in turn is based on Diewert and Smith (1994).

 <sup>(1994).
 &</sup>lt;sup>6</sup> Some useful website addresses are: <u>http://www.esri.cao.go.jp/en/sna/</u> (Statistics Information Site; Economic and Social Research Institute (ESRI), Cabinet Office, Government of Japan); <u>http://www.stat.go.jp/english/data/chouki/index.htm</u> (Historical Statistics of Japan; Statistical Research and Training Institute, Ministry of Internal Affairs and Communications);

asset data in Nomura (2004) and many additional unpublished tables that he made available for this study. One limitation of our study must be noted and that is the fact that we have no industry detail in our data base and thus we cannot locate the contributions of TFP growth in individual industries to the aggregate market sector TFP growth.⁷ An Appendix lists our final data set covering some 30 market sector inputs and outputs over the years 1955-2003.

Also in section 2, we address an important issue in productivity measurement: namely, what are the effects of ignoring the contributions of inventories and business sector land services to both the market sector real rate of return and to the market sector rates of TFP growth. We find that these effects are quite large in the case of Japan.

In section 3, we change our focus from measuring TFP growth to another measurement problem; i.e., we attempt to measure the determinants of *real income growth* in Japan. In this section and in sections 4 and 5, we adapt the analytic framework for productivity measurement that was developed by Diewert and Morrison (1986) and Kohli (1990) to the real income growth context. The main determinants of growth in real income generated by the market sector of the economy are:

- Technical progress or improvements in Total Factor Productivity;
- Growth in domestic output prices or the prices of internationally traded goods and services relative to the price of consumption; and
- Growth in primary inputs.

We show how each of these effects can be quantified.

In section 6, the methodology developed in the previous sections is implemented using our Japanese market sector data base for the years 1955-2003.

In section 7, we argue that the models developed in previous sections are not quite appropriate for determining what factors cause real income to grow. The problem is that up to this point, we have been using *gross output* as our output concept and a user cost of capital that contains a depreciation term. In this section, we argue that the depreciation

http://www.stat.go.jp/english/index.htm (Statistics Bureau, Ministry of Internal Affairs and Communications; the Monthly Statistics of Japan and the Japan Statistical Yearbooks can be found at this site). The Historical Statistics of Japan (HSOJ) presents two sets of national accounts: one for the years 1955-1998 using the System of National Accounts (SNA) 1968 and another for the years 1980-2003 using SNA 1993. We encountered some major difficulties in reconciling the two sets of accounts. Basically, we used SNA 1993 data back to 1980 and then spliced on as best we could the SNA 1968 data to extend the SNA 1993 series back to 1955. Shuji Hasegawa from the Cabinet Office provided some valuable assistance to us in reconciling the two sets of accounts.

⁷ Moreover, we cannot measure the contributions to aggregate TFP growth of shifts in labour resources from less productive sectors to more productive sectors. Thus the productivity boom over the years 1955-1973 was no doubt fuelled by the shift of labour resources from agriculture to industry and we are unable to measure the exact contribution to aggregate TFP growth of this shift (although its effects are reflected in the aggregate TFP growth). On the other hand, industry price and quantity data are notoriously unreliable due to the lack of detailed surveys on gross outputs and intermediate inputs, for service industries in particular.

term in the user cost should be taken out of the user cost and regarded as a negative output that will act as an offset to gross investment. Thus we move from a gross product concept to a net product framework. This "new" approach is illustrated using our Japanese data base.

Section 8 concludes.

2. Japanese Productivity Growth: A Conventional Approach

In this section, we measure the productivity growth of the market oriented sector of the Japanese economy using a conventional chained Fisher index number approach. Basically, TFP growth is set equal to a chained Fisher output index divided by a chained Fisher primary input index. The output aggregate is an aggregate of the familiar $C + I + G + X - M^8$ and the input aggregate is an aggregate of L + K, labour and capital services components. The production theory framework will be explained more fully in section 3 below when we shift our focus to real income measures.

There are 16 net outputs in our data base:

- Domestic final consumption expenditures of households,⁹ excluding residential housing (both owner occupied and rental properties) at producer's prices;
- Final consumption expenditures of private non-profit institutions serving households;
- General government purchases from the market sector;
- Exports of goods and services (excluding direct purchases in the domestic market by non-resident households);
- Minus imports of goods and services (excluding direct purchases abroad by resident households);
- Residential construction investment;
- Nonresidential construction investment (investment in business structures);
- Other construction investment (engineering construction investment);
- Computers, electronic and electrical equipment investment;
- Motor vehicles and other transport equipment investment;
- Other machinery and equipment investment;
- Software investment;
- Mineral exploration investment;
- Changes in finished goods (wholesale and retail trade) inventory;
- Changes in work in progress inventory;
- Changes in materials inventory.

⁸ G here is the usual government consumption aggregate found in the national accounts except general government labour input and government consumption of fixed capital (depreciation) are subtracted from the national accounts aggregate. The resulting aggregate should be equal to the (net) purchases of the general government sector from the market sector. Note that commodity tax wedges that fall on the outputs of the market sector have been removed from prices.

⁹ This aggregate is equal to domestic final consumption expenditures of households less direct purchases abroad plus purchases of non-residents in the domestic market. The sum of the last two items is small and so we will treat domestic final consumption as a good proxy for household final consumption expenditures.

There are 14 primary inputs in our data base:

- Market sector labour input;¹⁰
- Nonresidential construction (structures) services;
- Other construction (engineering construction) services;
- Computers, electronic and electrical equipment services;
- Motor vehicles and other transport equipment services;
- Other machinery and equipment services;
- Software services;
- Mineral exploration services;
- Finished goods (wholesale and retail trade) inventory services;
- Work in progress inventory services;
- Materials inventory services;
- Agricultural land services;
- Business land services (commercial and industrial land services excluding commercial and industrial land used in the general government sector);
- Forested land services (including the services of the timber standing on the land).

The prices and quantities of the above 30 inputs and outputs are listed in the Data Appendix. For each of the capital service inputs n, a beginning of the year t user cost U_n^{t} of the form $U_n^{t} \equiv [r^t + \delta_n + \tau_n^t] P_n^{t} / (1 + r^t)$ was computed, where r^t is the year t balancing real interest rate¹¹ that makes the value of market sector outputs in year t equal to the corresponding value of primary inputs, δ_n is the constant geometric rate of depreciation for asset n, τ_n^{t} is the sum of specific and general capital tax rates on asset n in year t and P_n^{t} is the stock price of asset n in year t, which is assumed to equal the corresponding investment price. The above components of the user costs are also listed in the Data Appendix.

In Table 1 below, we list the Fisher year to year output and input growth factors, y^t/y^{t-1} and x^t/x^{t-1} respectively along with their ratios, $\tau^t = [y^t/y^{t-1}]/[x^t/x^{t-1}]$ (the TFP growth rates or more accurately, one plus the TFP growth rates), and the balancing year t real interest rate r^{t} .¹²

¹⁰ The labour input measure excludes general government employees. The market sector labour input is an hours based aggregate of market sector employees, the self employed and family workers. We assumed that self employed workers earned .67 times the wage of a market sector employee and family workers earned .33 times the wage of a market sector employee. This is rather unsatisfactory and we hope to improve these estimates in the future.

¹¹ We assume that producer's expect asset price inflation in year t to equal the general inflation rate. This assumption causes the expected capital gains term in the user cost formula to cancel out with the Fisher effect on nominal interest rates, leaving a real interest rate in the user cost formula. This assumption is not entirely satisfactory but other assumptions generally lead to negative user costs for land in Japan, which we wanted to avoid. Our treatment of capital taxes is an average approach as opposed to the usual marginal approach. We could not implement the marginal approach due to the complexity of modeling the Japanese tax code. Moreover, a case can be made for using the average approach in any case.

¹² Note that r^t was slightly negative in the years 1974 and 1978. This did not cause a problem in our user costs since (positive) capital taxes always outweighed these two negative terms. However, the existence of negative balancing real interest rates does illustrate a problem with using these endogenous rates rather than

 Table 1: Fisher Chained Indexes of Output, Input and Productivity Growth and the

 Balancing Real Rate of Return in the Japanese Economy, 1956-2003.

Year t	y ^t /y ^{t-1}	$\mathbf{x}^{t} / \mathbf{x}^{t-1}$	τ^{t}	\mathbf{r}^{t}
1956.	1.11089	1.05055	1.05744	0.02993
1950.	1.13045	1.05055	1.07531	0.03838
1958.	1.00747	1.03533	0.97310	0.02170
1959.	1.13767	1.03643	1.09769	0.02837
1960.	1.15791	1.04420	1.10890	0.03667
1961.	1.17788	1.04165	1.13079	0.04748
1962.	1.04966	1.04448	1.00496	0.03219
1963.	1.13036	1.03441	1.09275	0.03843
1964.	1.14388	1.04365	1.09604	0.04473
1965.	1.04259	1.04262	0.99997	0.03428
1966.	1.12095	1.05268	1.06486	0.03974
1967.	1.12901	1.04769	1.07762	0.04413
1968.	1.14330	1.03862	1.10078	0.05125
1969.	1.13089	1.04689	1.08024	0.05083
1970.	1.10212	1.04407	1.05560	0.04586
1971.	1.03201	1.03818	0.99406	0.02824
1972.	1.06591	1.03196	1.03290	0.02054
1973.	1.09695	1.04567	1.04904	0.01648
1974.	1.04474	1.00797	1.03648	0.01490
1975.	0.94364	0.99872	0.94485	-0.00121
1976.	1.06655	1.03347	1.03201	0.00516
1977.	1.03411	1.02770	1.00624	0.00182
1978.	1.02873	1.01936	1.00919	-0.00168
1979.	1.10179	1.02089	1.07925	0.00430
1980.	1.07161	1.02340	1.04711	0.01616
1981.	1.03011	1.02309	1.00685	0.01387
1982.	1.03933	1.02047	1.01849	0.01363
1983.	1.01051	1.03019	0.98090	0.00841
1984.	1.04736	1.01238	1.03456	0.01218
1985.	1.07489	1.02129	1.05248	0.01979
1986.	1.00342	1.02200	0.98183	0.01289
1987.	1.05007	1.02399	1.02547	0.01334
1988.	1.06141	1.02941	1.03108	0.01322
1989.	1.06592	1.02684	1.03806	0.01546
1990.	1.06747	1.02445	1.04199	0.01687
1991.	1.01770	1.02274	0.99507	0.01085
1992.	1.00688	1.01349	0.99348	0.01098
1993.	0.98630	1.00666	0.97977	0.00705
1994.	1.01604	1.00765	1.00833	0.00896

an exogenous expected real rate. However, for Japan, when we replaced the endogenous balancing real rates r^t with a constant real interest rate of 2%, we found very little change in our results overall.

1995.	1.01505	1.00979	1.00521	0.00751
1996.	1.04013	1.00788	1.03200	0.01021
1997.	1.02844	1.00544	1.02287	0.01088
1998.	0.97323	1.00219	0.97110	0.00566
1999.	0.99736	1.00142	0.99595	0.00517
2000.	1.03810	1.01692	1.02084	0.00823
2001.	0.99320	0.99501	0.99819	0.00373
2002.	0.99371	0.99834	0.99537	0.00336
2003.	1.01977	1.00072	1.01903	0.00599
Average	1.0579	1.0255	1.0312	0.0193

Thus the (Fisher) average annual growth rates for market sector outputs and inputs in Japan were 5.79% and 2.55% per year respectively and the average rate of TFP growth over the period 1955-2003 was an excellent 3.12% per year. The real rates of return averaged 1.93% per year.¹³

However, there are some big variations in the averages over various subperiods. Over the period 1956-1973 (up to the first oil shock), the average real rate of return was a very reasonable 3.61% per year and the average rate of TFP growth was a spectacular 6.07% per year. The average real rate of return dropped to 0.39% per year over the oil shock years 1974-1979 inclusive and the rate of TFP growth dropped to 1.80% per year, which is still very good by international standards. Over the bubble years, 1980-1990 inclusive, the real rate of return recovered to an average of 1.42% per year and TFP growth increased to 2.35% per year. However, for the post bubble years 1991-2003, the real rate of return fell to 0.76% per year on average and the rate of TFP growth fell to a rather disappointing 0.29% per year.

It is important to include land and inventory services in the list of capital services. If these primary inputs are omitted from a productivity or real rate of return analysis, very distorted results can be obtained. We illustrate this point by recomputing the balancing real rates of return and the Fisher TFP growth rates for Japan by omitting first land and then land and inventories from the asset base. The results when land is dropped may be found in Table 2 below.

Table 2: Fisher Chained Indexes of Output, Input and Productivity Growth and theBalancing Real Rate of Return in the Japanese Economy with Land Omitted, 1956-2003.

Year t	y^{t}/y^{t-1}	$\mathbf{x}^{t} / \mathbf{x}^{t-1}$	$\boldsymbol{\tau}^t$	\mathbf{r}^{t}
1956.	1.11089	1.05515	1.05282	0.08642
1957.	1.13045	1.05935	1.06711	0.10884
1958.	1.00747	1.04622	0.96297	0.06718
1959.	1.13767	1.04148	1.09236	0.08650
1960.	1.15791	1.05346	1.09915	0.11397
1961.	1.17788	1.05316	1.11842	0.14791

¹³ When we include 1955 in the average, the average real rate of return increases slightly to 1.94% per year.

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1962.	1.04966	1.06103	0.98929	0.10569
1963.	1.13036	1.04471	1.08198	0.12734
1964.	1.14388	1.05637	1.08284	0.14322
1965.	1.04259	1.05420	0.98898	0.11036
1966.	1.12095	1.05367	1.06386	0.12718
1967.	1.12901	1.05107	1.07416	0.14463
1968.	1.14330	1.05296	1.08580	0.16540
1969.	1.13089	1.05561	1.07131	0.16683
1970.	1.10212	1.06036	1.03937	0.15304
1971.	1.03201	1.05231	0.98070	0.09924
1972.	1.06591	1.03953	1.02538	0.07805
1973.	1.09695	1.04681	1.04790	0.06946
1974.	1.04474	1.01153	1.03282	0.06073
1975.	0.94364	1.00190	0.94185	0.01363
1976.	1.06655	1.03452	1.03097	0.02733
1977.	1.03411	1.02808	1.00586	0.01972
1978.	1.02873	1.02009	1.00847	0.01419
1979.	1.10179	1.02170	1.07839	0.02643
1980.	1.07161	1.02527	1.04519	0.05161
1981.	1.03011	1.02652	1.00350	0.04673
1982.	1.03933	1.02272	1.01624	0.04664
1983.	1.01051	1.03255	0.97865	0.03549
1984.	1.04736	1.01472	1.03217	0.04494
1985.	1.07489	1.02250	1.05124	0.06504
1986.	1.00342	1.02661	0.97741	0.05125
1987.	1.05007	1.02615	1.02331	0.05687
1988.	1.06141	1.03135	1.02914	0.06109
1989.	1.06592	1.03052	1.03435	0.06757
1990.	1.06747	1.02942	1.03697	0.07088
1991.	1.01770	1.02936	0.98868	0.05351
1992.	1.00688	1.01669	0.99035	0.04673
1993.	0.98630	1.00840	0.97808	0.03262
1994.	1.01604	1.00809	1.00789	0.03289
1995.	1.01505	1.01072	1.00429	0.02877
1996.	1.04013	1.00849	1.03138	0.03357
1997.	1.02844	1.00621	1.02210	0.03264
1998.	0.97323	1.00307	0.97025	0.02028
1999.	0.99736	1.00213	0.99525	0.01749
2000.	1.03810	1.01702	1.02073	0.02238
2001.	0.99320	0.99534	0.99785	0.01377
2002.	0.99371	0.99820	0.99551	0.01240
2003.	1.01977	1.00016	1.01960	0.01575
Average		1.0302	1.0265	0.0672
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Note that all of the real rates of return are now positive and the average real rate of return has increased from 1.93% to 6.72% per year, an extremely large increase! Moreover, the

average rate of TFP growth has decreased from 3.12% per year to 2.65% per year, a drop of almost 0.5% per year, which is a very large change indeed.¹⁴

When both land and inventories are dropped, there are further changes as Table 3 below indicates.

 Table 3: Fisher Chained Indexes of Output, Input and Productivity Growth and the

 Balancing Real Rate of Return in the Japanese Economy with Land and Inventories

 Omitted, 1956-2003.

Year t	y^t / y^{t-1}	$\mathbf{x}^{t} / \mathbf{x}^{t-1}$	$\mathbf{\tau}^{\mathrm{t}}$	r^t
1956.	1.11089	1.05435	1.05362	0.13736
1957.	1.13045	1.05801	1.06847	0.17433
1958.	1.00747	1.04263	0.96628	0.10939
1959.	1.13767	1.04467	1.08902	0.13644
1960.	1.15791	1.05543	1.09709	0.17840
1961.	1.17788	1.05911	1.11215	0.22450
1962.	1.04966	1.06224	0.98816	0.15736
1963.	1.13036	1.05236	1.07412	0.18335
1964.	1.14388	1.05947	1.07967	0.20283
1965.	1.04259	1.05599	0.98731	0.15330
1966.	1.12095	1.05656	1.06094	0.17349
1967.	1.12901	1.05276	1.07244	0.19585
1968.	1.14330	1.05305	1.08570	0.22543
1969.	1.13089	1.05686	1.07004	0.22575
1970.	1.10212	1.06241	1.03738	0.20496
1971.	1.03201	1.05413	0.97902	0.13156
1972.	1.06591	1.04291	1.02205	0.10093
1973.	1.09695	1.05065	1.04407	0.08712
1974.	1.04474	1.01384	1.03047	0.07605
1975.	0.94364	1.00055	0.94313	0.01911
1976.	1.06655	1.03564	1.02986	0.03516
1977.	1.03411	1.02829	1.00566	0.02595
1978.	1.02873	1.02046	1.00810	0.01934
1979.	1.10179	1.02290	1.07713	0.03298
1980.	1.07161	1.02551	1.04495	0.06285
1981.	1.03011	1.02620	1.00381	0.05752
1982.	1.03933	1.02292	1.01604	0.05754
1983.	1.01051	1.03216	0.97903	0.04448
1984.	1.04736	1.01513	1.03175	0.05574
1985.	1.07489	1.02234	1.05139	0.08028
1986.	1.00342	1.02556	0.97841	0.06400
1987.	1.05007	1.02722	1.02225	0.07028

¹⁴ The intuitive explanation for this result is not hard to explain: dropping land (which grows slowly or not at all) from the list of inputs has the effect of increasing the average rate of input growth, thus decreasing the rate of productivity growth. This effect is particularly pronounced in Japan with its high land prices.

1988.	1.06141	1.03150	1.02899	0.07513
1989.	1.06592	1.03109	1.03377	0.08230
1990.	1.06747	1.02950	1.03688	0.08588
1991.	1.01770	1.02911	0.98891	0.06523
1992.	1.00688	1.01773	0.98934	0.05619
1993.	0.98630	1.00923	0.97727	0.03919
1994.	1.01604	1.00904	1.00694	0.03875
1995.	1.01505	1.01129	1.00372	0.03386
1996.	1.04013	1.00887	1.03098	0.03924
1997.	1.02844	1.00643	1.02187	0.03809
1998.	0.97323	1.00311	0.97021	0.02401
1999.	0.99736	1.00266	0.99472	0.02056
2000.	1.03810	1.01760	1.02015	0.02585
2001.	0.99320	0.99556	0.99763	0.01627
2002.	0.99371	0.99855	0.99516	0.01461
2003.	1.01977	1.00043	1.01933	0.01812
Average	1.0579	1.0311	1.0255	0.0912

Compared to the results in Table 2, the average rate of TFP growth has dropped another 0.1% per year when we omit inventories (in addition to omitting land services) and the average real rate of return has increased to 9.12% per year, compared to the 6.72% when land is dropped and the original 1.93% when both land and inventories were present in the productivity asset base.

The results presented in this section illustrate the importance of including both land and inventories in the asset base when doing productivity and rate of return studies.

