An Economic Approach to the Measurement of Productivity Growth Using Differences Instead of Ratios

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Discussion Paper 09-03
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Revised May 12, 2009

Abstract

Traditional index number theory decomposes a value ratio into the product of a price index times a quantity index. Growth accounting is based on this traditional approach to index number theory. This paper takes an alternative approach which decomposes a value difference into the sum of a price difference plus a quantity difference. We apply this new exact difference methodology in order to decompose the growth of new measure of labour productivity into additive explanatory factors. The new measure of labour productivity takes into account changes in the terms of trade. We apply our methodology to investigate the growth in living standards per unit of labour for the Japanese economy over the years 1955-2006. The paper also introduces a new flexible functional form for a GDP function that is based on the normalized quadratic functional form pioneered by Diewert and Wales.

Journal of Economic Literature Numbers

C14, D24, O47, O53

1. Introduction

The recent boom in the prices of natural resources and the low prices of manufactured goods produced by some developing countries has stimulated interest in the effects of changes in the prices of exports and imports on living standards for a country. An improvement in a country's terms of trade has much the same effect as an improvement in a country's productivity growth. Diewert (1983), Diewert and Morrison (1986), Morrison and Diewert (1990), Kohli (1990) (1991) (2003) (2004a) (2004b) (2006) (2007), Diewert, Mizobuchi and Nomura (2005) and Diewert and Lawrence (2006) have all developed production theory methodologies which enable one to obtain exact index number estimates of the contributions of productivity growth and changes in a country's terms of trade. The present paper is yet another contribution to this exact index number literature.

[1] The authors gratefully acknowledge the financial support of the SSHRC and the Australian Research Council (DP0559033).

Many observers use labour productivity (real output divided by labour input) as an approximate welfare measure. However, improvements in a country's terms of trade (an increase in the price of exports relative to the price of imports) do not show up in labour productivity measures, because the effects of changes in output and intermediate input prices are removed from the measure of output growth. The primary contribution of the present paper is a proposed improved measure of labour productivity that will allow us to assess the relative contributions to welfare of an improvement in Total Factor Productivity and of changes in real international prices. Our proposed approximate welfare measure is equal to the nominal income generated by the market sector of the economy divided by the product of the price of consumption times the quantity of labour input.[2] This proposed welfare measure can be modeled using production theory and exact index number techniques. This approach will be implemented for the Japanese business sector in section 5 of the paper. The main determinants of growth for this measure of approximate welfare are:

- Technical progress or improvements in the Total Factor Productivity of the market sector of the economy;
- Changes in domestic output prices or the prices of internationally traded goods and services relative to the price of consumption; and
- The effects of capital deepening; i.e., growth in market sector capital input relative to the growth of market sector labour input.

Section 2 introduces the market sector nominal output function. With a constant returns to scale technology, the value of output is distributed to the primary inputs that produced the market sector outputs. Thus the nominal output function can also be interpreted as a nominal income function. In section 3, this income function is used to provide theoretical definitions of the effects of real output price and relative input quantity changes on deflated market sector real income. A formal definition of productivity change is also provided. Section 4 introduces the normalized quadratic income function. Using this functional form to represent the technology of the market sector in each period enables us to obtain empirically observable exact measures of the effects of real output price and relative input quantity changes on deflated market sector real income. Appendix A shows that this functional form is a flexible functional form.

It turns out that our empirically observable measures of price, quantity and productivity change are equal to measures of price and quantity change that were originally suggested by Bennet (1920). Thus our paper is also a contribution to the recent literature on the Bennet indicators of price and quantity change.[3]

[2] The analysis presented in sections 2-4 below is somewhat more general. Instead of deflating nominal income by a single price, we deflate by a fixed weight price index of outputs and instead of deflating nominal income by labour input, we deflate by a fixed weight quantity index of inputs. However, in our empirical work in section 5, we will specialize these indexes as indicated.

[3] Our results in section 4 are similar in part to the results obtained by Balk, Färe and Grosskopf (2004). See Diewert (1992) (2005), Chambers (2001) (2002) and Balk (2003) (2007) for additional material on the Bennet indicators of price and quantity change.