We turn now to a more theoretical framework where we will be able to determine the factors that explain real income growth in the Japanese economy.

3. The Production Theory Framework for the Analysis of Real Income Growth

In this section, we present the production theory framework which will be used in the remainder of the paper. The main references are Diewert and Morrison (1986) and Kohli (1990).¹⁵

Initially, we assume that the market sector of the economy produces quantities of M $(net)^{16}$ outputs, $y = [y_1,...,y_M]$, which are sold at the positive producer prices P =

¹⁵ The theory also draws on Samuelson (1953), Diewert (1974; 133-141) (1980) (1983; 1077-1100), Fox and Kohli (1998), Kohli (1978) (1991) (2003) (2004a) (2004b), Morrison and Diewert (1990), Samuelson (1953) and Sato (1976). The theoretical framework explained in this section was recently used by Diewert (2005b) and Diewert and Lawrence (2005b).

¹⁶ If the mth commodity is an import (or other produced input) into the market sector of the economy, then the corresponding quantity y_m is indexed with a negative sign. We will follow Kohli (1978) (1991) and Woodland (1982) in assuming that imports flow through the domestic production sector and are "transformed" (perhaps only by adding transportation, wholesaling and retailing margins) by the domestic production sector. The recent textbook by Feenstra (2004; 76) also uses this approach.

 $[P_1,...,P_M]$. We further assume that the market sector of the economy uses positive quantities of N primary inputs, $x = [x_1,...,x_N]$ which are purchased at the positive primary input prices $W = [W_1,...,W_N]$. In period t, we assume that there is a feasible set of output vectors y that can be produced by the market sector if the vector of primary inputs x is utilized by the market sector of the economy; denote this period t production possibilities set by S^t. We assume that S^t is a closed convex cone that exhibits a free disposal property.¹⁷

Given a vector of output prices P and a vector of available primary inputs x, we define *the period t market sector GDP function*, $g^{t}(P,x)$, as follows:¹⁸

(1) $g^{t}(P,x) = \max_{y} \{P \cdot y : (y,x) \text{ belongs to } S^{t}\};$ t = 0,1,2,...

Thus market sector GDP depends on t (which represents the period t technology set S^t), on the vector of output prices P that the market sector faces and on x, the vector of primary inputs that is available to the market sector.

If P^t is the period t output price vector and x^t is the vector of inputs used by the market sector during period t and if the GDP function is differentiable with respect to the components of P at the point P^t, x^t , then the period t vector of market sector outputs y^t will be equal to the vector of first order partial derivatives of $g^t(P^t, x^t)$ with respect to the components of P; i.e., we will have the following equations for each period t:¹⁹

(2) $y^t = \nabla_P g^t(P^t, x^t)$; t = 0, 1, 2, ...

Thus the period t market sector supply vector y^t can be obtained by differentiating the period t market sector GDP function with respect to the components of the period t output price vector P^t .

If the GDP function is differentiable with respect to the components of x at the point P^t, x^t , then the period t vector of input prices W^t will be equal to the vector of first order partial

¹⁷ For a more explanation for the meaning of these properties, see Diewert (1973) (1974; 134) or Woodland (1982) or Kohli (1978) (1991). The assumption that S^t is a cone means that the technology is subject to constant returns to scale. This is an important assumption since it implies that the value of outputs should equal the value of inputs in equilibrium. In our empirical work, we use an expost rate of return in our user costs of capital, which forces the value of inputs to equal the value of outputs for each period. The function g^t is known as the *GDP function* or the *national product function* in the international trade literature (see Kohli (1978)(1991), Woodland (1982) and Feenstra (2004; 76). It was introduced into the economics literature by Samuelson (1953). Alternative terms for this function include: (i) the *gross profit function*; see Gorman (1968); (ii) the *restricted profit function*; see Lau (1976) and McFadden (1978); and (iii) the *variable profit function*; see Diewert (1973) (1974) (1993).

¹⁸ The function $g^{t}(P,x)$ will be linearly homogeneous and convex in the components of P and linearly homogeneous and concave in the components of x; see Diewert (1973) (1974; 136). Notation: $P \cdot y = \sum_{m=1}^{M} P_m y_m$.

 $P_m y_m$. ¹⁹ These relationships are due to Hotelling (1932; 594). Note that $\nabla_P g^t(P^t, x^t) = [\partial g^t(P^t, x^t)/\partial P_1, ..., \partial g^t(P^t, x^t)/\partial P_M]$.

derivatives of $g^t(P^t,x^t)$ with respect to the components of x; i.e., we will have the following equations for each period t:²⁰

(3)
$$W^{t} = \nabla_{x} g^{t}(P^{t}, x^{t});$$
 $t = 0, 1, 2, ..., t = 0$

Thus the period t market sector input prices W^t paid to primary inputs can be obtained by differentiating the period t market sector GDP function with respect to the components of the period t input quantity vector x^t.

The constant returns to scale assumption on the technology sets S^t implies that the value of outputs will equal the value of inputs in period t; i.e., we have the following relationships:

(4)
$$g^{t}(P^{t},x^{t}) = P^{t} \cdot y^{t} = W^{t} \cdot x^{t}$$
; $t = 0,1,2,...$

The above material will be useful in what follows but of course, our focus is not on GDP; instead our focus is on the income generated by the market sector or more precisely, on *the real income generated by the market sector*. However, since market sector GDP (the value of market sector production) is distributed to the factors of production used by the market sector, nominal market sector GDP will be equal to nominal market sector income; i.e., from (4), we have $g^t(P^t, x^t) = P^t \cdot y^t = W^t \cdot x^t$. As an approximate welfare measure that can be associated with market sector production,²¹ we will choose to measure the *real income generated by the market sector in period t*, r^t , in terms of the number of consumption bundles that the nominal income could purchase in period t; i.e., define ρ^t as follows:

(5)
$$\rho^{t} = W^{t} \cdot x^{t} / P_{C}^{t}$$
;
 $= w^{t} \cdot x^{t}$
 $= p^{t} \cdot y^{t}$
 $= g^{t}(p^{t}, x^{t})$
 $t = 0, 1, 2, ...$

where $P_C^t > 0$ is the *period t consumption expenditures deflator* and the market sector period t *real output price* p^t and *real input price* w^t vectors are defined as the corresponding nominal price vectors deflated by the consumption expenditures price index; i.e., we have the following definitions:²²

²⁰ These relationships are due to Samuelson (1953) and Diewert (1974; 140). Note that $\nabla x g^t(P^t, x^t) = [\partial g^t(P^t, x^t)/\partial x_1, ..., \partial g^t(P^t, x^t)/\partial x_N].$ ²¹ Since some of the primary inputs used by the market sector can be owned by foreigners, our measure of

²¹ Since some of the primary inputs used by the market sector can be owned by foreigners, our measure of *domestic* welfare generated by the market production sector is only an approximate one. Moreover, our suggested welfare measure is not sensitive to the distribution of the income that is generated by the market sector.

²² Our approach is similar to the approach advocated by Kohli (2004b; 92), except he essentially deflates nominal GDP by the domestic expenditures deflator rather than just the domestic (household) expenditures deflator; i.e., he deflates by the deflator for C+G+I, whereas we suggest deflating by the deflator for C. Another difference in his approach compared to the present approach is that we restrict our analysis to the market sector GDP, whereas Kohli deflates all of GDP (probably due to data limitations). Our treatment of the balance of trade surplus or deficit is also different.

(6)
$$p^{t} = P^{t}/P_{C}^{t}$$
; $w^{t} = W^{t}/P_{C}^{t}$; $t = 0, 1, 2, ...$

The first and last equality in (5) imply that period t real income, ρ^t , is equal to the period t GDP function, evaluated at the period t real output price vector p^t and the period t input vector x^t , $g^t(p^t,x^t)$. Thus the growth in real income over time can be explained by three main factors: t (Technical Progress or Total Factor Productivity growth), growth in real output prices and the growth of primary inputs. We will shortly give formal definitions for these three growth factors.

Using the linear homogeneity properties of the GDP functions $g^{t}(P,x)$ in P and x separately, we can show that the following counterparts to the relations (2) and (3) hold using the deflated prices p and w:²³

(7)
$$y^t = \nabla_p g^t(p^t, x^t)$$
; $t = 0, 1, 2, ...$
(8) $w^t = \nabla_x g^t(p^t, x^t)$; $t = 0, 1, 2, ...$

Now we are ready to define a family of *period t productivity growth factors or technical progress shift factors* $\tau(p,x,t)$:²⁴

(9)
$$\tau(p,x,t) = g^{t}(p,x)/g^{t-1}(p,x)$$
; $t = 1,2,...$

Thus $\tau(p,x,t)$ measures the proportional change in the real income produced by the market sector at the reference real output prices p and reference input quantities used by the market sector x where the numerator in (9) uses the period t technology and the denominator in (9) uses the period t–1 technology. Thus each choice of reference vectors p and x will generate a possibly different measure of the shift in technology going from period t–1 to period t. Note that we are using the chain system to measure the shift in technology.

It is natural to choose special reference vectors for the measure of technical progress defined by (9): a *Laspeyres type measure* τ_L^t that chooses the period t–1 reference vectors p^{t-1} and x^{t-1} and a *Paasche type measure* τ_P^t that chooses the period t reference vectors p^t and x^t :

$$(10) \tau_{L}^{t} = \tau(p^{t-1}, x^{t-1}, t) = g^{t}(p^{t-1}, x^{t-1})/g^{t-1}(p^{t-1}, x^{t-1});$$

$$(11) \tau_{P}^{t} = \tau(p^{t}, x^{t}, t) = g^{t}(p^{t}, x^{t})/g^{t-1}(p^{t}, x^{t});$$

$$t = 1, 2, ...;$$

$$t = 1, 2, ...;$$

Since both measures of technical progress are equally valid, it is natural to average them to obtain an overall measure of technical change. If we want to treat the two measures in

²³ If producers in the market sector of the economy are solving the profit maximization problem that is associated with $g^t(P,x)$, which uses the original output prices P, then they will also solve the profit maximization problem that uses the normalized output prices $p = P/P_C$; i.e., they will also solve the problem defined by $g^t(p,x)$.

²⁴ This measure of technical progress is due to Diewert and Morrison (1986; 662).

a symmetric manner and we want the measure to satisfy the time reversal property from index number theory²⁵ (so that the estimate going backwards is equal to the reciprocal of the estimate going forwards), then the geometric mean will be the best simple average to take.²⁶ Thus we define the geometric mean of (10) and (11) as follows:²⁷

(12)
$$\tau^{t} = [\tau_{L}^{t} \tau_{P}^{t}]^{1/2}$$
; $t = 1, 2, ...$

At this point, it is not clear how we will obtain empirical estimates for the theoretical productivity growth indexes defined by (10)-(12). One obvious way would be to assume a functional form for the GDP function $g^{t}(p,x)$, collect data on output and input prices and quantities for the market sector for a number of years (and for the consumption expenditures deflator), add error terms to equations (7) and (8) and use econometric techniques to estimate the unknown parameters in the assumed functional form. However, econometric techniques are generally not completely straightforward: different functional forms.²⁸ Moreover, as the number of outputs and inputs grows, it will be impossible to estimate a flexible functional form. Thus we will suggest methods for implementing measures like (12) in this paper that are based on exact index number techniques.

We turn now to the problem of defining theoretical indexes for the effects on real income due to changes in real output prices. Define a family of *period t real output price growth* factors $\alpha(p^{t-1},p^t,x,s)$:²⁹

(13)
$$\alpha(p^{t-1}, p^t, x, s) = g^s(p^t, x)/g^s(p^{t-1}, x);$$

 $s = 1, 2, ...$

Thus $\alpha(p^{t-1}, p^t, x, s)$ measures the proportional change in the real income produced by the market sector that is induced by the change in real output prices going from period t–1 to t, using the technology that is available during period s and using the reference input quantities x. Thus each choice of the reference technology s and the reference input vector x will generate a possibly different measure of the effect on real income of a change in real output prices going from period t–1 to period t.

²⁵ See Fisher (1922; 64).

²⁶ See the discussion in Diewert (1997) on choosing the "best" symmetric average of Laspeyres and Paasche indexes that will lead to the satisfaction of the time reversal test by the resulting average index.
²⁷ The theoretical productivity change indexes defined by (10)-(12) were first defined by Diewert and

²⁷ The theoretical productivity change indexes defined by (10)-(12) were first defined by Diewert and Morrison (1968; 662-663). See Diewert (1993) for properties of symmetric means.

²⁸ "The estimation of GDP functions such as (19) can be controversial, however, since it raises issues such as estimation technique and stochastic specification. ... We therefore prefer to opt for a more straightforward index number approach." Ulrich Kohli (2004a; 344).

²⁹ This measure of real output price change was essentially defined by Fisher and Shell (1972; 56-58), Samuelson and Swamy (1974; 588-592), Archibald (1977; 60-61), Diewert (1980; 460-461) (1983; 1055) and Balk (1998; 83-89). Readers who are familiar with the theory of the true cost of living index will note that the real output price index defined by (13) is analogous to the Konüs (1924) *true cost of living index* which is a ratio of cost functions, say $C(u,p^t)/C(u,p^{t-1})$ where u is a reference utility level: g^s replaces C and the reference utility level u is replaced by the vector of reference variables x.

Again, it is natural to choose special reference vectors for the measures defined by (13): a *Laspeyres type measure* α_L^t that chooses the period t–1 reference technology and reference input vector x^{t-1} and a *Paasche type measure* α_P^t that chooses the period t reference technology and reference input vector x^t :

$$\begin{array}{ll} (14) \ \alpha_L{}^t \equiv \alpha(p^{t-1},p^t,x^{t-1},t-1) = g^{t-1}(p^t,x^{t-1})/g^{t-1}(p^{t-1},x^{t-1}) \ ; & t = 1,2, \dots \ ; \\ (15) \ \alpha_P{}^t \equiv \alpha(p^{t-1},p^t,x^t,t) & = g^t(p^t,x^t)/g^t(p^{t-1},x^t) \ ; & t = 1,2, \dots \ ; \\ \end{array}$$

Since both measures of real output price change are equally valid, it is natural to average them to obtain an overall measure of the effects on real income of the change in real output prices:³⁰

(16)
$$\alpha^{t} = [\alpha_{L}^{t} \alpha_{P}^{t}]^{1/2};$$
 $t = 1, 2, ..., t = 1, 2, ..., t$

Finally, we look at the problem of defining theoretical indexes for the effects on real income due to changes in real output prices. Define a family of *period t real input quantity growth factors* $\beta(x^{t-1},x^t,p,s)$.³¹

(17)
$$\beta(x^{t-1}, x^t, p, s) = g^s(p, x^t)/g^s(p, x^{t-1});$$
 $s = 1, 2, ..., s =$

Thus $\beta(x^{t-1}, x^t, p, s)$ measures the proportional change in the real income produced by the market sector that is induced by the change in input quantities used by the market sector going from period t–1 to t, using the technology that is available during period s and using the reference real output prices p. Thus each choice of the reference technology s and the reference real output price vector p will generate a possibly different measure of the effect on real income of a change in input quantities going from period t–1 to period t.

Again, it is natural to choose special reference vectors for the measures defined by (17): a *Laspeyres type measure* β_L^t that chooses the period t–1 reference technology and reference real output price vector p^{t-1} and a *Paasche type measure* β_P^t that chooses the period t reference technology and reference real output price vector p^t :

$$(18) \ \beta_{L}{}^{t} = \beta(x^{t-1}, x^{t}, p^{t-1}, t-1) = g^{t-1}(p^{t-1}, x^{t})/g^{t-1}(p^{t-1}, x^{t-1}); \qquad t = 1, 2, ...;$$

$$(19) \ \beta_{P}{}^{t} = \beta(x^{t-1}, x^{t}, p^{t}, t) = g^{t}(p^{t}, x^{t})/g^{t}(p^{t}, x^{t-1}); \qquad t = 1, 2, ...;$$

Since both measures of real input growth are equally valid, it is natural to average them to obtain an overall measure of the effects of input growth on real income:³²

(20)
$$\beta^{t} = [\beta_{L}^{t} \beta_{P}^{t}]^{1/2};$$
 $t = 1, 2,$

 $^{^{30}}$ The indexes defined by (13)-(16) were defined by Diewert and Morrison (1986; 664) in the nominal GDP function context.

 ³¹ This type of index was defined as a true index of value added by Sato (1976; 438) and as a real input index by Diewert (1980; 456).
 ³² The theoretical indexes defined by (17)-(20) were defined in Diewert and Morrison (1986; 665) in the

³² The theoretical indexes defined by (17)-(20) were defined in Diewert and Morrison (1986; 665) in the nominal GDP context.

Recall that market sector real income for period t was defined by (5) as ρ^t equal to nominal period t factor payments $W^t \cdot x^t$ deflated by the household consumption price deflator P_C^t . It is convenient to define γ^t as the *period t chain rate of growth factor for real income*:

(21)
$$\gamma^{t} = \rho^{t} / \rho^{t-1}$$
; $t = 1, 2, ...$

It turns out that the definitions for γ^t and the technology, output price and input quantity growth factors $\tau(p,x,t)$, $\alpha(p^{t-1},p^t,x,s)$, $\beta(x^{t-1},x^t,p,s)$ defined by (9), (13) and (17) respectively satisfy some interesting identities, which we will now develop. We have:

$$\begin{array}{ll} (22) \ \gamma^t &\equiv \rho^t / \rho^{t-1} \ ; & t = 1,2, \dots . \\ &= g^t (p^t, x^t) / g^{t-1} (p^{t-1}, x^{t-1}) & \text{using definitions (4) and (5)} \\ &= [g^t (p^t, x^t) / g^{t-1} (p^t, x^t)] [g^{t-1} (p^t, x^t) / g^{t-1} (p^{t-1}, x^t)] [g^{t-1} (p^{t-1}, x^{t-1})] \\ &= \tau_P^t \ \alpha (p^{t-1}, p^t, x^t, t-1) \ \beta_L^t & \text{using definitions (11), (13) and (18).} \end{array}$$

In a similar fashion, we can establish the following companion identity:

(23)
$$\gamma^t = \tau_L^t \alpha(p^{t-1}, p^t, x^{t-1}, t) \beta_P^t$$
 using definitions (10), (13) and (19)

Thus multiplying (22) and (23) together and taking positive square roots of both sides of the resulting identity and using definitions (12) and (20), we obtain the following identity:

(24)
$$\gamma^{t} = \tau^{t} \left[\alpha(p^{t-1}, p^{t}, x^{t}, t-1) \alpha(p^{t-1}, p^{t}, x^{t-1}, t) \right]^{1/2} \beta^{t}; \qquad t = 1, 2, ...,$$

In a similar fashion, we can derive the following alternative decomposition for γ^t into growth factors:

(25)
$$\gamma^{t} = \tau^{t} \alpha^{t} \left[\beta(x^{t-1}, x^{t}, p^{t}, t-1)\beta(x^{t-1}, x^{t}, p^{t-1}, t)\right]^{1/2}$$
; $t = 1, 2, ...$

It is quite likely that the real output price growth factor $[\alpha(p^{t-1},p^t,x^t,t-1)\alpha(p^{t-1},p^t,x^{t-1},t)]^{1/2}$ is fairly close to α^t defined by (16) and it is quite likely that the input growth factor $[\beta(x^{t-1},x^t,p^t,t-1)\beta(x^{t-1},x^t,p^{t-1},t)]^{1/2}$ is quite close to β^t defined by (20); i.e., we have the following approximate equalities:

$$\begin{array}{ll} (26) \left[\alpha(p^{t-1},p^t,x^t,t-1)\alpha(p^{t-1},p^t,x^{t-1},t) \right]^{1/2} \approx \alpha^t ; & t = 1,2, \dots ; \\ (27) \left[\beta(x^{t-1},x^t,p^t,t-1)\beta(x^{t-1},x^t,p^{t-1},t) \right]^{1/2} \approx \beta^t ; & t = 1,2, \dots . \end{array}$$

Substituting (26) and (27) into (24) and (25) respectively leads to the following approximate decompositions for the growth of real income into explanatory factors:

(28)
$$\gamma^{t} \approx \tau^{t} \alpha^{t} \beta^{t}$$
; $t = 1, 2, ...$

where τ^t is a technology growth factor, α^t is a growth in real output prices factor and β^t is a growth in primary inputs factor.

Rather than look at explanatory factors for the growth in real market sector income, it is sometimes convenient to express the level of real income in period t in terms of an *index* of the technology level or of Total Factor Productivity in period t, T^t, of the level of real output prices in period t, A^t, and of the level of primary input quantities in period t, B^t.³³ Thus we use the growth factors τ^t , α^t and β^t as follows to define the levels T^t, A^t and B^t:

 $\begin{array}{l} (29) \ T^0 \equiv 1 \ ; \ T^t \equiv T^{t-1} \ \tau^t \ ; \ t = 1,2, \ \ldots \ ; \\ (30) \ A^0 \equiv 1 \ ; \ A^t \equiv A^{t-1} \alpha^t \ ; \ t = 1,2, \ \ldots \ ; \\ (31) \ B^0 \equiv 1 \ ; \ B^t \equiv B^{t-1} \beta^t \ ; \ t = 1,2, \ \ldots \ . \end{array}$

Using the approximate equalities (28) for the chain links that appear in (29)-(31), we can establish the following approximate relationship for the level of real income in period t, ρ^{t} , and the period t levels for technology, real output prices and input quantities:

(32)
$$\rho^{t} / \rho^{0} \approx T^{t} A^{t} B^{t}$$
; $t = 0, 1, 2, ...$

In the following section, we note a set of assumptions on the technology sets that will ensure that the approximate real income growth decompositions (28) and (32) hold as exact equalities.

4. The Translog GDP Function Approach

We now follow the example of Diewert and Morrison (1986; 663) and assume that the log of the period t (deflated) GDP function, $g^{t}(p,x)$, has the following translog functional form:³⁴

$$(33) \ln g^{t}(p,x) = a_{0}^{t} + \sum_{m=1}^{M} a_{m}^{t} \ln p_{m}^{t} + (1/2) \sum_{m=1}^{M} \sum_{k=1}^{M} a_{mk} \ln p_{m}^{t} \ln p_{k}^{t} + \sum_{n=1}^{N} b_{n}^{t} \ln x_{n}^{t} + (1/2) \sum_{n=1}^{N} \sum_{j=1}^{N} b_{nj} \ln x_{n}^{t} \ln x_{j}^{t} + \sum_{m=1}^{M} \sum_{n=1}^{M} c_{mn} \ln p_{m}^{t} \ln x_{n}^{t}; t = 0, 1, 2, ...$$

Note that the coefficients for the quadratic terms are assumed to be constant over time. The coefficients must satisfy the following restrictions in order for g^t to satisfy the linear homogeneity properties that we have assumed in section 2 above:³⁵

(34) $\sum_{m=1}^{M} a_m^{t} = 1$ for t = 0, 1, 2, ...;

³³ This type of levels presentation of the data is quite instructive when presented in graphical form. It was suggested by Kohli (1990) and used extensively by him; see Kohli (1991), (2003) (2004a) (2004b) and Fox and Kohli (1998).

³⁴ This functional form was first suggested by Diewert (1974; 139) as a generalization of the translog functional form introduced by Christensen, Jorgenson and Lau (1971). Diewert (1974; 139) indicated that this functional form was flexible.

 $^{^{35}}$ There are additional restrictions on the parameters which are necessary to ensure that $g^t(p,x)$ is convex in p and concave in x.

 $\begin{array}{l} (35) \sum_{n=1}^{N} b_{n}^{\ t} = 1 \ for \ t = 0,1,2, \ ...; \\ (36) \ a_{mk} = a_{km} \ for \ all \ k,m \ ; \\ (37) \ b_{nj} = b_{jn} \ for \ all \ n,j \ ; \\ (38) \ \sum_{k=1}^{M} a_{mk} = 0 \ for \ m = 1, \ ..., M \ ; \\ (39) \ \sum_{j=1}^{N} b_{nj} = 0 \ for \ n = 1, \ ..., N \ ; \\ (40) \ \sum_{n=1}^{N} c_{mn} = 0 \ for \ m = 1, \ ..., M \ ; \\ (41) \ \sum_{m=1}^{M} c_{mn} = 0 \ for \ n = 1, \ ..., N \ . \end{array}$

Recall the approximate decomposition of real income growth going from period t–1 to t given by (28) above, $\gamma^t \approx \tau^t \alpha^t \beta^t$. Diewert and Morrison (1986; 663) showed that³⁶ if g^{t-1} and g^t are defined by (33)-(41) above and there is competitive profit maximizing behavior on the part of all market sector producers for all periods t, then (28) holds as an exact equality; i.e., we have

(42)
$$\gamma^{t} = \tau^{t} \alpha^{t} \beta^{t}$$
; $t = 1, 2, ...$

In addition, Diewert and Morrison (1986; 663-665) showed that τ^t , α^t and β^t could be calculated using empirically observable price and quantity data for periods t–1 and t as follows:

$$\begin{array}{ll} (43) \ \ln \alpha^{t} &= \sum_{m=1}^{M} (1/2) [(p_{m}^{t-1} y_{m}^{t-1} / p^{t-1} \cdot y^{t-1}) + (p_{m}^{t} y_{m}^{t} / p^{t} \cdot y^{t})] \ \ln (p_{m}^{t} / p_{m}^{t-1}) \\ &= \ln P_{T} (p^{t-1}, p^{t}, y^{t-1}, y^{t}); \\ (44) \ \ln \beta^{t} &= \sum_{n=1}^{N} (1/2) [(w_{n}^{t-1} x_{n}^{t-1} / w^{t-1} \cdot x^{t-1}) + (w_{n}^{t} x_{n}^{t} / w^{t} \cdot x^{t})] \ \ln (x_{n}^{t} / x_{n}^{t-1}) \\ &= \ln Q_{T} (w^{t-1}, w^{t}, x^{t-1}, x^{t}); \\ (45) \qquad \tau^{t} &= \gamma^{t} / \alpha^{t} \beta^{t} \end{array}$$

where $P_T(p^{t-1}, p^t, y^{t-1}, y^t)$ is the Törnqvist (1936) and Törnqvist and Törnqvist (1937) output price index and $Q_T(w^{t-1}, w^t, x^{t-1}, x^t)$ is the Törnqvist input quantity index.

Since equations (42) now hold as exact identities under our present assumptions, equations (32), the cumulated counterparts to equations (28), will also hold as exact decompositions; i.e., under our present assumptions, we have

(46)
$$\rho^t / \rho^0 = T^t A^t B^t$$
; $t = 1, 2, ...$

We will implement the real income decompositions (42) and (46) in section 6 using our Japanese data base for the years 1955-2003.

5. The Translog GDP Function Approach and Specific Price and Quantity Contribution Factors

³⁶ Diewert and Morrison established their proof using the nominal GDP function $g^t(P,x)$. However, it is easy to rework their proof using the deflated GDP function $g^t(p,x)$ using the fact that $g^t(p,x) = g^t(P/P_C,x) =$ $g^t(P,x)/P_C$ using the linear homogeneity property of $g^t(P,x)$ in P.

For some purposes, it is convenient to decompose the aggregate period t contribution factor due to changes in all deflated output prices α^t into separate effects for each change in each output price. Similarly, it can sometimes be useful to decompose the aggregate period t contribution factor due to changes in all market sector primary input quantities β^t into separate effects for each change in each input quantity. In this section, we indicate how this can be done, making the same assumptions on the technology that were made in the previous section.

We first model the effects of a change in a single (deflated) output price, say p_m , going from period t–1 to t. Counterparts to the theoretical Laspeyres and Paasche type price indexes defined by (14) and (15) above for changes in all (deflated) output prices are the following *Laspeyres type measure* α_{Lm}^{t} that chooses the period t–1 reference technology and holds constant other output prices at their period t–1 levels and holds inputs constant at their period t–1 levels x^{t-1} and a *Paasche type measure* α_{Pm}^{t} that chooses the period t reference technology and reference input vector x^{t} and holds constant other output prices at their period t levels:

$$(47) \ \alpha_{Lm}{}^{t} \equiv g^{t-1}(p_{1}{}^{t-1},...,p_{m-1}{}^{t-1},p_{m}{}^{t},p_{m+1}{}^{t-1},...,p_{M}{}^{t-1},x^{t-1})/g^{t-1}(p^{t-1},x^{t-1}) ; \qquad m = 1,...,M; \\ (48) \ \alpha_{Pm}{}^{t} \equiv g^{t}(p^{t},x^{t})/g^{t}(p_{1}{}^{t},...,p_{m-1}{}^{t},p_{m}{}^{t-1},p_{m+1}{}^{t},...,p_{M}{}^{t},x^{t}) ; \qquad m = 1,...,M; \\ t = 1,2,...; \\ m = 1,...,M; \\ t = 1,2,...; \\ t = 1,2,...; \\ n = 1,...,M; \\ t = 1,..$$

Since both measures of real output price change are equally valid, it is natural to average them to obtain an *overall measure of the effects on real income of the change in the real price of output m*:³⁷

(49)
$$\alpha_{\rm m}^{\rm t} = [\alpha_{\rm Lm}^{\rm t} \alpha_{\rm Pm}^{\rm t}]^{1/2};$$
 m = 1,...,M; t = 1,2,

Under the assumption that the deflated GDP functions $g^{t}(p,x)$ have the translog functional forms as defined by (33)-(41) in the previous section, the arguments of Diewert and Morrison (1986; 666) can be adapted to give us the following result:

(50)
$$\ln\alpha_{m}^{t} = (1/2)[(p_{m}^{t-1}y_{m}^{t-1}/p^{t-1}\cdot y^{t-1}) + (p_{m}^{t}y_{m}^{t}/p^{t}\cdot y^{t})] \ln(p_{m}^{t}/p_{m}^{t-1});$$

 $m = 1,...,M; t = 1,2,....$

Note that $\ln \alpha_m^{t}$ is equal to the mth term in the summation of the terms on the right hand side of (43). This observation means that we have the following exact decomposition of the period t aggregate real output price contribution factor α^{t} into a product of separate price contribution factors; i.e., we have under present assumptions:

(51)
$$\alpha^{t} = \alpha_{1}^{t} \alpha_{2}^{t} \dots \alpha_{M}^{t};$$
 $t = 1, 2, \dots$

 $^{^{37}}$ The indexes defined by (47)-(49) were defined by Diewert and Morrison (1986; 666) in the nominal GDP function context.

The above decomposition is useful for analyzing how real changes in the price of exports (i.e., a change in the price of exports relative to the price of domestic consumption) and in the price of imports impact on the real income generated by the market sector. In our empirical work which follows later, we let M equal three. The three net outputs are:

- Domestic sales (C+I+G);
- Exports (X) and
- Imports (M).

Since commodities 1 and 2 are outputs, y_1 and y_2 will be positive but since commodity 3 is an input into the market sector, y_3 will be negative. Hence an increase in the real price of exports will *increase* real income but an increase in the real price of imports will *decrease* the real income generated by the market sector, as is evident by looking at the contribution terms defined by (50) for m = 2 (where $y_m^t > 0$) and for m = 3 (where $y_m^t < 0$).

As mentioned above, it is also useful to have a decomposition of the aggregate contribution of input growth to the growth of real income into separate contributions for each important class of primary input that is used by the market sector. We now model the effects of a change in a single input quantity, say x_n , going from period t–1 to t. Counterparts to the theoretical Laspeyres and Paasche type quantity indexes defined by (18) and (19) above for changes in input n are the following *Laspeyres type measure* β_{Ln}^{t} that chooses the period t–1 reference technology and holds constant other input quantities at their period t–1 levels and holds real output prices at their period t–1 levels p^{t-1} and a *Paasche type measure* β_{Pn}^{t} that chooses the period t reference technology and reference real output price vector p^{t} and holds constant other input quantities at their period t levels:

$$(52) \ \beta_{Ln}{}^{t} \equiv g^{t-1}(p^{t-1}, x_{1}{}^{t-1}, ..., x_{n-1}{}^{t-1}, x_{n}{}^{t}, x_{n+1}{}^{t-1}, ..., x_{N}{}^{t-1})/g^{t-1}(p^{t-1}, x^{t-1}); \qquad n = 1, ..., N; \\ (53) \ \beta_{Pn}{}^{t} \equiv g^{t}(p^{t}, x^{t})/g^{t}(p^{t}, x_{1}{}^{t}, ..., x_{n-1}{}^{t}, x_{n}{}^{t-1}, x_{n+1}{}^{t}, ..., p_{N}{}^{t}); \qquad m = 1, ..., M; \\ (53) \ \beta_{Pn}{}^{t} \equiv g^{t}(p^{t}, x^{t})/g^{t}(p^{t}, x_{1}{}^{t}, ..., x_{n-1}{}^{t}, x_{n}{}^{t-1}, x_{n+1}{}^{t}, ..., p_{N}{}^{t}); \qquad m = 1, ..., M; \\ t = 1, 2, ...; \\ t = 1, 2, ...;$$

Since both measures of input change are equally valid, as usual, we average them to obtain an overall measure of the effects on real income of the change in the quantity of input n.³⁸

(54)
$$\beta_n^t = [\beta_{Pn}^t \beta_{Pn}^t]^{1/2}$$
; $n = 1,...,N$; $t = 1,2,...$

Under the assumption that the deflated GDP functions $g^t(p,x)$ have the translog functional forms as defined by (33)-(41) in the previous section, the arguments of Diewert and Morrison (1986; 667) can be adapted to give us the following result:

(55)
$$\ln\beta_n^t = (1/2)[(w_n^{t-1}x_n^{t-1}/w^{t-1}\cdot x^{t-1}) + (w_n^t x_n^t/w^t\cdot x^t)] \ln(x_n^t/x_n^{t-1});$$

 $^{^{38}}$ The indexes defined by (52)-(54) were defined by Diewert and Morrison (1986; 667) in the nominal GDP function context.

n = 1,...,N; t = 1,2,...

Note that $\ln\beta_n^t$ is equal to the nth term in the summation of the terms on the right hand side of (44). This observation means that we have the following exact decomposition of the period t aggregate input growth contribution factor β^t into a product of separate input quantity contribution factors; i.e., we have under present assumptions:

(56)
$$\beta^{t} = \beta_{1}^{t} \beta_{2}^{t} \dots \beta_{N}^{t};$$
 $t = 1, 2,$

It should be noted that there is another approach to contribution analysis that was suggested in Diewert and Morrison (1986; 674-676). This approach is completely nonparametric but it relies on taking averages of first order approximations to various theoretical indexes and so it does not lead to exact decompositions as is the case for the translog approach outlined above.³⁹

In the following section, we implement the translog approach using our Japanese data base.

6. The Deflated GDP Translog Approach for Japan

The basic price and quantity data for market sector net output (C + G + I + X - M) are listed in Tables 4 (prices) and 5 (quantities) below. The 11 investment aggregates were aggregated using a chained Törnqvist price index and the first two consumption aggregates were aggregated into C again using a chained Törnqvist price index. Tables 4 and 5 also list the prices (W_L and W_K) and quantities (x_L and x_K) for primary inputs. The 13 capital services were aggregated into aggregate capital services using a chained Törnqvist price index.

Year	P_{C}	P_{G}	P_I	P_X	P_M	W_L	W_K
1955.	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1956.	1.00190	1.09315	1.08966	1.04421	1.05901	1.06015	1.17140
1957.	1.02873	1.23234	1.11530	1.06042	1.09455	1.12002	1.43223
1958.	1.01383	1.26337	1.08517	1.00108	0.91553	1.19115	1.13384
1959.	1.02195	1.22298	1.09671	1.03344	1.00774	1.28144	1.30374
1960.	1.04856	1.36779	1.12536	1.06001	1.06312	1.41223	1.59503
1961.	1.09325	1.52944	1.18842	1.05194	1.10701	1.62223	1.99404
1962.	1.17064	1.64530	1.22206	1.03432	1.07171	1.87672	1.79187
1963.	1.25381	1.73978	1.23440	1.05967	1.09622	2.14445	2.04410
1964.	1.29698	1.95976	1.26520	1.07653	1.12296	2.43452	2.30762
1965.	1.38477	2.13180	1.29157	1.07065	1.11517	2.74574	2.10611

Table 4: Market Sector Output and Input Prices for Japan 1955-2003	Table 4	: Market	Sector Out	tput and In	put Prices for	r Japan 1955-2003
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³⁹ This nonparametric approach was implemented in Diewert and Lawrence (2005b) and compared with the translog approach using Australian data. The results were virtually identical. We did a similar exercise using our Japanese data and obtained the same result: the two approaches give virtually the same answers. This is consistent with the results found in Diewert and Morrison (1986) and Morrison and Diewert (1990).

1066	1 45260	2 26020	1 24057	1 06005	1 1 40 2 2	2 04664	2 22400
1966. 1967	1.45269	2.26038	1.34057 1.39509	1.06905	1.14023	3.04664	2.33408
1967.	1.49762	2.40298		1.07055	1.10140	3.42006	2.61490
1968.	1.57607	2.58273	1.43578	1.07173	1.10662	3.92083	3.00409
1969.	1.64561	2.84017	1.50321	1.08749	1.13111	4.56194	3.23524
1970.	1.76977	3.12254	1.57495	1.11814	1.13727	5.34943	3.38966
1971.	1.89247	3.40542	1.60231	1.14929	1.10554	6.20088	2.90747
1972.	1.99647	3.72525	1.67822	1.14170	1.11476	7.10272	2.78679
1973.	2.23464	4.25365	1.96572	1.25249	1.36931	8.58472	3.13160
1974.	2.80206	5.78630	2.35593	1.64495	2.04418	11.03768	3.68297
1975.	3.15037	5.63242	2.41893	1.72635	1.78354	13.09550	2.63197
1976.	3.43844	5.76826	2.54487	1.76102	1.80385	14.23948	3.09905
1977.	3.68333	5.92539	2.64465	1.69598	1.68457	15.53244	3.05002
1978.	3.81976	5.91166	2.73713	1.58930	1.42793	16.62487	3.05990
1979.	3.93302	6.05542	2.93269	1.71824	2.02039	17.64361	3.62211
1980.	4.26628	6.19212	3.15185	1.88495	2.35311	18.61965	4.72252
1981.	4.46801	6.31713	3.18702	1.94589	2.36895	19.75436	4.69731
1982.	4.57835	6.53242	3.20514	2.01615	2.43043	20.70442	4.76947
1983.	4.66641	6.97581	3.19698	1.95276	2.29927	21.25655	4.43906
1984.	4.78201	6.73432	3.21541	1.98380	2.23942	22.03585	4.87588
1985.	4.85840	6.73374	3.16742	1.92525	2.10905	22.54127	5.59623
1986.	4.87985	6.72082	3.11760	1.69025	1.47229	23.17142	5.20419
1987.	4.86923	6.08128	3.11969	1.61953	1.34625	23.19048	5.49981
1988.	4.87836	5.58044	3.15686	1.59133	1.30063	23.68318	5.77112
1989.	4.95370	5.39242	3.27383	1.64384	1.35828	24.76920	6.25115
1990.	5.06249	5.07531	3.36684	1.67158	1.45497	26.20684	6.56146
1991.	5.18191	4.95820	3.42432	1.63227	1.37810	27.83529	6.08265
1992.	5.24955	5.02302	3.44849	1.59096	1.30988	28.74286	5.78199
1993.	5.28757	4.98001	3.44956	1.48418	1.20267	29.40147	5.18721
1994.	5.28737	4.90085	3.42826	1.43813	1.14816	29.86487	5.08605
1995.	5.23635	4.83749	3.39526	1.40880	1.12748	30.04179	4.92699
1996.	5.19400	4.74903	3.35480	1.45407	1.21544	30.31299	5.10866
1997.	5.20041	4.70165	3.36631	1.47750	1.28242	31.23549	5.08518
1998.	5.10753	4.65071	3.31394	1.48593	1.23523	31.06780	4.49354
1999.	5.04513	4.49804	3.24440	1.35901	1.13217	30.69501	4.26091
2000.	4.98695	4.40474	3.20669	1.30905	1.15802	30.29806	4.40641
2001.	4.89207	4.31181	3.12045	1.32538	1.19583	30.47848	3.96985
2002.	4.80561	4.25010	3.03741	1.31236	1.17625	29.98659	3.79841
2002.	4.73080	4.19481	2.97040	1.26867	1.16837	29.92703	3.78084
							2