In section 4, our decomposition of real income per unit labour input change into explanatory factors largely parallels the corresponding decomposition of nominal income change obtained by Diewert and Morrison (1986) and Kohli (1990), who derived exact results using the Translog functional form. An advantage of the present approach over the Translog approach is that our present approach is valid even if some individual prices or quantities are zero whereas the Translog approach fails if an exogenous price or quantity approaches zero. Since zero prices and quantities do occur empirically, applied welfare economists may find our present approach useful in these situations. Another advantage of our suggested difference approach is that it is valid even if value subaggregates (such as net exports or inventory change) change sign over the two periods being compared where as the traditional ratio approach to index number theory breaks down under these conditions.

Section 5 applies our methodology based on the difference approach to the analysis of the market sector in Japanese economy. We decompose changes in the (gross) real income per unit of labour and the net real income of the market sector per unit of labour input for the years 1955-2006. We found that productivity growth and the growth in capital services were the two main contributors to the growth in real income per unit of labour and net real income per unit of labour. It is also shown that changes in terms of trade had smaller effects on them on average.

2. The Production Theory Framework

In this section, we outline the economic approach to production theory which will be used in the remainder of the paper.[4] The main reference is Diewert and Morrison (1986).[5] The economic approach to production theory relies on the assumption of (competitive) *optimizing behavior* on the part of producers. In our empirical work, we will apply the economic approach to the market sector of the Japanese economy. Thus we will consider only that part of the Japanese economy that is motivated by profit maximizing behavior.[6]

We assume that the market sector of the economy produces quantities of M (net)[7] outputs, $Y \equiv [Y_1,...,Y_M]$, which are sold at the positive producer prices $P \equiv [P_1,...,P_M]$. We further assume that the market sector of the economy uses positive quantities of N

[4] This material is drawn from Diewert, Mizobuchi and Nomura (2005) and Diewert and Lawrence (2006).
[5] The theory also draws on Samuelson (1953), Fisher and Shell (1972), Diewert (1974; 133-141) (1980) (1983; 1077-1100), Archibald (1977), Fox and Kohli (1998), Kohli (1978) (1990) (1991) (2003) (2004a) (2004b) (2006) (2007) and Morrison and Diewert (1990).
[6] The Japanese business sector excludes all of the general government sectors such as schools, hospitals, universities, defence and public administration where no independent measures of output can be obtained. For owner occupied housing, output is equal to input and hence no productivity improvements can be generated by this sector according to SNA conventions. However, we do include the consumption of residential housing services in our model.
[7] If the mth commodity is an import (or other produced input) into the market sector of the economy, then the corresponding quantity y_m is indexed with a negative sign. We will follow Kohli (1978) (1991) and Woodland (1982) in assuming that imports flow through the domestic production sector and are "transformed" (perhaps only by adding transportation, wholesaling and retailing margins) by the domestic production sector. The recent textbook by Feenstra (2004; 76) also uses this approach.

primary inputs, $X \equiv [X_1,...,X_N]$ which are purchased at the positive primary input prices $W \equiv [W_1,...,W_N]$. In period t, we assume that there is a feasible set of output vectors Y that can be produced by the market sector if the vector of primary inputs X is utilized by the market sector of the economy; denote this period t production possibilities set by S^t. We assume that S^t is a closed convex cone that exhibits a free disposal property.[8]

Given a vector of output prices P and a vector of available primary inputs X, we define *the period t market sector income function*, $g^t(P,X)$, as follows:[9]

$$(1)\ g^t(P,X) \equiv \max_Y \{P \cdot Y : (Y,X) \text{ belongs to } S^t\} ; \qquad t = 0,1,2, \ldots .$$

Thus market sector nominal income depends on t (which represents the period t technology set S^t), on the vector of output prices P that the market sector faces and on X, the vector of primary inputs that is available to the market sector.

If P^t is the period t output price vector and X^t is the vector of inputs used by the market sector during period t and if the income function is differentiable with respect to the components of P at the point P^t,X^t, then the period t vector of market sector outputs Y^t will be equal to the vector of first order partial derivatives of $g^t(P^t,X^t)$ with respect to the components of P; i.e., we will have the following equations for each period t:[10]

$$(2)\ Y^t = \nabla_P\, g^t(P^t,X^t) ; \qquad t = 0,1,2, \ldots .$$

Thus the period t market sector supply vector Y^t can be obtained by differentiating the period t market sector income function with respect to the components of the period t output price vector P^t.