Table 5: Market Sector Outputs and Inputs for Japan 1955-2003

Year	Ус	УG	Уı	Уx	Ум	\mathbf{x}_{L}	X _K
1955.	4552.6	405.7	2004.0	902.1	-899.2	5048.4	1916.8
1956.	4932.8	380.1	2498.0	1062.0	-1141.4	5332.9	1985.8
1957.	5242.4	363.6	3335.2	1186.1	-1414.8	5596.9	2095.6
1958.	5623.3	382.9	2822.9	1249.6	-1260.2	5692.7	2261.8

1959.	6034.9	431.9	3523.4	1415.9	-1393.0	5839.6	2405.5
1960.	6678.7	446.9	4485.4	1596.7	-1629.6	6041.9	2563.8
1961.	7274.0	463.4	6171.1	1680.0	-2003.4	6135.8	2803.7
1962.	7971.0	505.0	5878.6	1974.6	-2018.2	6187.9	3132.3
1963.	8796.7	570.4	7062.9	2115.8	-2403.0	6222.5	3426.5
1964.	9840.4	575.0	8251.0	2578.6	-2710.8	6353.4	3723.8
1965.	10378.1	603.7	8072.4	3199.6	-2860.9	6458.2	4077.2
1966.	11206.4	639.8	9439.1	3744.0	-3202.9	6672.2	4456.5
1967.	12342.9	664.4	11606.3	3996.5	-4027.3	6831.3	4876.9
1968.	13463.0	713.3	13688.9	4951.4	-4506.2	6917.2	5297.4
1969.	14792.4	753.7	15806.9	5988.7	-5149.0	6987.4	5890.4
1970.	15822.9	824.6	18191.1	7033.7	-6402.4	7092.7	6451.7
1971.	16606.5	887.4	17846.7	8186.3	-6789.7	7145.5	7098.6
1972.	18124.4	949.0	18530.2	8528.9	-7051.0	7218.7	7687.2
1973.	19718.2	1031.3	21232.8	8982.6	-8400.6	7426.5	8360.2
1974.	19433.8	997.1	23148.5	11068.9	-9496.1	7270.4	9043.9
1975.	20231.9	1236.7	19802.9	10964.3	-10683.6	7098.4	9696.9
1976.	20751.4	1311.8	21819.2	12787.5	-11884.1	7307.3	10156.6
1977.	21509.8	1402.7	21979.1	14288.2	-12735.6	7464.2	10658.2
1978.	22555.2	1514.5	22362.3	14260.5	-13494.8	7559.4	11123.7
1979.	24097.4	1620.8	25537.4	14869.0	-13701.4	7657.8	11661.4
1980.	24272.4	3079.5	26117.4	17360.5	-14976.9	7762.8	12256.2
1981.	24589.6	3274.6	26235.2	19396.4	-15263.6	7876.0	12808.2
1982.	25810.6	3464.0	26988.1	19376.9	-15489.3	7965.5	13376.5
1983.	26593.3	3676.1	25436.8	19966.1	-14992.8	8180.1	13898.4
1984.	27203.0	3803.4	27308.4	22553.6	-16574.8	8223.6	14339.1
1985.	28317.3	3802.5	30781.7	23887.3	-16735.2	8318.5	14985.9
1986.	29207.5	3998.9	30162.5	22415.0	-16795.8	8418.0	15668.2
1987.	30389.6	4186.8	33385.3	22207.6	-18840.3	8542.5	16379.6
1988.	31819.7	4434.0	37010.9	23353.5	-21819.7	8727.5	17136.8
1989.	33267.6	4690.2	41092.9	25522.9	-25445.8	8871.8	17959.3
1990.	34783.4	5045.8	45022.2	27208.2	-27402.8	8956.5	18924.3
1991.	35646.7	5433.3	44370.2	28379.4	-27170.8	9033.3	19897.5
1992.	36387.6	5730.0	42894.0	29509.6	-26809.5	8982.6	20990.2
1993.	36641.7	6074.3	40375.6	29534.1	-26507.5	8905.0	21863.0
1994.	37590.2	6407.9	40435.3	30612.6	-28578.5	8906.7	22413.2
1995.	38227.9	6885.5	41142.3	31958.5	-32376.7	8969.5	22778.9
1996.	39260.1	7304.7	44229.4	33869.9	-36934.7	8986.3	23282.8
1997.	39574.5	7452.5	45398.4	37699.6	-37548.0	8950.7	23919.8
1998.	39426.6	7639.6	41700.2	36808.2	-35263.4	8856.8	24717.5
1999.	39212.3	8243.1	40973.8	37423.3	-36249.5	8785.1	25347.6
2000.	39472.0	8943.8	43353.1	42009.9	-39712.0	8918.3	25884.7
2001.	40104.9	9538.7	41865.9	39439.5	-39890.3	8763.2	26561.9
2002.	40210.9	10037.9	39187.9	42329.6	-40342.4	8659.3	27212.9
2003.	40502.6	10165.4	39988.0	45903.7	-41915.9	8622.9	27569.8

We now form domestic absorption y_D using a chained Törnqvist price index; i.e., an aggregate of C + I + G, which are outputs 1,2,3 and 6-16 in our data base. This variable is listed in column 6 of Table 6 below. We deflate the corresponding price, P_D , by the price of our consumption aggregate, P_C , to form the real price $p_D = P_D/P_C$. The nominal prices of exports, imports, labour and capital services are also deflated by P_C and the resulting real prices are listed in the first 5 columns of Table 6. Finally, GDP deflated by P_C is listed in the last column of Table 6. This is our target real income variable, ρ^t .

Table 6: Real Prices for Outputs and Inputs, Domestic Absorption and Real Income Produced by the Market Sector of Japan 1955-2003

Year	$p_{\rm D}$	p_X	p_{M}	\mathbf{W}_{L}	WK	УD	ρ^t
1955.	1.00000	1.00000	1.00000	1.00000	1.00000	6962.3	6965.2
1956.	1.03146	1.04223	1.05699	1.05813	1.16917	7818.4	7964.7
1957.	1.03498	1.03080	1.06398	1.08874	1.39223	8979.7	9011.1
1958.	1.03233	0.98742	0.90304	1.17490	1.11837	8836.3	9217.9
1959.	1.03112	1.01124	0.98610	1.25391	1.27573	10021.1	10391.2
1960.	1.03551	1.01092	1.01389	1.34683	1.52116	11661.4	12037.4
1961.	1.04449	0.96221	1.01258	1.48386	1.82396	14007.4	14218.5
1962.	1.02645	0.88355	0.91549	1.60315	1.53067	14435.9	14714.6
1963.	1.00105	0.84516	0.87431	1.71035	1.63032	16524.4	16228.9
1964.	1.00115	0.83003	0.86583	1.87707	1.77923	18736.6	18551.3
1965.	0.98359	0.77316	0.80531	1.98281	1.52091	19150.8	19006.4
1966.	0.97983	0.73591	0.78491	2.09725	1.60673	21342.8	21153.7
1967.	0.98520	0.71484	0.73544	2.28367	1.74604	24584.6	24115.7
1968.	0.97618	0.68000	0.70214	2.48773	1.90606	27763.7	27305.4
1969.	0.97959	0.66084	0.68735	2.77219	1.96598	31168.5	30950.8
1970.	0.96835	0.63180	0.64261	3.02267	1.91531	34560.3	33796.0
1971.	0.94686	0.60730	0.58418	3.27660	1.53634	35183.3	34318.8
1972.	0.94551	0.57186	0.55837	3.55764	1.39586	37515.6	36411.8
1973.	0.96606	0.56049	0.61277	3.84166	1.40139	41776.9	40245.9
1974.	0.94957	0.58705	0.72953	3.93912	1.31438	43131.0	40526.1
1975.	0.90539	0.54798	0.56614	4.15682	0.83545	41582.5	37608.0
1976.	0.88864	0.51216	0.52461	4.14127	0.90130	44001.0	39415.6
1977.	0.87568	0.46045	0.45735	4.21695	0.82806	45162.1	40301.8
1978.	0.87307	0.41607	0.37383	4.35234	0.80107	46872.9	41812.0
1979.	0.88702	0.43688	0.51370	4.48602	0.92095	51447.7	45092.6
1980.	0.87982	0.44183	0.55156	4.36438	1.10694	54598.8	47446.7
1981.	0.86552	0.43552	0.53020	4.42129	1.05132	55380.3	48287.5
1982.	0.86007	0.44037	0.53085	4.52224	1.04174	57723.7	49956.8
1983.	0.85749	0.41847	0.49273	4.55523	0.95128	57744.6	50483.5
1984.	0.84631	0.41485	0.46830	4.60807	1.01963	60168.4	52515.5
1985.	0.83538	0.39627	0.43410	4.63965	1.15187	64229.3	55856.7
1986.	0.82868	0.34637	0.30171	4.74839	1.06647	65145.6	56681.5
1987.	0.82172	0.33260	0.27648	4.76266	1.12950	69376.7	59185.7
1988.	0.81856	0.32620	0.26661	4.85474	1.18300	74328.4	62642.7

1991.0.814340.314990.265945.371631.1738286162.971879.61992.0.812360.303070.249525.475301.1014386227.072301.6	1989.	0.82226	0.33184	0.27419	5.00014	1.26192	79696.3	67023.3
1992. 0.81236 0.30307 0.24952 5.47530 1.10143 86227.0 72301.6	1990.	0.81935	0.33019	0.28740	5.17667	1.29609	85170.3	70892.5
	1991.	0.81434	0.31499	0.26594	5.37163	1.17382	86162.9	71879.6
1993 0.80913 0.28069 0.22745 5.56049 0.98102 84910.3 70964	1992.	0.81236	0.30307	0.24952	5.47530	1.10143	86227.0	72301.6
1775. 0.00715 0.20007 0.22745 5.50047 0.70102 04710.5 70704.1	1993.	0.80913	0.28069	0.22745	5.56049	0.98102	84910.3	70964.1
1994. 0.80615 0.27199 0.21715 5.64835 0.96192 86518.8 71867.9	1994.	0.80615	0.27199	0.21715	5.64835	0.96192	86518.8	71867.9
1995. 0.80593 0.26904 0.21532 5.73716 0.94092 88427.0 72892.0	1995.	0.80593	0.26904	0.21532	5.73716	0.94092	88427.0	72892.6
1996. 0.80400 0.27995 0.23401 5.83615 0.98357 92669.9 75345.7	1996.	0.80400	0.27995	0.23401	5.83615	0.98357	92669.9	75345.7
1997. 0.80387 0.28411 0.24660 6.00635 0.97784 94168.4 77150.9	1997.	0.80387	0.28411	0.24660	6.00635	0.97784	94168.4	77150.9
1998. 0.80511 0.29093 0.24185 6.08274 0.87979 91216.7 75619.7	1998.	0.80511	0.29093	0.24185	6.08274	0.87979	91216.7	75619.7
1999. 0.80084 0.26937 0.22441 6.08408 0.84456 91042.5 74856.8	1999.	0.80084	0.26937	0.22441	6.08408	0.84456	91042.5	74856.8
2000. 0.80005 0.26250 0.23221 6.07547 0.88359 94055.1 77054.3	2000.	0.80005	0.26250	0.23221	6.07547	0.88359	94055.1	77054.3
2001. 0.79753 0.27092 0.24444 6.23018 0.81149 94312.5 76151.0	2001.	0.79753	0.27092	0.24444	6.23018	0.81149	94312.5	76151.0
2002. 0.79532 0.27309 0.24477 6.23991 0.79041 92865.3 75542.7	2002.	0.79532	0.27309	0.24477	6.23991	0.79041	92865.3	75542.7
2003. 0.79380 0.26817 0.24697 6.32600 0.79920 94008.9 76582.2	2003.	0.79380	0.26817	0.24697	6.32600	0.79920	94008.9	76582.2

The chain link information on period by period changes in real income that corresponds to (42) (generalized to include separate contribution factors for changes in real domestic, export and import prices, α_D^t , α_X^t and α_M^t respectively and separate contribution factors for growth in labour and capital input, β_L^t and β_K^t respectively) is given in Table 7 below. The last column in Table 5, α_T^t , is the contribution factor for *real changes in the terms of trade* and is simply the product of the export and import price factors, α_X^t and α_M^t .

Table 7: The Decomposition of Market Sector (Gross) Real Income Growth intoTranslog Productivity, Real Output Price Change and Input Quantity ContributionFactors

Year t	γ^t	$\boldsymbol{\tau}^t$	${\alpha_{\mathrm{D}}}^{\mathrm{t}}$	α_X^{t}	α_{M}^{t}	${\beta_L}^t$	β_{K}^{t}	α_{T}^{t}
1956.	1.14350	1.05743	1.03165	1.00557	0.99225	1.04007	1.01007	0.99778
1957.	1.13139	1.07520	1.00349	0.99849	0.99895	1.03402	1.01669	0.99744
1958.	1.02294	0.97290	0.99741	0.99422	1.02411	1.01197	1.02309	1.01819
1959.	1.12729	1.09770	0.99884	1.00324	0.98882	1.01839	1.01770	0.99202
1960.	1.15843	1.10889	1.00425	0.99996	0.99626	1.02379	1.01993	0.99622
1961.	1.18119	1.13085	1.00881	0.99390	1.00018	1.01020	1.03105	0.99408
1962.	1.03489	1.00490	0.98241	0.99014	1.01361	1.00557	1.03871	1.00362
1963.	1.10291	1.09282	0.97494	0.99493	1.00589	1.00371	1.03053	1.00079
1964.	1.14310	1.09607	1.00010	0.99796	1.00125	1.01361	1.02961	0.99921
1965.	1.02453	0.99994	0.98244	0.99132	1.00902	1.01083	1.03146	1.00026
1966.	1.11298	1.06486	0.99622	0.99359	1.00308	1.02200	1.03001	0.99666
1967.	1.14002	1.07763	1.00545	0.99639	1.00790	1.01554	1.03166	1.00426
1968.	1.13227	1.10080	0.99086	0.99398	1.00555	1.00801	1.03035	0.99949
1969.	1.13351	1.08025	1.00346	0.99642	1.00245	1.00636	1.04025	0.99886
1970.	1.09193	1.05564	0.98865	0.99419	1.00798	1.00947	1.03424	1.00212
1971.	1.01547	0.99399	0.97824	0.99455	1.01138	1.00489	1.03319	1.00586
1972.	1.06099	1.03298	0.99862	0.99165	1.00507	1.00710	1.02469	0.99668
1973.	1.10530	1.04895	1.02147	0.99740	0.98909	1.02027	1.02489	0.98652

1974.	1.00696	1.03644	0.98282	1.00663	0.97428	0.98508	1.02324	0.98073
1975.	0.92800	0.94391	0.95321	0.98904	1.04296	0.98231	1.01789	1.03153
1976.	1.04806	1.03204	0.98157	0.98904	1.01222	1.02277	1.01042	1.00113
1977.	1.02248	1.00618	0.98561	0.98262	1.02099	1.01659	1.01094	1.00324
1978.	1.03747	1.00931	0.99708	0.98466	1.02710	1.00998	1.00928	1.01134
1979.	1.07846	1.07907	1.01590	1.00700	0.95697	1.01007	1.01071	0.96367
1980.	1.05221	1.04715	0.99179	1.00172	0.98833	1.01010	1.01312	0.99003
1981.	1.01772	1.00686	0.98371	0.99758	1.00677	1.01044	1.01252	1.00434
1982.	1.03457	1.01846	0.99374	1.00192	0.99980	1.00818	1.01218	1.00171
1983.	1.01054	0.98088	0.99704	0.99146	1.01165	1.01959	1.01040	1.00302
1984.	1.04025	1.03456	0.98728	0.99851	1.00751	1.00388	1.00847	1.00600
1985.	1.06362	1.05190	0.98753	0.99207	1.01059	1.00814	1.01304	1.00258
1986.	1.01477	0.98148	0.99233	0.97959	1.04073	1.00833	1.01353	1.01949
1987.	1.04418	1.02549	0.99196	0.99471	1.00778	1.01028	1.01358	1.00244
1988.	1.05841	1.03109	0.99628	0.99761	1.00329	1.01472	1.01448	1.00089
1989.	1.06993	1.03806	1.00440	1.00213	0.99724	1.01103	1.01563	0.99936
1990.	1.05773	1.04200	0.99653	0.99937	0.99495	1.00627	1.01806	0.99432
1991.	1.01392	0.99506	0.99401	0.99410	1.00824	1.00569	1.01696	1.00230
1992.	1.00587	0.99346	0.99764	0.99522	1.00617	0.99619	1.01738	1.00137
1993.	0.98150	0.97972	0.99614	0.99082	1.00825	0.99404	1.01275	0.99900
1994.	1.01274	1.00834	0.99644	0.99634	1.00398	1.00013	1.00751	1.00031
1995.	1.01426	1.00520	0.99973	0.99873	1.00077	1.00495	1.00482	0.99950
1996.	1.03365	1.03201	0.99765	1.00486	0.99128	1.00131	1.00656	0.99610
1997.	1.02396	1.02286	0.99984	1.00195	0.99387	0.99724	1.00823	0.99581
1998.	0.98015	0.97103	1.00150	1.00333	1.00227	0.99260	1.00974	1.00561
1999.	0.98991	0.99596	0.99485	0.98942	1.00832	0.99422	1.00725	0.99765
2000.	1.02936	1.02084	0.99903	0.99642	0.99611	1.01072	1.00613	0.99254
2001.	0.98828	0.99816	0.99691	1.00449	0.99366	0.98762	1.00752	0.99812
2002.	0.99201	0.99539	0.99728	1.00117	0.99983	0.99150	1.00690	1.00100
2003.	1.01376	1.01903	0.99814	0.99715	0.99881	0.99700	1.00374	0.99596
Averag	ge 1.0527	1.0311	0.99532	0.99620	1.0037	1.0078	1.0175	0.99982

From the average contribution factors listed in the last row of Table 7, it can be seen that real market sector income in Japan grew at a very high average annual rate of 5.27% per year. Productivity growth accounted for 3.11% of this growth,⁴⁰ labour input growth for 0.78% and capital input 1.75% of this growth on average while the real price effects were -0.47% per year (due to domestic prices falling faster than consumption prices), -0.38% per year due to export prices falling faster than domestic consumption prices. Thus the effects of changes in the terms of trade on living standards was negligible for Japan over the entire sample period.

⁴⁰ Compare this average rate of TFP growth with the corresponding Fisher TFP growth rate of 3.12% per year; see section 2 above.

The annual change information in the previous table can be converted into cumulative changes using equations (46) (with obvious extensions to multiple inputs and outputs).⁴¹ Table 6 gives this levels growth information.

Table 8: The Decomposition of Market Sector (Gross) Real Income Levels into Productivity, Real Output Price Change and Input Quantity Contribution Factors

Year t	$ ho^t/ ho^0$	T^t	${A_D}^t$	A_X^{t}	A_M^t	${\rm B_L}^t$	${B_K}^t$	A_X^{t}
1955.	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1956.	1.14350	1.05743	1.03165	1.00557	0.99225	1.04007	1.01007	0.99778
1957.	1.29374	1.13696	1.03525	1.00405	0.99121	1.07545	1.02693	0.99522
1958.	1.32342	1.10614	1.03257	0.99825	1.01511	1.08832	1.05065	1.01333
1959.	1.49187	1.21421	1.03137	1.00148	1.00376	1.10833	1.06925	1.00525
1960.	1.72822	1.34643	1.03576	1.00144	1.00001	1.13470	1.09056	1.00145
1961.	2.04137	1.52260	1.04489	0.99533	1.00019	1.14628	1.12442	0.99552
1962.	2.11259	1.53006	1.02651	0.98552	1.01380	1.15266	1.16794	0.99912
1963.	2.33000	1.67208	1.00078	0.98053	1.01977	1.15695	1.20361	0.99991
1964.	2.66343	1.83272	1.00089	0.97853	1.02104	1.17269	1.23925	0.99912
1965.	2.72877	1.83262	0.98331	0.97004	1.03025	1.18539	1.27824	0.99938
1966.	3.03706	1.95147	0.97960	0.96383	1.03342	1.21147	1.31660	0.99604
1967.	3.46231	2.10298	0.98494	0.96035	1.04158	1.23029	1.35828	1.00029
1968.	3.92026	2.31495	0.97594	0.95457	1.04736	1.24015	1.39951	0.99978
1969.	4.44363	2.50073	0.97931	0.95115	1.04993	1.24804	1.45584	0.99864
1970.	4.85212	2.63986	0.96820	0.94562	1.05830	1.25986	1.50570	1.00075
1971.	4.92718	2.62399	0.94713	0.94047	1.07034	1.26602	1.55567	1.00662
1972.	5.22768	2.71053	0.94582	0.93262	1.07577	1.27501	1.59408	1.00328
1973.	5.77815	2.84322	0.96613	0.93019	1.06403	1.30085	1.63375	0.98975
1974.	5.81837	2.94682	0.94953	0.93636	1.03666	1.28144	1.67172	0.97069
1975.	5.39942	2.78152	0.90510	0.92610	1.08119	1.25876	1.70163	1.00129
1976.	5.65893	2.87065	0.88842	0.91595	1.09441	1.28742	1.71936	1.00242
1977.	5.78617	2.88840	0.87563	0.90003	1.11737	1.30878	1.73816	1.00567
1978.	6.00298	2.91528	0.87307	0.88622	1.14765	1.32185	1.75429	1.01708
1979.	6.47399	3.14577	0.88696	0.89243	1.09827	1.33515	1.77308	0.98013
1980.	6.81197	3.29409	0.87967	0.89397	1.08545	1.34864	1.79634	0.97036
1981.	6.93269	3.31669	0.86534	0.89180	1.09280	1.36272	1.81883	0.97457
1982.	7.17235	3.37792	0.85993	0.89351	1.09258	1.37387	1.84099	0.97623
1983.	7.24796	3.31333	0.85738	0.88588	1.10531	1.40078	1.86014	0.97918
1984.	7.53970	3.42784	0.84648	0.88456	1.11361	1.40621	1.87589	0.98505
1985.	8.01939	3.60574	0.83592	0.87755	1.12540	1.41765	1.90036	0.98759
1986.	8.13781	3.53898	0.82951	0.85963	1.17124	1.42947	1.92608	1.00684
1987.	8.49735	3.62917	0.82284	0.85508	1.18035	1.44416	1.95222	1.00929
1988.	8.99367	3.74201	0.81977	0.85304	1.18423	1.46541	1.98050	1.01020
1989.	9.62260	3.88443	0.82338	0.85485	1.18097	1.48158	2.01145	1.00955
1990.	10.17810	4.04756	0.82053	0.85431	1.17500	1.49087	2.04779	1.00382

⁴¹ The last column in Table 8 denoted by A_T^{t} is the cumulative effect of changes in the real prices of exports and imports and is equal to the product of the entries denoted by A_X^{t} and A_M^{t} .

1991. 10.31983	4.02755	0.81561	0.84927	1.18469	1.49935	2.08253	1.00613
1992. 10.38040	4.00120	0.81368	0.84522	1.19200	1.49364	2.11873	1.00750
1993. 10.18839	3.92003	0.81054	0.83746	1.20184	1.48474	2.14574	1.00649
1994. 10.31814	3.95271	0.80766	0.83440	1.20662	1.48494	2.16186	1.00680
1995. 10.46526	3.97327	0.80743	0.83333	1.20755	1.49229	2.17228	1.00629
1996. 10.81744	4.10045	0.80554	0.83738	1.19703	1.49425	2.18654	1.00237
1997. 11.07663	4.19420	0.80541	0.83902	1.18969	1.49013	2.20453	0.99817
1998. 10.85678	4.07270	0.80662	0.84181	1.19238	1.47909	2.22599	1.00376
1999. 10.74726	4.05626	0.80246	0.83291	1.20230	1.47054	2.24212	1.00141
2000. 11.06275	4.14078	0.80168	0.82992	1.19762	1.48631	2.25586	0.99393
2001. 10.93307	4.13315	0.79920	0.83365	1.19003	1.46791	2.27281	0.99207
2002. 10.84573	4.11409	0.79703	0.83462	1.18983	1.45542	2.28849	0.99305
2003. 10.99497	4.19238	0.79554	0.83224	1.18841	1.45105	2.29704	0.98905

Thus over the 49 year period, real income ρ^t/ρ^0 (from the gross domestic product point of view) grew almost eleven fold (10.99), which is spectacular growth. From the above Table, it can be seen that productivity growth T^t contributes the most to the overall growth in market sector real income (the growth factor is 4.19), the growth in capital services input B_K^t makes the next largest contribution (the growth factor is 2.30) followed by the growth in labour input B_L^t (the growth factor is 1.45) while the change in domestic real prices A_D^t makes a negative contribution (.795) as does the growth in real export prices A_X^t (.832) and the change in import prices A_M^t makes a modest positive contribution (1.188).

7. The Deflated NDP Translog Approach

There is a severe flaw with all of the analysis presented in the previous sections. The problem is that depreciation payments are part of the user cost of capital for each asset but depreciation does not provide households with any sustainable purchasing power. Hence our real income measure defined by (5) above is overstated.

To see why Gross Domestic Product overstates income, consider the model of production that is described by the following quotations:

"We must look at the production process during a period of time, with a beginning and an end. It starts, at the commencement of the Period, with an Initial Capital Stock; to this there is applied a Flow Input of labour, and from it there emerges a Flow Output called Consumption; then there is a Closing Stock of Capital left over at the end. If Inputs are the things that are put in, the Outputs are the things that are got out, and the production of the Period is considered in isolation, then the Initial Capital Stock is an Input. A Stock Input to the Flow Input of labour; and further (what is less well recognized in the tradition, but is equally clear when we are strict with translation), the Closing Capital Stock is an Output, a Stock Output to match the Flow Output of Consumption Goods. Both input and output have stock and flow components; capital appears both as input and as output" John R. Hicks (1961; 23).

"The business firm can be viewed as a receptacle into which factors of production, or inputs, flow and out of which outputs flow...The total of the inputs with which the firm can work within the time period specified includes those inherited from the previous period and those acquired during the current period. The total of the outputs of the business firm in the same period includes the amounts of outputs currently sold and the amounts of inputs which are bequeathed to the firm in its succeeding period of activity." Edgar O. Edwards and Philip W. Bell (1961; 71-72).

Hicks and Edwards and Bell obviously had the same model of production in mind: in each accounting period, the business unit combines the capital stocks and goods in process that it has inherited from the previous period with "flow" inputs purchased in the current period (such as labour, materials, services and additional durable inputs) to produce current period "flow" outputs as well as end of the period depreciated capital stock components which are regarded as outputs from the perspective of the current period (but will be regarded as inputs from the perspective of the next period).⁴²

All of the "flow" inputs that are purchased during the period and all of the "flow" outputs that are sold during the period are the inputs and outputs that appear in the usual definition of cash flow. These are the flow inputs and outputs that are very familiar to national income accountants. But this is not the end of the story: the firm inherits an endowment of assets at the beginning of the production period and at the end of the period, the firm will have the net profit or loss that has occurred due to its sales of outputs and its purchases of inputs during the period. As well, *it will have a stock of assets that it can use when it starts production in the following period*. Just focusing on the flow transactions that occur within the production period will not give a complete picture of the firm's productive activities. Hence, to get a complete picture of the firm's production activities over the course of a period, it is necessary to add the value of the closing stock of assets less the beginning of the period stock of assets to the cash flow that accrued to the firm from its sales and purchases of market goods and services during the accounting period.

We illustrate the above theory by considering a very simple two output, two input model of the market sector. One of the outputs is output in period t, Y^t and the other output is an investment good, I^t . One of the inputs is the flow of noncapital primary input X^t and the other input is K^t , capital services. Suppose that the average prices during period t of a unit of Y^t , X^t and I^t are P_Y^t , P_X^t and P_I^t respectively. Suppose further that the interest rate prevailing at the beginning of period t is r^t . The value of the beginning of period t capital stock is assumed to be P_I^t , the investment price for period t. In order to induce households to let the business sector use the initial stock of capital, firms have to pay households interest equal to $r^t P_I^t K^t$. Then neglecting balance sheet items, the market sector's period t *cash flow* is:⁴³

(57) $CF^{t} = P_{Y}^{t}Y^{t} + P_{I}^{t}I^{t} - P_{X}^{t}X^{t} - r^{t}P_{I}^{t}K^{t}$.

 K^{t} is interpreted as the firm's beginning of period t stock of capital it has at its disposal and its end of period stock of capital is defined to be K^{t+1} . These capital stocks are

⁴² For more on this model of production and additional references to the literature, see the Appendices in Diewert (1977) (1980). The usual user cost of capital can be derived from this framework if depreciation is independent of use.

⁴³ For equity financed firms, we need to include an imputed return for equity capital.

valued at the balance sheet prices prevailing at the beginning and end of period t, P_I^t and P_I^{t+1} respectively.

The market sector period t *pure profit* is defined as its cash flow plus the value of its end of period t capital stock less the value of its beginning of period t capital stock:

(58)
$$\prod^{t} = CF^{t} + P_{I}^{t+1} K^{t+1} - P_{I}^{t} K^{t}.$$

Now the end of period depreciated stock of capital is related to the beginning of the period stock by the following equation:

(59) $K^{t+1} = (1 - \delta)K^t + I^t$

where $0 < \delta < 1$ denotes the depreciation rate.

Now substitute (57) and (59) into the definition of pure profits (58) and we obtain the following expression:

$$(60) \prod^{t} = P_{Y}^{t} Y^{t} + P_{I}^{t} I^{t} - P_{X}^{t} X^{t} - r^{t} P_{I}^{t} K^{t} + P_{I}^{t+1} (1 - \delta) K^{t} - P_{I}^{t} K^{t} = P_{Y}^{t} Y^{t} + P_{I}^{t} I^{t} - P_{X}^{t} X^{t} - \{r^{t} P_{I}^{t} + \delta P_{I}^{t+1} - (P_{I}^{t+1} - P_{I}^{t})\} K^{t}.$$

The expression that precedes the capital stock K^t , $\{r^t P_I^t + \delta P_I^{t+1} - (P_I^{t+1} - P_I^t)\}$, can be recognized as the *user cost of capital*;⁴⁴ it is the gross rental price that must be paid to a capitalist in order to induce him or her to loan the services of a unit of the capital stock to the production sector.

Some simplifications for (60) occur if we make two additional assumptions:

- Assume that producers and households expect price level stability so that the end of the period price for a new unit of capital P₁^{t+1} is expected to be equal to the beginning of the period price for a new unit of capital P₁^t; in this case, we can interpret r^t as the period t real interest rate;
- Assume that pure profits are zero so that Π^t equals zero.

Substituting these two assumptions into equation (60) leads to the following expression:

$$(61) \prod^{t} = P_{Y}^{t} Y^{t} + P_{I}^{t} I^{t} - P_{X}^{t} X^{t} - \{r^{t} P_{I}^{t} + \delta P_{I}^{t}\} K^{t} = 0.$$

Equation (61) can be rearranged to yield the following value of output equals value of input equation:

(62)
$$P_Y^t Y^t + P_I^t I^t = P_X^t X^t + \{r^t P_I^t + \delta P_I^t\} K^t$$
.

⁴⁴ See Christensen and Jorgenson (1969) for a derivation in continuous time and Diewert (1980; 471) for a derivation in discrete time.

Equation (62) is essentially the closed economy counterpart to the (gross) value of outputs equals (gross) value of primary inputs equation (4), $P^{t}\cdot y^{t} = W^{t}\cdot x^{t}$, that we have been using thus far in this study. We now come to the point of this rather long digression: *the (gross) payments to primary inputs that is defined by the right hand side of (62) is not income*, in the sense of Hicks.⁴⁵ The owner of a unit of capital cannot spend the entire period t gross rental income $\{r^{t} P_{I}^{t} + \delta P_{I}^{t}\}$ on consumption during period t because the depreciation portion of the rental, δP_{I}^{t} , is required in order to keep his or her capital intact. Thus the owner of a new unit of capital at the beginning of period t loans the unit to the market sector and gets the gross return $\{r^{t} P_{I}^{t} + \delta P_{I}^{t}\}$ at the end of the period plus the depreciated unit of the initial capital stock, which is worth only $(1 - \delta)P_{I}^{t}$. Thus δP_{I}^{t} of this gross return must be set aside in order to restore the lender of the capital services to his or her original wealth position at the beginning of period t. This means that *period t Hicksian market sector income* is not the value of payments to primary inputs, $P_{X}^{t} X^{t} + \{r^{t} P_{I}^{t} + \delta P_{I}^{t}\}K^{t}$; instead it is the value of payments to labour $P_{X}^{t} X^{t}$ plus the reward for waiting, $r^{t} P_{I}^{t} K^{t}$. Using this definition of market sector (net) Hicksian income, we can rearrange equation (62) as follows:

(63) Hicksian market sector income = $P_X^t X^t + r^t P_I^t K^t$ = $P_Y^t Y^t + P_I^t I^t - \delta P_I^t K^t$ = Value of consumption + value of gross investment – value of depreciation.

Thus in this Hicksian net income framework, our new output concept is equal to our old output concept less the value of depreciation. We take the price of depreciation to be the corresponding investment price P_I^t and the quantity of depreciation is taken to be the depreciation rate times the beginning of the period stock, δK^t .

Hence the overstatement of income problem that is implicit in the approaches used in previous sections can readily be remedied: all we need to do is to take the user cost formula for an asset and decompose it into two parts:

- One part that represents depreciation and foreseen obsolescence, $\delta P_I^t K^t$, and
- The remaining part that is the reward for postponing consumption, $r^t P_I^t K^t$.

In our data base used in the previous sections, the user costs had the following form:⁴⁶

⁴⁵ We will use Hicks' third concept of income here: "Income No. 3 must be defined as the maximum amount of money which the individual can spend this week, and still be able to expect to spend this week, and still be able to expect to spend the same amount *in real terms* in each ensuing week." J.R. Hicks (1946; 174).

^{(1946; 174).} ⁴⁶ There is no actual or smoothed capital gains term in the user cost formula (64). If such terms are introduced into the user cost formula, due to the rapid increases in the price of land, the resulting user costs inevitably turn out to be negative for at least some periods. Thus to avoid these negative user costs, we assumed that producers expected the rates of asset price inflation to equal general price inflation, which would be reflected in the nominal beginning of the period interest rate. Under these assumptions, the capital gains term vanishes from the user cost formula but the nominal interest rate is replaced by a real interest rate. For more discussion on this approach to user costs, see Diewert (2005c).

where r^{t} was the balancing period t real rate of interest, δ^{t} was a geometric depreciation rate for period t, τ^{t} was an average capital taxation rate on the asset and P₁^t was the period t investment price for the asset. Thus in this section, we split up each user cost times the beginning of the period stock K^t into the depreciation component $\delta^t P_I^t K^t/(1+r^t)$ and the remaining term $(r^{t} + \tau^{t})P_{1}^{t}K^{t}/(1+r^{t})$ and we regard the second term as a genuine income component but the first term is treated as an intermediate input cost for the market sector and is an offset to gross investment made by the market sector during the period under consideration. Thus in this section, we use a net product approach instead of a gross product approach. In this section, our investment aggregate I is a net investment aggregate (gross investment components are indexed with a positive sign in the aggregate and depreciation components are indexed with a negative sign in the aggregate). Our capital services aggregate is now a "reward for waiting" capital services aggregate rather than the gross return aggregate that was used in the previous section.⁴⁷ Using chained Törnqvist price indexes to do the aggregation, our old gross investment price index P_I is listed in Table 9 below along with the new price for "waiting" capital services P_{KW} , the price of the depreciation aggregate P_{DEP} and the price of the new net investment aggregate P_{NI} . The quantity aggregates that correspond to the price data listed in Table 9 are also listed in Table 9.48

Year	УI	X _{KW}	Y DEP	Уni	P_{I}	P_{KW}	P _{DEP}	P _{NI}
1955.	2004.0	1260.4	656.4	1347.6	1.00000	1.00000	1.00000	1.00000
1956.	2498.0	1307.6	678.1	1819.6	1.08966	1.21419	1.08902	1.09006
1957.	3335.2	1367.5	730.9	2608.9	1.11530	1.57182	1.16561	1.09925
1958.	2822.9	1453.3	814.5	1996.9	1.08517	1.10923	1.16954	1.05701
1959.	3523.4	1514.0	899.9	2619.7	1.09671	1.38217	1.15964	1.07670
1960.	4485.4	1585.4	998.0	3491.8	1.12536	1.84236	1.17083	1.11091
1961.	6171.1	1688.0	1174.4	5008.8	1.18842	2.47911	1.19725	1.18348
1962.	5878.6	1823.9	1427.6	4458.1	1.22206	2.10831	1.23796	1.21505
1963.	7062.9	1928.5	1687.7	5383.7	1.23440	2.56114	1.22349	1.23586
1964.	8251.0	2043.3	1953.3	6307.3	1.26520	3.02788	1.23186	1.27360
1965.	8072.4	2180.0	2268.1	5825.5	1.29157	2.63055	1.25756	1.30011
1966.	9439.1	2355.2	2540.9	6917.3	1.34057	3.04542	1.27085	1.36247
1967.	11606.3	2543.3	2868.0	8741.5	1.39509	3.54975	1.29869	1.42620
1968.	13688.9	2697.7	3308.9	10376.7	1.43578	4.28408	1.31676	1.47421
1969.	15806.9	2941.5	3873.6	11935.4	1.50321	4.68522	1.36184	1.54884
1970.	18191.1	3130.9	4556.9	13648.9	1.57495	4.92284	1.41676	1.62606

Table 9: The Quantity and Price of Gross I	Investment, Waitin	ng Capital Services,
Depreciation and Net Investment in Japan		

⁴⁷ This approach seems to be broadly consistent with an approach advocated by Rymes (1968) (1983), who stressed the role of waiting services: "Second, one can consider the 'waiting' or 'abstinence' associated with the net returns to capital as the nonlabour primary input." T.K. Rymes (1968; 362). Denison (1974) also advocated a net product approach to productivity measurement.

 $^{^{48}}$ The y_{DEP} entries should have a negative sign attached to them.