If the income function is differentiable with respect to the components of X at the point P^t,X^t, then the period t vector of input prices W^t will be equal to the vector of first order

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[8] For a more explanation for the meaning of these properties, see Diewert (1973) (1974; 134) or Woodland (1982) or Kohli (1978) (1991). The assumption that S^t is a cone means that the technology is subject to constant returns to scale. This is an important assumption since it implies that the value of outputs should equal the value of inputs in equilibrium. In our empirical work, we use an ex post rate of return in our user costs of capital, which forces the value of inputs to equal the value of outputs for each period.

[9] The function g^t is known as the *GDP function* or the *national product function* in the international trade literature (see Kohli (1978)(1991), Woodland (1982) and Feenstra (2004; 76)). It was introduced into the economics literature by Samuelson (1953). Alternative terms for this function include: (i) the *gross profit function*; see Gorman (1968); (ii) the *restricted profit function*; see Lau (1976) and McFadden (1978); and (iii) the *variable profit function*; see Diewert (1973) (1974). However, we will call it the (nominal) income function, since it also defines the amount of income that is distributed to the vector of primary inputs that is used by the market sector. The function $g^t(P,X)$ will be linearly homogeneous and convex in the components of P and linearly homogeneous and concave in the components of X; see Diewert (1973) (1974; 136). Notation: $P \cdot Y \equiv \sum_{m=1}^{M} P_m Y_m$.

[10] These relationships are due to Hotelling (1932; 594). Note that $\nabla_P g^t(P^t,X^t) \equiv [\partial g^t(P^t,X^t)/\partial P_1, \ldots, \partial g^t(P^t,x^t)/\partial P_M]$.

partial derivatives of $g^t(P^t,X^t)$ with respect to the components of X; i.e., we will have the following equations for each period t:[11]

(3) $W^t = \nabla_X g^t(P^t,X^t)$; $\quad t = 0,1,2, \ldots$.

Thus the period t market sector input prices W^t paid to primary inputs can be obtained by differentiating the period t market sector income function with respect to the components of the period t input quantity vector X^t.

The constant returns to scale assumption on the technology sets S^t implies that the value of outputs will equal the value of inputs in period t; i.e., we have the following relationships:

(4) $g^t(P^t,X^t) = P^t \cdot Y^t = W^t \cdot X^t$; $\quad t = 0,1,2, \ldots$.

The above material will be useful in what follows. Note that our focus is not on the value of outputs generated by the market sector; instead our focus is on the amount of nominal income generated by the market sector. Since the value of market sector production is distributed to the factors of production used by the market sector, nominal market sector output will be equal to nominal market sector income; i.e., from (4), we have $g(P^t,X^t) = P^t \cdot Y^t = W^t \cdot X^t$. As an approximate welfare measure that can be associated with market sector production,[12] we will choose to measure the *real income generated by the market sector in period t*, r^t, in terms of the number of fixed basket consumption bundles (with weights represented by the nonnegative, nonzero vector $\mu > 0_M$) that the nominal income generated by the market sector could purchase in period t; i.e., we define r^t as follows:

(5) $r^t \equiv W^t \cdot X^t / P^t \cdot \mu$ $\quad t = 0,1,2, \ldots$
$= (1/P^t \cdot \mu) g^t(P^t,X^t)$ $\quad$ using (4)
$= g^t(P^t/P^t \cdot \mu, X^t)$ $\quad$ using the linear homogeneity of $g^t(P,X)$ in P
$= g^t(p^t,X^t)$ $\quad$ using definition (6) below
$= p^t \cdot Y^t$ $\quad$ using (4) and (6)

where $P^t \cdot \mu > 0$ is a *period t consumption expenditures deflator*[13] and the market sector period t *real output price* p^t and *real input price* w^t vectors are defined as the

[11] These relationships are due to Samuelson (1953) and Diewert (1974; 140). Note that $\nabla_X g^t(P^t,X^t) \equiv [\partial g^t(P^t,X^t)/\partial X_1, \ldots, \partial g^t(P^t,X^t)/\partial X_N]$.

[12] Since some of the primary inputs used by the market sector can be owned by foreigners, our measure of *domestic* welfare generated by the market production sector is only an approximate one. Moreover, our suggested welfare measure is not sensitive to the distribution of the income that is generated by the market sector.

[13] In our empirical work, we will form 7 subaggregates of Japanese net outputs where the first subaggregate is consumption. We will choose our 7 dimensional μ vector to be the first unit vector. Thus we simply deflate the period t P^t and W^t price vectors by $P_1^{\,t}$, the price of consumption in period t.

corresponding nominal price vectors deflated by the consumption expenditures deflator; i.e., we have the following definitions:[14]

(6) $p^t \equiv P^t/P^t \cdot \mu$; $w^t \equiv W^t/P^t \cdot \mu$; $t = 0,1,2, \ldots$.