1971.	17846.7	3331.9	5353.4	12613.5	1.60231	3.84068	1.46490	1.64535
1972.	18530.2	3504.5	6052.7	12661.9	1.67822	3.50404	1.51056	1.73393
1973.	21232.8	3749.9	6723.9	14688.7	1.96572	3.91239	1.71178	2.05790
1974.	23148.5	3976.0	7450.9	15913.7	2.35593	4.44257	2.09973	2.44389
1975.	19802.9	4225.7	7966.6	12230.4	2.41893	1.89808	2.19683	2.48566
1976.	21819.2	4403.1	8372.2	13826.1	2.54487	2.86907	2.25065	2.65324
1977.	21979.1	4640.2	8761.8	13648.3	2.64465	2.57144	2.34833	2.75137
1978.	22362.3	4829.4	9158.8	13680.0	2.73713	2.48796	2.40449	2.86448
1979.	25537.4	4985.9	9697.0	16252.3	2.93269	3.58206	2.51407	3.10815
1980.	26117.4	5193.2	10282.4	16330.9	3.15185	5.91268	2.64280	3.37666
1981.	26235.2	5373.7	10862.7	15985.7	3.18702	5.77219	2.68314	3.40717
1982.	26988.1	5567.6	11441.3	16234.3	3.20514	5.90713	2.70166	3.42424
1983.	25436.8	5753.5	11950.7	14376.8	3.19698	5.09743	2.70844	3.40501
1984.	27308.4	5876.8	12452.9	15731.7	3.21541	6.18888	2.69376	3.44926
1985.	30781.7	6102.0	13121.7	18442.0	3.16742	8.05201	2.64683	3.40352
1986.	30162.5	6279.2	14009.1	17172.4	3.11760	7.15259	2.61459	3.34297
1987.	33385.3	6473.0	14909.6	19470.0	3.11969	7.94594	2.59232	3.36421
1988.	37010.9	6696.5	15839.6	22113.9	3.15686	8.58520	2.61422	3.41096
1989.	41092.9	6908.0	16977.3	25016.3	3.27383	9.65533	2.68399	3.55625
1990.	45022.2	7162.4	18318.9	27620.1	3.36684	10.31194	2.74649	3.66652
1991.	44370.2	7388.1	19752.8	25945.3	3.42432	8.90276	2.79732	3.72641
1992.	42894.0	7667.1	21220.2	23494.1	3.44849	8.07551	2.80155	3.76563
1993.	40375.6	7901.7	22312.8	20376.2	3.44956	6.44767	2.79930	3.76999
1994.	40435.3	8078.7	22924.5	19980.2	3.42826	6.29172	2.75538	3.77657
1995.	41142.3	8189.1	23346.2	20314.5	3.39526	5.95079	2.71993	3.75047
1996.	44229.4	8322.9	23973.0	22647.3	3.35480	6.62063	2.66305	3.73287
1997.	45398.4	8457.9	24857.9	23067.2	3.36631	6.55404	2.66325	3.75521
1998.	41700.2	8643.4	25882.6	19008.0	3.31394	4.93024	2.64481	3.66883
1999.	40973.8	8756.6	26733.4	17741.4	3.24440	4.44664	2.58351	3.60002
2000.	43353.1	8869.8	27434.2	19376.5	3.20669	5.03591	2.52937	3.59347
2001.	41865.9	9030.2	28274.0	17458.1	3.12045	3.97072	2.46129	3.49693
2002.	39187.9	9194.0	29057.9	14523.5	3.03741	3.70469	2.38506	3.42376
2003.	39988.0	9344.3	29390.5	14998.4	2.97040	3.91078	2.30324	3.40616

Note that the price of waiting capital services increases much more rapidly than the other investment prices. This is due to the fact that land services are included in the capital services aggregate but there is very little investment in land. Hence the situation is explained by the fact that land prices in Japan have been increasing much more rapidly than the prices of investment goods over most of the sample period.

Note that gross investment in Japan grew 19.95 fold over the sample period whereas net investment grew only 11.13 fold. Note also that gross investment is well above depreciation or replacement investment for every year.

All of the analysis presented in sections 3 and 4 above applies to the new situation with obvious modifications. The counterpart to Table 7 in the previous section using the new framework is Table 10 below.

Table 10: The Decomposition of Market Sector (Net) Real Income Growth into Translog Productivity, Real Output Price Change and Input Quantity Contribution Factors

Year t	γ^t	$\boldsymbol{\tau}^t$	${\alpha_{\mathrm{D}}}^{\mathrm{t}}$	$\alpha_{\mathrm{X}}{}^{\mathrm{t}}$	α_{M}^{t}	${\beta_L}^t$	β_{K}^{t}	$\alpha_{\mathrm{T}}{}^{\mathrm{t}}$
1956.	1.14565	1.06353	1.02611	1.00614	0.99146	1.04429	1.00774	0.99755
1957.	1.13218	1.08315	0.99963	0.99833	0.99884	1.03754	1.01067	0.99718
1958.	1.01165	0.97013	0.99520	0.99360	1.02671	1.01326	1.01370	1.02014
1959.	1.13188	1.10913	1.00054	1.00360	0.98758	1.02045	1.00845	0.99114
1960.	1.16573	1.12103	1.00641	0.99995	0.99587	1.02633	1.01096	0.99582
1961.	1.18396	1.14492	1.01169	0.99329	1.00020	1.01124	1.01742	0.99349
1962.	1.02108	1.00552	0.98423	0.98910	1.01507	1.00617	1.02132	1.00400
1963.	1.10428	1.10390	0.98105	0.99436	1.00656	1.00414	1.01457	1.00088
1964.	1.14498	1.10740	1.00315	0.99774	1.00139	1.01515	1.01620	0.99912
1965.	1.01501	0.99997	0.98555	0.99032	1.01007	1.01210	1.01731	1.00029
1966.	1.11708	1.07289	1.00023	0.99283	1.00345	1.02466	1.01972	0.99626
1967.	1.14251	1.08704	1.00712	0.99597	1.00882	1.01736	1.02093	1.00476
1968.	1.13465	1.11290	0.99403	0.99330	1.00618	1.00893	1.01716	0.99943
1969.	1.13057	1.08981	1.00494	0.99601	1.00273	1.00709	1.02633	0.99873
1970.	1.08660	1.06242	0.99120	0.99350	1.00892	1.01060	1.01860	1.00237
1971.	1.00089	0.99333	0.97972	0.99384	1.01285	1.00553	1.01609	1.00662
1972.	1.05492	1.03774	1.00162	0.99049	1.00578	1.00810	1.01059	0.99621
1973.	1.10251	1.05626	1.02280	0.99703	0.98751	1.02325	1.01295	0.98457
1974.	0.99565	1.04219	0.98349	1.00766	0.97037	0.98281	1.01082	0.97780
1975.	0.91729	0.93640	0.95704	0.98723	1.05028	0.97938	1.00794	1.03686
1976.	1.05874	1.03742	0.98894	0.98722	1.01429	1.02663	1.00387	1.00132
1977.	1.02299	1.00720	0.98749	0.97985	1.02441	1.01929	1.00530	1.00377
1978.	1.03834	1.01081	0.99865	0.98222	1.03151	1.01160	1.00361	1.01317
1979.	1.07899	1.09224	1.01597	1.00812	0.95028	1.01168	1.00326	0.95800
1980.	1.05613	1.05467	0.99540	1.00199	0.98651	1.01170	1.00597	0.98847
1981.	1.01673	1.00793	0.98595	0.99721	1.00783	1.01208	1.00585	1.00501
1982.	1.03451	1.02138	0.99549	1.00222	0.99976	1.00947	1.00591	1.00198
1983.	1.00791	0.97790	0.99918	0.99012	1.01350	1.02271	1.00511	1.00349
1984.	1.04486	1.04008	0.98993	0.99827	1.00869	1.00449	1.00331	1.00694
1985.	1.07049	1.05992	0.99064	0.99088	1.01220	1.00937	1.00707	1.00297
1986.	1.00960	0.97876	0.99367	0.97658	1.04695	1.00959	1.00566	1.02242
1987.	1.04215	1.02946	0.99170	0.99389	1.00898	1.01186	1.00600	1.00282
1988.	1.05671	1.03602	0.99468	0.99724	1.00381	1.01703	1.00722	1.00103
1989. 1000	1.06777	1.04420	1.00335	1.00246	0.99681 0.99414	1.01279	1.00703 1.00857	0.99926
1990. 1991.	1.05412	1.04892 0.99425	0.99576 0.99384	0.99927 0.99311	1.00964	1.00728 1.00665	1.00857	0.99341 1.00268
1991. 1992.	1.00431 0.99608	0.99423	0.99384	0.99311	1.00964	0.99551	1.00695	1.00268
1772.	0.77008	0.77230	0.7774/	0.7743/	1.00/28	0.77551	1.00/43	1.00101

1993.	0.97006	0.97592	0.99694	0.98906	1.00985	0.99289	1.00538	0.99880
1994.	1.01302	1.01001	0.99887	0.99562	1.00477	1.00016	1.00359	1.00037
1995.	1.01409	1.00624	1.00032	0.99847	1.00093	1.00594	1.00213	0.99940
1996.	1.03766	1.03844	0.99975	1.00582	0.98957	1.00157	1.00261	0.99533
1997.	1.02167	1.02741	1.00004	1.00234	0.99267	0.99670	1.00269	0.99499
1998.	0.96579	0.96526	0.99953	1.00402	1.00274	0.99107	1.00325	1.00677
1999.	0.98313	0.99509	0.99616	0.98711	1.01016	0.99295	1.00170	0.99714
2000.	1.03225	1.02552	1.00094	0.99562	0.99524	1.01312	1.00172	0.99089
2001.	0.98077	0.99780	0.99802	1.00550	0.99224	0.98485	1.00233	0.99770
2002.	0.98700	0.99432	0.99981	1.00144	0.99979	0.98953	1.00211	1.00123
2003.	1.01885	1.02351	1.00218	0.99649	0.99853	0.99630	1.00195	0.99503
Averag	ge 1.0505	1.0353	0.99684	0.99564	1.0042	1.0088	1.0089	0.99978

The new results are quite interesting. In the previous GDP model, the average rate of increase in real income was 5.27% per year, which has now decreased by 0.2% per year to 5.05%. Moreover, there are some big shifts in the explanatory factors: productivity growth now accounts for an average contribution of 3.53% per year compared to the old 3.11%; the contribution of labour input growth has marginally increased from 0.78% per year to 0.88% and the contribution of capital services input growth has dramatically decreased from 1.75% per year to 0.89% per year. The contributions of real output price changes (including changes in real export and import prices) are little changed and are generally small.

The above period to period results can be cumulated and Table 11 is the net product counterpart to the gross product cumulative results found in Table 8 above.

Table 11: The Decomposition of Market Sector Real Income Levels intoProductivity, Real Output Price Change and Input Quantity Contribution Factorsusing the Translog Net Product Approach

Year t	$ ho^t / ho^0$	T^{t}	A_D^t	A_X^t	A_M^t	${\rm B_L}^{\rm t}$	${B_K}^t$	A_T^t
1955.	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1956.	1.14565	1.06353	1.02611	1.00614	0.99146	1.04429	1.00774	0.99755
1957.	1.29708	1.15196	1.02573	1.00447	0.99031	1.08349	1.01850	0.99474
1958.	1.31219	1.11755	1.02080	0.99804	1.01677	1.09786	1.03246	1.01477
1959.	1.48524	1.23951	1.02136	1.00164	1.00414	1.12032	1.04118	1.00578
1960.	1.73140	1.38953	1.02791	1.00159	0.99999	1.14982	1.05259	1.00158
1961.	2.04991	1.59090	1.03992	0.99487	1.00019	1.16274	1.07093	0.99506
1962.	2.09311	1.59969	1.02352	0.98402	1.01526	1.16991	1.09376	0.99904
1963.	2.31139	1.76590	1.00413	0.97847	1.02192	1.17475	1.10971	0.99991
1964.	2.64648	1.95556	1.00729	0.97625	1.02334	1.19255	1.12769	0.99904
1965.	2.68621	1.95550	0.99273	0.96680	1.03364	1.20697	1.14721	0.99932
1966.	3.00071	2.09803	0.99296	0.95987	1.03721	1.23674	1.16983	0.99558
1967.	3.42834	2.28065	1.00003	0.95600	1.04636	1.25821	1.19432	1.00032
1968.	3.88997	2.53814	0.99406	0.94960	1.05282	1.26944	1.21482	0.99976
1969.	4.39787	2.76609	0.99897	0.94581	1.05570	1.27844	1.24681	0.99849

1970.	4.77875	2.93875	0.99019	0.93966	1.06512	1.29199	1.27000	1.00085
1971.	4.78301	2.91916	0.97011	0.93388	1.07881	1.29913	1.29043	1.00748
1972.	5.04572	3.02932	0.97168	0.92499	1.08505	1.30965	1.30410	1.00366
1973.	5.56293	3.19977	0.99383	0.92224	1.07150	1.34010	1.32099	0.98818
1974.	5.53875	3.33477	0.97743	0.92930	1.03975	1.31706	1.33528	0.96624
1975.	5.08065	3.12269	0.93544	0.91743	1.09203	1.28991	1.34588	1.00186
1976.	5.37910	3.23955	0.92509	0.90571	1.10763	1.32426	1.35109	1.00318
1977.	5.50276	3.26286	0.91352	0.88746	1.13466	1.34980	1.35825	1.00696
1978.	5.71373	3.29812	0.91229	0.87168	1.17042	1.36545	1.36316	1.02023
1979.	6.16508	3.60233	0.92686	0.87876	1.11223	1.38140	1.36760	0.97737
1980.	6.51112	3.79929	0.92260	0.88051	1.09722	1.39756	1.37576	0.96611
1981.	6.62003	3.82943	0.90963	0.87805	1.10581	1.41444	1.38381	0.97095
1982.	6.84846	3.91129	0.90553	0.87999	1.10555	1.42783	1.39199	0.97287
1983.	6.90264	3.82486	0.90479	0.87130	1.12048	1.46026	1.39911	0.97627
1984.	7.21229	3.97818	0.89568	0.86979	1.13021	1.46681	1.40373	0.98305
1985.	7.72069	4.21653	0.88730	0.86186	1.14399	1.48055	1.41366	0.98597
1986.	7.79479	4.12699	0.88168	0.84168	1.19770	1.49475	1.42167	1.00808
1987.	8.12330	4.24858	0.87436	0.83654	1.20845	1.51248	1.43019	1.01091
1988.	8.58401	4.40162	0.86971	0.83422	1.21305	1.53824	1.44051	1.01196
1989.	9.16577	4.59618	0.87263	0.83628	1.20918	1.55791	1.45064	1.01121
1990.	9.66181	4.82104	0.86893	0.83567	1.20209	1.56926	1.46308	1.00455
1991.	9.70341	4.79332	0.86358	0.82991	1.21368	1.57970	1.47325	1.00725
1992.	9.66542	4.75641	0.86295	0.82524	1.22252	1.57260	1.48424	1.00887
1993.	9.37607	4.64190	0.86031	0.81621	1.23456	1.56143	1.49223	1.00766
1994.	9.49811	4.68836	0.85933	0.81263	1.24045	1.56168	1.49758	1.00803
1995.	9.63196	4.71763	0.85961	0.81139	1.24160	1.57096	1.50077	1.00742
1996.	9.99470	4.89898	0.85939	0.81611	1.22866	1.57343	1.50468	1.00272
1997.	10.21127	5.03328	0.85943	0.81802	1.21965	1.56823	1.50873	0.99769
1998.	9.86198	4.85842	0.85902	0.82131	1.22299	1.55424	1.51364	1.00445
1999.	9.69557	4.83456	0.85572	0.81072	1.23541	1.54329	1.51621	1.00157
2000.	10.00824	4.95795	0.85652	0.80717	1.22953	1.56353	1.51882	0.99244
2001.	9.81583	4.94705	0.85483	0.81161	1.21999	1.53985	1.52236	0.99016
2002.	9.68827	4.91894	0.85467	0.81278	1.21973	1.52372	1.52558	0.99138
2003.	9.87088	5.03460	0.85653	0.80993	1.21794	1.51808	1.52855	0.98645

Thus the overall growth in real net income in Japan over the 49 year period was a 9.87 fold increase. The main (multiplicative) explanatory factors were productivity growth 5.034), increases in labour input (1.518) and increases in (waiting) capital services (1.5128). There were smaller effects due to the relative fall in the price of domestic C + G + I relative to the price of C (0.856), the relative fall in the price of exports (0.810) and the relative fall in the price of c imports (1.218). The combined effects of changes in the prices of exports and imports relative to the price of consumption were negligible (0.986) over the sample period.

8. Conclusion

It seems clear to us that it is time that statistical agencies focused their attentions on producing estimates of net national product rather than GDP since it is net product that is sustainable and is of direct interest for purposes of measuring welfare change.⁴⁹ For those statistical agencies that produce estimates of TFP growth, it would also be desirable to move to a net concept of capital services and a corresponding net output measure of TFP growth. Judging from our experience with the Japanese data, a move to net measures will change our perception of the growth process: TFP growth becomes more important in the net framework and the effects of capital deepening become less important.

The computations presented in this paper also illustrated the importance of including land and inventory stocks in the asset base when measuring real rates of return and when computing TFP growth.

On a theoretical level, the results of Diewert and Morrison (1986) were modified to give an exact decomposition of the growth in real incomes generated by the market sector of the economy.

Finally, the paper documented the well known productivity slowdown that occurred in the Japanese economy over the past 15 years. In future work, a more detailed sectoral breakdown of the Japanese economy may be helpful in determining the industrial origins of the overall market sector productivity improvements. It is clear that the spectacular six percent plus TFP growth rates that took place over the period 1955-1973 were partly driven by the shift of the labour force from agriculture to industry (and by the shift from self employment and family work to paid employment that took place during that period). It seems that the same phenomenon is taking place in China now and it will be interesting to see if China can sustain its rapid rate of growth once the shift from agriculture to industry has been completed. On the other hand, if Japan wishes to improve on its productivity performance in recent years, it seems evident that structural reforms in agriculture and services will have to take place.

Data Appendix

The prices of the 16 net outputs listed in section 2 above are listed in Tables A1 and A2 below and the corresponding quantities are listed in Tables A3 and A4. The units of measurement for Tables A3 and A4 are billions of constant 1955 yen.

Table A1: Market Sector Producer Prices for Japanese Net Outputs 1-8, 1955-2003

Year	\mathbf{P}_1	P_2	P ₃	\mathbf{P}_4	P ₅	P_6	\mathbf{P}_7	P_8
1955.	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1956.	1.00091	1.04615	1.09315	1.04421	1.05901	1.10674	1.09550	1.13292
1957.	1.02553	1.17402	1.23234	1.06042	1.09455	1.09355	1.09076	1.17526
1958.	1.00955	1.21289	1.26337	1.00108	0.91553	1.02925	1.00381	1.13489

⁴⁹ Diewert and Fox (2005) also recently advocated a change to net output and the corresponding measures of TFP growth.

1050	1 01552	1 22227	1 22200	1 02244	1 00774	1.05200	1 017(7	1 10101
	1.01553							
	1.04535		1.36779			1.07549		1.25949
1961.	1.08920	1.28583	1.52944	1.05194	1.10701	1.21291	1.07861	1.39173
	1.16632	1.37610		1.03432	1.07171	1.24461	1.09003	1.48771
	1.24931					1.29607	1.10101	1.54629
1964.	1.29122		1.95976		1.12296	1.33257	1.13829	1.64757
1965.			2.13180					1.71794
1966.	1.44610						1.22966	
1967.	1.48971					1.57400		1.94642
1968.	1.56743				1.10662		1.32822	
	1.63573		2.84017				1.40252	
1970.	1.75697				1.13727		1.47959	
1971.			3.40542				1.50187	
	1.97780		3.72525		1.11476		1.61574	2.48215
1973.	2.21196				1.36931		2.00264	3.12303
1974.	2.77293	4.58741			2.04418		2.37877	3.86180
					1.78354			3.97611
1976.	3.40207							
1977.	3.64388		5.92539		1.68457		2.70634	
	3.77883				1.42793			
1979.	3.88894						3.13596	5.24033
1980.			6.19212	1.88495	2.35311	4.44380	3.40411	5.77787
1981.		7.52015			2.36895			5.89303
1982.	4.52948	7.69026	6.53242	2.01615	2.43043	4.46069	3.47203	5.92231
	4.61708						3.49475	5.89459
	4.73160						3.57409	5.96348
	4.80666						3.57719	
1986.	4.82738						3.55824	5.88928
1987.		8.26421			1.34625		3.60516	5.96528
	4.82460				1.30063		3.67548	
1989.	4.89836	8.52905	5.39242	1.64384	1.35828	4.96679	3.85442	6.38175
1990.	5.00305	8.96291	5.07531	1.67158	1.45497	5.14210	3.99359	6.61961
	5.12029							
1992.	5.18734	9.34334	5.02302	1.59096	1.30988	5.35192	4.15254	6.88955
1993.	5.22592	9.33467	4.98001	1.48418	1.20267	5.43693	4.17118	6.89443
1994.	5.22487	9.39690	4.90085	1.43813	1.14816	5.45534	4.16974	6.91833
1995.	5.17328	9.39213	4.83749	1.40880	1.12748	5.41307	4.17920	6.95029
1996.	5.12960	9.44597	4.74903	1.45407	1.21544	5.44492	4.18912	6.94591
1997.	5.13427	9.57329	4.70165	1.47750	1.28242	5.54647	4.22240	7.00572
1998.	5.04020	9.55649	4.65071	1.48593	1.23523	5.42516	4.16065	6.89268
	4.97857							
	4.91978							
	4.82340							
	4.73984							
2003.	4.66586	9.00536	4.19481	1.26867	1.16837	5.23241	4.02869	6.57900

Table A2: Market Sector Producer Prices for Japanese Net Outputs 9-16, 1955-2003

Year	P ₉	P ₁₀	P ₁₁	P ₁₂	P ₁₃	P_{14}	P ₁₅	P ₁₆
1955.	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1956.	1.00363	1.09622	1.10278	1.01020	1.10750	1.02692	1.01904	1.02742
1957.	1.04741	1.16074	1.30181	1.01327	1.17301	1.01153	0.98224	0.93289
1958.	1.09650	1.12895	1.33136	1.01378	1.07671	1.00000	0.99092	0.94518
1959.	1.03285	1.16016	1.28603	1.01888	1.12934	1.02692	1.02013	0.94518
1960.	1.03597	1.13538	1.33644	1.04541	1.15118	1.06538	1.08920	1.00005
1961.	1.03573	1.12708	1.36452	1.14429	1.27380	1.11538	1.13591	0.97761
1962.	1.03568	1.10959	1.41014	1.15712	1.29563	1.17307	1.18232	1.01501
1963.	0.97892	1.10439	1.38393	1.17222	1.33483	1.21154	1.22841	1.03247
1964.	0.94980	1.09557				1.24231	1.26402	1.03746
	0.96051					1.27692	1.32611	1.09980
	0.94335	1.09293		1.28178			1.38392	1.15717
	0.92550				1.62934			1.17213
-/	0.93184			1.41341		1.41923	1.51353	1.20455
	0.94629		1.55994		1.79619		1.61153	1.23447
	0.95914			1.56249		1.60000	1.67167	1.22450
	0.94573				1.94121			1.20953
	0.90896			1.68880		1.86538		1.51877
	0.93029				2.66965			2.49887
	1.10856							
					3.20381			
1976.	1.09307				3.47032			
					3.61534			
					3.81523			
	1.09184							
	1.11784							
					4.62150			
1982.	1.09129	1.86591			4.63830			
1983.		1.88451			4.63830			
	1.03619							
	0.99599							
	0.93211							
	0.87869							
	0.85936							
	0.86182							
	0.86255							
	0.84644							
	0.83056							
	0.81483							
	0.78990							
	0.75111							
	0.69924							
1997.	0.67379	1./3/83	3.20011	3.30108	3.03510	3.8/092	4.43001	2.32430

1998.0.652441.733853.245815.496085.569433.869234.414832.184641999.0.618941.705463.163525.530615.477043.846154.408852.493882000.0.587791.687083.083825.544115.501123.773074.341232.568702001.0.533111.667243.006915.611405.458323.692304.268472.586162002.0.477771.654502.952685.639715.372493.646154.237952.685922003.0.428171.607722.869205.668165.339723.603024.187822.95276

Table A3: Market Sector Net Outputs 1-8 for Japan, 1955-2003

Year	\mathbf{y}_1	y ₂	y ₃	y 4	y 5	y 6	y 7	y 8
1955.	4452.8	99.8	405.7	902.1	-899.2	265.4	383.6	530.0
1956.	4824.3	108.5	380.1	1062.0	-1141.4	292.8	464.4	575.5
1957.	5134.5	108.5	363.6	1186.1	-1414.8	347.3	585.6	661.7
1958.	5518.5	107.1	382.9	1249.6	-1260.2	410.5	548.3	688.4
1959.	5925.5	112.5	431.9	1415.9	-1393.0	450.5	726.5	803.5
1960.	6529.2	147.6	446.9	1596.7	-1629.6	576.3	981.2	974.8
1961.	7106.3	165.0	463.4	1680.0	-2003.4	653.0	1174.3	1140.4
1962.	7795.0	174.2	505.0	1974.6	-2018.2	765.7	1246.5	1246.8
1963.	8607.3	188.1	570.4	2115.8	-2403.0	892.7	1370.6	1362.9
1964.	9605.8	229.5	575.0	2578.6	-2710.8	1125.7	1670.1	1384.9
1965.	10115.7	254.4	603.7	3199.6	-2860.9	1351.9	1525.3	1429.0
1966.	10940.0	260.7	639.8	3744.0	-3202.9	1432.4	1625.6	1661.6
1967.	12085.0	258.4	664.4	3996.5	-4027.3	1660.8	1957.7	1775.8
1968.	13204.9	263.4	713.3	4951.4	-4506.2	1975.1	2214.7	2056.5
1969.	14530.0	272.8	753.7	5988.7	-5149.0	2257.0	2517.5	2334.5
1970.	15637.8	219.8	824.6	7033.7	-6402.4	2550.8	3058.9	2464.8
1971.	16453.7	201.1	887.4	8186.3	-6789.7	2713.6	3020.6	2818.3
1972.	17992.6	196.0	949.0	8528.9	-7051.0	3008.1	2847.2	3298.4
1973.	19623.4	182.6	1031.3	8982.6	-8400.6	3399.9	3030.6	3409.4
1974.		175.2	997.1	11068.9	-9496.1	3245.3	3455.3	3129.2
1975.		165.0	1236.7	10964.3		3355.8	3634.5	3260.0
	20697.5	164.3	1311.8	12787.5		3585.7	3509.0	3336.1
1977.		195.4	1402.7	14288.2		3671.6	3319.3	3560.8
	22460.7	199.9	1514.5	14260.5		3865.6	3138.0	4055.0
	23979.9	223.2	1620.8	14869.0		3744.0	3883.3	3883.6
	24105.7	252.5	3079.5	17360.5		3499.9	4068.5	3865.1
	24411.6	261.1	3274.6	19396.4		3391.8	4178.1	3948.2
	25643.1	262.7	3464.0	19376.9		3381.2	4091.6	3948.3
	26400.2	282.8	3676.1	19966.1		3233.3	4071.2	3744.7
	26985.7	301.0		22553.6		3125.5	4237.3	3660.0
1985.		315.5		23887.3		3195.4	4742.9	3713.3
1986.		338.4		22415.0		3437.0	4789.9	4040.4
1987.		349.7		22207.6		4082.0	4840.1	4195.9
1988.		366.1		23353.5		4619.1	5328.6	4615.2
1989.		385.4		25522.9		4599.0	6254.5	4871.4
1990.	34477.2	401.6	5045.8	27208.2	-27402.8	4834.6	6684.9	5341.3

1991. 35306.6	426.2	5433.3	28379.4 -27170.8	4627.7	7041.1	5699.4
1992. 35987.1	464.6	5730.0	29509.6 - 26809.5	4397.0	6427.3	5998.8
1993. 36206.4	485.7	6074.3	29534.1 - 26507.5	4476.7	5448.3	6426.7
1994. 37140.2	500.2	6407.9	30612.6 - 28578.5	4934.8	4914.9	6430.4
1995. 37727.6	532.3	6885.5	31958.5 - 32376.7	4741.6	4152.0	6342.4
1996. 38735.1	552.8	7304.7	33869.9 - 36934.7	5267.1	3697.9	6446.8
1997. 39044.0	557.9	7452.5	37699.6 - 37548.0	4664.0	4815.0	6136.7
1998. 38742.3	638.7	7639.6	36808.2 - 35263.4	4012.4	4855.4	6121.8
1999. 38441.7	682.7	8243.1	37423.3 - 36249.5	4000.4	4529.6	6053.1
2000. 38784.5	640.9	8943.8	42009.9 - 39712.0	3992.8	4906.9	5832.6
2001. 39386.3	661.6	9538.7	39439.5 - 39890.3	3772.7	4807.1	5677.8
2002. 39447.6	685.4	10037.9	42329.6 -40342.4	3624.3	4663.0	5402.9
2003. 39711.9	701.7	10165.4	45903.7 -41915.9	3565.7	4874.5	4869.5

Table A4: Market Sector Net Outputs 9-16 for Japan, 1955-2003

Year	y 9	y ₁₀	y ₁₁	y12	y ₁₃	y ₁₄	y 15	y ₁₆
1955.	90.5	177.3	223.4	14.4	1.2	203.4	25.5	89.3
1956.	135.5	224.5	311.2	18.8	1.3	114.3	150.5	214.8
1957.	181.2	303.6	378.1	23.2	1.5	318.5	145.4	427.6
1958.	200.2	374.3	375.8	27.4	1.5	250.9	-105.7	-9.3
1959.	271.3	343.5	452.6	33.3	1.4	121.7	82.5	206.9
1960.	381.7	536.1	639.6	41.0	1.5	217.0	64.5	-0.2
1961.	513.0	765.5	884.9	51.4	1.4	454.6	174.4	328.1
1962.	610.7	870.1	977.7	59.9	1.6	61.4	-10.7	-88.6
1963.	661.2	1000.5	1068.8	68.3	1.5	267.3	69.7	210.0
1964.	772.0	1246.9	1190.4	80.3	1.6	416.2	152.6	171.5
1965.	800.8	1287.8	1186.2	86.1	1.6	362.9	-32.5	-23.3
1966.	969.5	1589.2	1369.1	99.6	1.8	449.5	99.1	62.6
1967.	1320.5	1866.9	1786.1	119.2	1.9	644.5	241.0	296.6
1968.	1782.7	2293.0	2204.0	145.8	2.0	886.3	119.6	165.0
1969.	2256.2	2659.9	2697.6	175.7	2.2	595.4	250.4	343.1
1970.	2974.5	2956.2	3252.3	207.6	2.3	637.8	305.4	317.6
1971.	3257.3	2919.2	3216.9	220.6	2.6	125.0	-4.6	-46.3
1972.	3912.3	3267.2	3153.7	246.9	3.1	-141.3	-113.4	-514.1
1973.	5113.8	3288.3	3558.2	280.9	3.4	-12.9	275.7	-401.0
1974.	4782.8	2539.8	3313.7	262.2	3.4	1571.4	409.3	492.3
1975.	4573.9	2477.5	2989.3	265.2	3.7	-258.8	-164.3	-132.0
1976.	5032.9	2472.9	3088.6	279.9	3.5	338.9	85.7	243.1
1977.	5553.7	2552.8	3102.4	293.5	3.4	-40.1	-102.2	459.7
1978.	6472.3	3096.2	3236.5	320.7	3.5	76.5	-281.4	-861.8
1979.	7040.4	3461.7	3500.4	341.6	3.2	835.4	70.4	-181.0
1980.	7211.4	3331.5	3574.5	342.9	2.9	532.2	415.5	257.8
1981.	8157.4	3450.8	3741.6	349.7	3.0	27.4	978.4	-244.9
1982.	8624.8	3104.4	3926.7	357.8	2.9	308.5	751.1	249.2
1983.	9229.3	3181.6	3933.9	350.1	2.9	-321.5	677.7	-57.1

1984.	11167.8	3273.0	4384.3	368.6	2.5	16.5	905.2	92.7
1985.	13582.2	3731.8	4829.4	426.1	2.4	153.8	787.1	1253.2
1986.	14932.7	3944.1	4878.8	446.4	2.4	-280.7	494.4	-414.9
1987.	16597.7	4465.7	4938.5	473.7	2.3	58.5	659.6	149.0
1988.	19304.9	4692.0	5739.3	520.5	2.2	144.8	591.0	-203.9
1989.	21311.2	5133.2	6499.8	596.7	2.1	700.2	667.6	-16.5
1990.	23108.5	5639.6	7381.5	524.0	1.9	647.8	809.5	651.6
1991.	24165.9	5347.5	7544.2	722.5	1.9	159.8	107.7	-121.0
1992.	23937.8	5273.0	7037.4	749.1	1.8	-106.1	-1.8	-107.8
1993.	23479.3	4964.6	6169.6	659.0	2.0	-248.7	-547.5	-393.6
1994.	22946.0	4876.7	5765.8	632.8	2.1	-105.0	-221.8	-416.7
1995.	26798.2	5495.7	6169.8	685.0	2.1	188.2	-199.7	-256.2
1996.	33704.2	5764.1	6514.8	799.6	2.2	325.1	-93.5	-37.0
1997.	36159.5	5579.7	6805.4	866.5	2.1	523.5	-124.5	539.2
1998.	34469.0	5075.0	6422.8	1098.5	2.0	-424.8	-571.9	5.1
1999.	36584.0	4806.4	6165.8	1179.9	1.9	-1059.6	18.7	-494.9
2000.	39817.8	4879.2	6422.7	1208.2	1.7	-20.6	-54.3	-100.6
2001.	39588.5	5214.7	6399.5	1291.0	1.6	115.7	-873.4	-54.1
2002.	32006.1	5254.9	5789.2	1304.2	1.6	-182.6	-423.5	-189.3
2003.	33990.8	5580.8	6148.2	1317.6	1.6	107.1	62.3	-285.0

Turning now to input prices and quantities used by the market sector in Japan, Tables A5 and A6 list the input prices for the 14 primary inputs in our data base (see section 2 above for a listing of these inputs). It should be noted that the price of labour, W_1 , grew the most over our sample period (a 29.9 fold increase) while the price of computers, electronic and electrical equipment, W_4 , fell to 0.42 times the initial level. It should be noted that while the price of labour W_1 are the beginning of the year stock prices of the assets 2-14.⁵⁰

Table A5: Market Sector Prices for Japanese Primary Inputs 1-7, 1955-2003

Year	W_1	W_2	W_3	W_4	W_5	W_6	W_7
1955.	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1956.	1.06015	1.09550	1.13292	1.00363	1.09622	1.10278	1.01020
1957.	1.12002	1.09076	1.17526	1.04741	1.16074	1.30181	1.01327
1958.	1.19115	1.00381	1.13489	1.09650	1.12895	1.33136	1.01378
1959.	1.28144	1.01767	1.18181	1.03285	1.16016	1.28603	1.01888
1960.	1.41223	1.01444	1.25949	1.03597	1.13538	1.33644	1.04541
1961.	1.62223	1.07861	1.39173	1.03573	1.12708	1.36452	1.14429
1962.	1.87672	1.09003	1.48771	1.03568	1.10959	1.41014	1.15712
1963.	2.14445	1.10484	1.54629	0.97892	1.10439	1.38393	1.17222
1964.	2.43452	1.13829	1.64757	0.94980	1.09557	1.40244	1.21342
1965.	2.74574	1.16444	1.71794	0.96051	1.09315	1.40403	1.21828
1966.	3.04664	1.22966	1.84471	0.94335	1.09293	1.39690	1.28178

⁵⁰ We approximated these beginning of the period asset prices by the corresponding annual average investment prices except for the land asset prices, which were obtained from other sources.

1967. 3.42006	1.29276	1.94642	0 92550	1.10649	1.43644	1.36439
1968. 3.92083	1.32822		0.93184	1.09800	1.49993	1.41341
1969. 4.56194	1.40252	2.14866	0.94629	1.10489	1.55994	1.47428
1970. 5.34943	1.47959	2.30850	0.95914	1.12416	1.61347	1.56249
1971. 6.20088	1.50187	2.35604	0.94573	1.14609	1.65680	1.60787
1972. 7.10272	1.61574	2.48215	0.90896	1.16109	1.70000	1.68880
1973. 8.58472	2.00264	3.12303	0.93029	1.25342	1.90376	1.98753
1974. 11.03768	2.37877	3.86180	1.10856	1.48117	2.44311	2.50467
1975. 13.09550	2.42566	3.97611	1.10610	1.57917	2.50750	2.63238
1976. 14.23948	2.61696	4.21563	1.09307	1.61520	2.56340	2.77662
1977. 15.53244	2.70634	4.45186	1.10890	1.69352	2.67557	2.94475
1978. 16.62487	2.86719	4.76309	1.08133	1.70556	2.69934	3.08668
1979. 17.64361	3.13596	5.24033	1.09184	1.72346	2.82978	3.37745
1980. 18.61965	3.40411	5.77787	1.11784	1.79342	3.00576	3.75307
1981. 19.75436	3.45028	5.89303	1.10841	1.81290	3.05955	3.95767
1982. 20.70442	3.47203	5.92231	1.09129	1.86591	3.08321	4.03131
1983. 21.25655	3.49475	5.89459	1.06028	1.88451	3.07114	4.18446
1984. 22.03585	3.57409	5.96348	1.03619	1.80400	3.09080	4.30422
1985. 22.54127	3.57719	5.92562	0.99599	1.79006	3.09547	4.16786
1986. 23.17142	3.55824	5.88928	0.93211	1.76097	3.05991	4.29576
1987. 23.19048	3.60516	5.96528	0.87869	1.74657	3.03876	4.51687
1988. 23.68318	3.67548	6.08975	0.85936	1.75098	3.07636	4.75239
1989. 24.76920		6.38175	0.86182	1.77114	3.15997	5.09978
1990. 26.20684	3.99359	6.61961	0.86255	1.78574	3.24625	5.52508
1991. 27.83529		6.79313	0.84644	1.79770	3.30897	5.72773
1992. 28.74286			0.83056	1.79461	3.32263	5.75550
1993. 29.40147			0.81483	1.79950	3.31038	5.64749
1994. 29.86487			0.78990	1.76806	3.27556	5.37770
1995. 30.04179			0.75111	1.75038	3.25175	5.19944
1996. 30.31299			0.69924	1.72862	3.22285	5.18479
1997. 31.23549		7.00572	0.67379	1.73783	3.26011	5.36168
1998. 31.06780		6.89268	0.65244	1.73385	3.24581	5.49608
1999. 30.69501	4.10895	6.81048	0.61894	1.70546	3.16352	5.53061
2000. 30.29806			0.58779	1.68708	3.08382	5.54411
2001. 30.47848			0.53311	1.66724	3.00691	5.61140
2002. 29.98659		6.61999	0.47777	1.65450	2.95268	5.63971
2003. 29.92703	4.02869	6.57900	0.42817	1.60772	2.86920	5.66816

Table A6: Market Sector Prices for Japanese Primary Inputs 8-14, 1955-2003

Year	W_8	W_9	W_{10}	W_{11}	W ₁₂	W ₁₃	W_{14}
1955.	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1956.	1.10750	0.99617	1.05533	1.04055	0.99029	1.17374	1.12385
1957.	1.17301	1.02299	1.07542	1.06908	1.00971	1.47500	1.26305
1958.	1.07671	1.00766	1.03659	0.97072	1.02913	1.79981	1.41948
1959.	1.12934	0.99617	1.04575	0.98351	1.04854	2.21126	1.59529

1960	1.15118	1 02299	1.07657	0 98351	1.07767	2.85319	1.79288
1961.	1.27380	1.06130	1.14947	1.04060	1.09709	3.95208	1.88544
1962.	1.29563	1.111111	1.19876	1.01725	1.21359	5.08853	1.98279
1963.	1.33483	1.16858	1.24774	1.05617	1.43689	6.09090	2.08516
1964.	1.39026	1.20690	1.29638	1.07434	1.52427	6.93877	2.19282
1965.	1.43505	1.23755	1.33396	1.07953	1.67961	7.86056	2.30604
1966.	1.54031	1.27203	1.39948	1.14440	1.85437	8.41532	2.52973
1967.	1.62934	1.30268	1.46049	1.20409	2.11650		2.77513
1968.	1.69093	1.35632	1.51547	1.21966		10.20130	3.04433
1969.	1.79619	1.41379	1.59727	1.25339		11.80269	3.33964
1970.	1.91321	1.50192	1.70070	1.23557		13.97532	3.66360
1970.	1.94121	1.59387	1.76416	1.27415		16.10288	4.20046
1972.	2.11366	1.65134	1.87366	1.25858		18.10110	4.81598
1972.		1.85824	2.23939	1.23838		21.67893	5.52170
1975.		2.37931	2.23939	2.60020		26.14064	6.33084
1974.	3.20381	2.55939	2.73214 2.87907	2.00020		26.42574	7.25854
1975.	3.47032	2.33939	3.08019	2.92458		26.21736	7.23834
1970.	3.61534	2.72797		2.92438		26.21/30	8.24024
	3.81523	2.80390	3.37159			26.86482	8.24024 9.04360
1978.							
1979.		3.03448	3.61277			27.60298	9.62586
1980.		3.26820	3.92139			29.13835	
1981.	4.62150	3.40996		4.43747		31.11412	
1982.	4.63830	3.49808				33.04778	
1983.		3.54406				34.58708	
1984.		3.60153				35.73703	
1985.	4.72172	3.63985				36.81809	
1986.	4.68869					38.10956	
1987.	4.77212					40.45276	
1988.	4.86114	3.59387				44.90033	
1989.	5.12206	3.65134	4.37841			49.46681	
1990.	5.29451					56.42712	
1991.	5.42833					63.28955	
						64.05983	
						60.54749	
						56.54404	
						53.09197	
						49.48009	
						46.07104	
						43.24371	
						40.32006	
						37.17151	
						34.07813	
						31.04654	
2003.	5.33972	3.63218	4.47244	2.79483	9.82604	28.14293	9.02647

Note that the price of agricultural land peaked in 1991 with a price equal to 19.7 times the 1955 price and then the price trended down to 9.8 in 2003; the price of business land peaked in 1992 with a price equal to 64.0 times the 1955 price and then the price trended down to 28.1 in 2003 and the price of forested land (and the timber standing on the land) peaked in 1990 with a price equal to 12.5 times the 1955 price and then the price trended down to 9.0 in 2003. The corresponding primary input quantities used are listed in Tables A7 and A8 below.

Year	X ₁	X ₂	X3	X 4	X 5	X 6	X 7
1955.	5048.4	2423.1	4794.9	468.4	798.8	1620.2	33.6
1956.	5332.9	2671.7	5054.5	480.9	806.8	1625.7	36.9
1957.	5596.9	2984.2	5333.0	537.8	866.4	1721.1	43.4
1958.	5692.7	3409.1	5681.4	630.4	991.6	1869.0	52.2
1959.	5839.6	3804.3	6016.0	726.0	1155.0	1983.9	62.3
1960.	6041.9	4360.5	6409.7	880.9	1259.4	2162.4	74.9
1961.	6135.8	5141.1	6920.7	1122.6	1535.8	2508.6	91.1
1962.	6187.9	6082.6	7512.2	1455.4	1981.7	3049.5	112.2
1963.	6222.5	7048.6	8119.8	1823.9	2429.6	3605.1	134.8
1964.	6353.4	8067.2	8766.9	2172.3	2905.8	4168.5	158.3
1965.	6458.2	9392.1	9326.7	2583.6	3535.4	4785.4	186.0
1966.	6672.2	10506.3	9886.4	2962.8	4069.6	5315.1	210.3
1967.	6831.3	11662.4	10543.8	3427.1	4762.0	5943.8	239.8
1968.	6917.2	13073.1	11316.7	4170.2	5586.3	6918.0	278.8
1969.	6987.4	14660.7	12225.2	5244.0	6630.5	8182.1	331.1
1970.	7092.7	16533.7	13285.5	6636.6	7808.9	9782.1	395.4
1971.	7145.5	18788.4	14329.3	8519.7	8991.9	11725.2	469.6
1972.	7218.7	20983.2	15465.1	10420.7	9831.9	13388.0	531.6
1973.	7426.5	22925.6	16878.9	12626.7	10625.6	14752.1	598.1
1974.	7270.4	24973.9	18417.5	15590.3	11124.0	16304.1	675.8
1975.	7098.4	27308.4	19861.4	17892.8	11005.8	17442.6	709.1
1976.	7307.3	29646.3	21283.7	19645.8	10952.3	18106.1	733.9
1977.	7464.2	31721.0	22689.1	21525.7	10798.0	18783.3	763.1
1978.	7559.4	33557.1	24125.8	23639.0	10749.9	19368.1	793.9
1979.	7657.8	35215.8	25603.6	26363.3	11360.8	20008.4	841.3
1980.	7762.8	37348.1	27027.2	29033.5	12057.7	20785.5	890.5
1981.	7876.0	39652.0	28411.9	31578.5	12789.7	21567.2	924.1
1982.	7965.5	41912.3	29828.6	34519.9	13320.1	22398.3	949.5
1983.	8180.1	43981.9	31299.3	37204.0	13474.5	23292.7	970.6
1984.	8223.6	45991.9	32455.2	40234.3	13748.0	24089.7	976.0
1985.	8318.5	48143.8	33422.8	44910.3	14151.3	25256.8	994.3
1986.	8418.0	50486.4	34416.4	51172.9	15005.2	26719.0	1056.3
1987.	8542.5	53073.2	35468.2	57817.5	15904.7	28076.5	1111.4
1988.	8727.5	55436.0	36458.3	64965.8	17129.1	29306.4	1176.1
1989.	8871.8	58072.9	37800.1	73686.3	18331.6	31161.2	1259.4

Table A7: Market Sector Primary Inputs 1-7 for Japan, 1955-2003

1990.	8956.5	61539.5	39425.8	82767.1	19723.0	33518.7	1386.4
1991.	9033.3	65276.5	41401.7	92217.5	21329.0	36450.0	1406.0
1992.	8982.6	69257.5	43571.2	101689.6	22335.1	39179.4	1602.2
1993.	8905.0	72289.3	45780.8	108653.4	23050.0	41003.8	1751.3
1994.	8906.7	74111.1	48088.4	113539.1	23316.7	41693.3	1762.4
1995.	8969.5	75393.4	50225.7	117690.5	23429.7	41908.7	1739.5
1996.	8986.3	76097.0	52237.6	124246.3	24100.5	42482.4	1764.1
1997.	8950.7	76210.3	54177.9	135676.2	24930.3	43307.6	1885.4
1998.	8856.8	77558.3	55945.8	149024.9	25422.1	44354.7	2030.2
1999.	8785.1	78947.8	57637.1	159025.7	25330.0	44899.9	2337.8
2000.	8918.3	79868.2	59119.4	169046.3	24981.1	45112.4	2622.2
2001.	8763.2	81330.7	60577.3	181944.8	24780.2	45600.7	2841.9
2002.	8659.3	82636.1	61920.8	192713.6	24955.3	46004.4	3075.0
2003.	8622.9	83749.2	63092.4	194390.7	25138.6	45748.1	3250.1

Table A8: Market Sector Primary Inputs 8-14 for Japan, 1955-2003

Year	X 8	X9	X ₁₀	X ₁₁	X ₁₂	X ₁₃	X14
1955.	1.9	2850.1	569.3	1144.9	11355.5	4453.6	2664.6
1956.	2.1	3054.3	593.5	1230.7	11493.2	4501.3	2692.1
1957.	2.3	3169.0	736.1	1437.1	11554.3	4545.1	2719.8
1958.	2.5	3488.7	873.9	1848.0	11592.6	4598.4	2752.1
1959.	2.6	3740.6	773.7	1839.1	11609.8	4695.1	2784.7
1960.	2.5	3862.8	851.9	2037.9	11605.9	4717.9	2817.8
1961.	2.7	4080.6	913.0	2037.7	11634.6	4890.4	2847.3
1962.	2.6	4536.9	1078.3	2353.0	11625.1	5064.8	2877.0
1963.	2.7	4598.5	1068.2	2267.9	11584.9	5207.8	2907.1
1964.	2.7	4866.8	1134.2	2469.7	11550.5	5352.1	2937.5
1965.	2.8	5284.6	1278.8	2634.5	11477.9	5569.9	2968.2
1966.	2.9	5648.9	1248.0	2612.1	11462.6	6227.1	2999.2
1967.	3.1	6100.1	1341.9	2672.3	11351.7	6896.6	3030.6
1968.	3.2	6747.1	1570.3	2957.3	11273.3	7088.3	3062.3
1969.	3.5	7636.8	1683.6	3115.9	11187.3	7752.9	3094.3
1970.	3.8	8234.5	1920.9	3445.6	11080.2	7938.1	3126.6
1971.	4.0	8874.8	2210.3	3750.8	10975.1	8099.1	3157.9
1972.	4.4	9000.3	2205.9	3706.3	10864.2	8278.3	3189.5
1973.	5.1	8858.5	2098.4	3212.2	10795.4	8956.3	3221.4
1974.	5.7	8845.6	2359.6	2826.8	10734.2	9414.5	3253.7
1975.	5.9	10423.0	2747.4	3299.9	10652.0	9870.7	3286.2
1976.	6.3	10163.2	2591.7	3173.0	10583.2	10155.1	3319.1
1977.	6.3	10503.4	2672.9	3406.6	10543.0	10791.7	3352.3
1978.	6.3	10463.1	2576.1	3848.4	10502.9	11180.2	3385.9
1979.	6.3	10539.9	2309.5	3020.2	10464.7	11455.2	3419.8
1980.	6.0	11378.5	2376.2	2846.3	10439.8	11803.0	3454.0
1981.	5.6	11912.7	2769.9	3094.1	10403.5	11921.3	3474.9
1982.	5.5	11940.2	3697.0	2858.7	10372.9	12198.2	3496.0

5 2 12240 0	1100 7	2008.2	10244.2	12414.0	3517.1
5.3 11927.2	5050.9	3043.3	10315.5	12406.5	3538.4
4.9 11943.8	5908.6	3132.4	10283.0	12900.0	3559.9
4.7 12098.2	6654.4	4336.8	10242.9	12888.2	3581.4
4.5 11816.4	7122.9	3938.1	10208.5	13149.5	3603.1
4.3 11875.1	7747.9	4081.3	10164.5	13554.3	3625.0
4.2 12020.5	8307.9	3885.3	10091.9	13784.2	3646.9
4.0 12723.4	8940.5	3869.4	10023.1	14021.1	3669.0
3.7 13373.7	9707.6	4495.6	9948.5	14013.9	3659.5
3.5 13534.1	9809.7	4379.3	9873.9	14338.5	3650.0
3.4 13427.6	9808.0	4275.7	9795.6	14651.8	3640.5
3.5 13177.9	9289.2	3897.4	9717.2	14996.3	3631.1
3.6 13072.5	9079.0	3496.9	9631.2	15100.6	3621.7
3.8 13261.4	8889.8	3250.7	9547.0	15340.3	3612.3
3.9 13587.7	8801.2	3215.1	9461.0	15572.6	3602.9
3.9 14113.2	8683.2	3733.3	9376.9	15820.1	3593.6
3.8 13686.8	8141.3	3738.2	9302.3	15835.6	3584.2
3.6 12623.1	8159.0	3262.6	9233.5	16075.7	3574.9
3.3 12602.4	8107.5	3165.9	9164.7	16324.0	3565.7
3.1 12718.5	7279.9	3113.9	9103.5	16790.5	3556.4
3.0 12535.2	6878.6	2932.0	9053.8	17468.0	3547.2
	 4.7 12098.2 4.5 11816.4 4.3 11875.1 4.2 12020.5 4.0 12723.4 3.7 13373.7 3.5 13534.1 3.4 13427.6 3.5 13177.9 3.6 13072.5 3.8 13261.4 3.9 14113.2 3.8 13686.8 3.6 12623.1 3.3 12602.4 3.1 12718.5 	5.3 11927.2 5050.9 4.9 11943.8 5908.6 4.7 12098.2 6654.4 4.5 11816.4 7122.9 4.3 11875.1 7747.9 4.2 12020.5 8307.9 4.0 12723.4 8940.5 3.7 13373.7 9707.6 3.5 13534.1 9809.7 3.4 13427.6 9808.0 3.5 13177.9 9289.2 3.6 13072.5 9079.0 3.8 13261.4 8889.8 3.9 14113.2 8683.2 3.8 13686.8 8141.3 3.6 12623.1 8159.0 3.3 12602.4 8107.5 3.1 12718.5 7279.9	5.3 11927.2 5050.9 3043.3 4.9 11943.8 5908.6 3132.4 4.7 12098.2 6654.4 4336.8 4.5 11816.4 7122.9 3938.1 4.3 11875.1 7747.9 4081.3 4.2 12020.5 8307.9 3885.3 4.0 12723.4 8940.5 3869.4 3.7 13373.7 9707.6 4495.6 3.5 13534.1 9809.7 4379.3 3.4 13427.6 9808.0 4275.7 3.5 13177.9 9289.2 3897.4 3.6 13072.5 9079.0 3496.9 3.8 13261.4 8889.8 3250.7 3.9 14113.2 8683.2 3733.3 3.8 13686.8 8141.3 3738.2 3.6 12623.1 8159.0 3262.6 3.3 12602.4 8107.5 3165.9 3.1 12718.5 7279.9 3113.9	5.3 11927.2 5050.9 3043.3 10315.5 4.9 11943.8 5908.6 3132.4 10283.0 4.7 12098.2 6654.4 4336.8 10242.9 4.5 11816.4 7122.9 3938.1 10208.5 4.3 11875.1 7747.9 4081.3 10164.5 4.2 12020.5 8307.9 3885.3 10091.9 4.0 12723.4 8940.5 3869.4 10023.1 3.7 13373.7 9707.6 4495.6 9948.5 3.5 13534.1 9809.7 4379.3 9873.9 3.4 13427.6 9808.0 4275.7 9795.6 3.5 13177.9 9289.2 3897.4 9717.2 3.6 13072.5 9079.0 3496.9 9631.2 3.8 13261.4 8889.8 3250.7 9547.0 3.9 13587.7 8801.2 3215.1 9461.0 3.9 14113.2 8683.2 3733.3 9376.9 3.8 13686.8 8141.3 3738.2 9302.3 3.6 12623.1 8159.0 3262.6 9233.5 3.3 12602.4 8107.5 3165.9 9164.7 3.1 12718.5 7279.9 3113.9 9103.5	5.3 11927.2 5050.9 3043.3 10315.5 12406.5 4.9 11943.8 5908.6 3132.4 10283.0 12900.0 4.7 12098.2 6654.4 4336.8 10242.9 12888.2 4.5 11816.4 7122.9 3938.1 10208.5 13149.5 4.3 11875.1 7747.9 4081.3 10164.5 13554.3 4.2 12020.5 8307.9 3885.3 10091.9 13784.2 4.0 12723.4 8940.5 3869.4 10023.1 14021.1 3.7 13373.7 9707.6 4495.6 9948.5 14013.9 3.5 13534.1 9809.7 4379.3 9873.9 14338.5 3.4 13427.6 9808.0 4275.7 9795.6 14651.8 3.5 13177.9 9289.2 3897.4 9717.2 14996.3 3.6 13072.5 9079.0 3496.9 9631.2 15100.6 3.8 13261.4 8889.8 3250.7 9547.0 15340.3 3.9 13587.7 8801.2 3215.1 9461.0 15572.6 3.9 14113.2 8683.2 3733.3 9376.9 15820.1 3.8 13686.8 8141.3 3738.2 9302.3 15835.6 3.6 12623.1 8159.0 3262.6 9233.5 16075.7 3.3 12602.4 8107.5 3165.9 9164.7 16324.0 3.1 12718.5 7279.9 </td

Note that the three types of inventory stock, x_9-x_{11} , peaked around 1991-92 and then declined fairly steadily to the end of the sample period. These declines no doubt reflect Japanese just-in-time inventory delivery innovations, made possible by the development of new IT products.

We now list the components that are used to construct the nominal user costs $U_m^t = (r^t + \delta_m + \tau_m^t) W_m^t/(1 + r^t)$ for m = 2, 3, ..., 14, where r^t is the year t (balancing) real interest rate, δ_m is the constant geometric depreciation rate for asset m, τ_m^t is the year t capital tax rate on asset m and W_m^t is the year t stock price for asset m listed in Tables A7 and A8 above.

The depreciation rates for assets 2-8 are $\delta_2 = 0.0415$, $\delta_3 = 0.0250$, $\delta_4 = 0.1500$, $\delta_5 = 0.2000$, $\delta_6 = 0.1300$, $\delta_7 = 0.3300$ and $\delta_8 = 0.5500$. The depreciation rates for the remaining 3 inventory and 3 land assets are assumed to be zero; i.e., $\delta_m = 0$ for m = 9,10, ..., 14.

The balancing real interest rates that make the value of inputs equal to the value of outputs in each year r^t and the capital tax rates τ_2^t and τ_3^t are listed in Table A9 below. The tax rates for the remaining assets are all equal to τ_3^t ; i.e., we have $\tau_m^t = \tau_3^t$ for m = 4,5, ..., 14.

Table A9: Real Interest Rates and Capital Taxation Rates for Japan, 1955-2003

Year r^t τ_2^t τ_3^t

1955.	0.02557	0.05467	0.01014
1956.	0.02993	0.05244	0.01161
1957.	0.03838	0.05359	0.01316
1958.	0.02170	0.05438	0.01201
1959.	0.02837	0.05292	0.01231
1960.	0.03667	0.05292	0.01465
1961.	0.04748	0.04937	0.01556
1962.	0.03219	0.04609	0.01467
1963.	0.03843	0.04382	0.01394
1964.	0.04473	0.04212	0.01374
1965.	0.03428	0.03869	0.01225
1966.	0.03974	0.03637	0.01150
1967.	0.04413	0.03593	0.01208
1968.	0.05125	0.03654	0.01292
1969.	0.05083	0.03693	0.01384
1970.	0.04586	0.03744	0.01473
1971.	0.02824	0.03750	0.01392
1972.	0.02054	0.03769	0.01426
1973.	0.01648	0.03768	0.01592
1974.	0.01490	0.03551	0.01503
1975.	-0.00121	0.03412	0.01182
1976.	0.00516	0.03389	0.01102
1977.	0.00182	0.03540	0.01223
1978.	-0.00162	0.03750	0.01225
1979.	0.00430	0.03662	0.01437
1980.	0.00430	0.03588	0.01430
1981.	0.01010	0.03599	0.01455
1982.	0.01367	0.03659	0.01435
1983.	0.01303	0.03037	0.01430
1985.	0.00341	0.03936	0.01447
1985.	0.01218	0.03930	0.01379
1985.	0.01979	0.04173	0.01721
1980. 1987.	0.01289	0.04472	0.01897
1987. 1988.	0.01334	0.04697	0.02121
1988. 1989.	0.01322	0.04789	0.02228
	0.01346		0.02228
1990.		0.04479 0.04276	
1991.	0.01085		0.01873
1992.	0.01098	0.03992	0.01550
1993.	0.00705	0.03879	0.01398
1994.	0.00896	0.03769	0.01219
1995.	0.00751	0.03912	0.01272
1996.	0.01021	0.04081	0.01347
1997.	0.01088	0.04049	0.01308
1998.	0.00566	0.03966	0.01169
1999.	0.00517	0.03912	0.01055
2000.	0.00823	0.03879	0.01106

2001.0.003730.038610.011092002.0.003360.038290.010822003.0.005990.037160.01003

We conclude this Appendix with a note on the Japanese data. A major problem that we encountered was that data from different sources often were not consistent with each other. Hence we were forced to make many guesses as to what the "truth" might be and so there is an unknown amount of measurement error in our data. A complete documentation of our data construction is available upon request.⁵¹

It seems to us that the present Japanese statistical system is too decentralized: approximately 50 separate agencies contribute to the construction of the Japanese national accounts. Great efficiencies could be achieved by centralizing these resources into a Statistics Japan.⁵²

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⁵¹ Please email: <u>diewert@econ.ubc.ca</u> for the various data files.

⁵² Suitable target agencies are the Australian Bureau of Statistics, Statistics Canada and Statistics Netherlands. Also, Japanese government employees are frequently rotated out of their jobs, which may be a wise strategy in general but is not recommended for workers in Statistical Agencies, where it may take years to train an employee to a high level of effectiveness.

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