The first and last equality in (5) imply that period t real income, r^t, is equal to the period t income function, evaluated at the period t real output price vector p^t and the period t input vector X^t, $G^t(p^t,X^t)$. Thus *the growth in real income over time can be explained by three main factors: t or technical progress or Total Factor Productivity growth* (shifts in g^t)*, growth in real output prices* (changes in p^t) *and growth in primary input quantities* (changes in X^t).

However, rather than find an exact decomposition for the change in real income over time into explanatory factors, the methodology to be developed in the following section only allows us to find an exact decomposition for the change in real income divided by an index of primary inputs. Thus define the *real income generated by the market sector in period t per unit primary input*, ρ^t, as our previous real income measure r^t divided by a period t index of primary inputs used, $X^t \cdot \theta$ (with weights represented by the nonnegative, nonzero vector $\theta > 0_N$); i.e., define ρ^t as follows:

$$
\begin{aligned}
(7)\ \rho^t &\equiv W^t \cdot X^t/[(P^t \cdot \mu)(X^t \cdot \theta)] && t = 0,1,2, \ldots \\
&= (W^t/P^t \cdot \mu) \cdot (X^t/ X^t \cdot \theta) && \text{rearranging terms} \\
&= w^t \cdot x^t && \text{using definitions (6) and (8)} \\
&= p^t \cdot y^t && \text{using (5) and (8)} \\
&= g^t(p^t,x^t)
\end{aligned}
$$

where the last equality follows using (4) and the linearly homogeneity of $g^t(P,X)$ in both P and X and where the market sector period t *deflated output quantity and input quantity* vectors y^t and x^t are defined as the corresponding quantity vectors Y^t and X^t deflated by the primary input index deflator $X^t \cdot \theta$ [15]; i.e., we have the following definitions:

(8) $y^t \equiv Y^t/X^t \cdot \theta$; $x^t \equiv X^t/X^t \cdot \theta$; $t = 0,1,2, \ldots$.

[14] Our approach to measuring real income is similar to the approach advocated by Kohli (2004b; 92), except he essentially deflates nominal GDP by the domestic expenditures deflator rather than just the domestic (household) expenditures deflator; i.e., he deflates by the deflator for C+G+I, whereas we suggest deflating by the deflator for C. Another difference in his approach compared to the present approach is that we restrict our analysis to the market sector GDP, whereas Kohli deflates all of GDP (probably due to data limitations). Our treatment of the balance of trade surplus or deficit is also different.

[15] In our empirical work, we will form the following aggregates of Japanese primary inputs: K (capital services), KIV (inventory services), LD (land services) L (labour input). We will choose our 4 dimensional θ vector to be the first unit vector. Thus we simply deflate the period t Y^t and X^t quantity vectors by $X_L{}^t$, the quantity of labour in period t. We regard the resulting "welfare" measure ρ^t defined by (7) as *an improved measure of labour productivity* since it takes into account changes in the prices of investment, exports and imports relative to the price of consumption. Traditional measures of labour productivity cannot take into account such price changes; in particular, they cannot take into account changes in the country's terms of trade.

Using the linear homogeneity properties of the income function $g^t(P,X)$ in P and X separately, we can show that the following counterparts to the relations (2) and (3) hold using the real prices p^t and w^t defined by (6) and the deflated quantities y^t and x^t defined by (8):[16]

$$(9)\quad y^t = \nabla_p g^t(p^t,x^t)\ ; \qquad t = 0,1,2, ...$$
$$(10)\quad w^t = \nabla_x g^t(p^t,x^t)\ ; \qquad t = 0,1,2, ...$$

In the following section, we will define various explanatory factors that will be used to explain the *change* in the real income generated by the market sector in period t per unit primary input over the previous period, $\rho^t - \rho^{t-1}$, into explanatory factors which are also differences. There will be three sets of explanatory factors that are associated with:

- Changes in real output prices p^t;
- Changes in deflated primary input quantities x^t and
- Changes in technology; i.e., shifts in the income functions g^t.

3. The Theoretical Explanation of Per Unit Primary Input Real Income Growth using Differences

Now we are ready to define a family of *period t productivity growth factors or technical progress shift factors* $\tau(p,x,t)$ using the difference approach as opposed to the usual ratio approach:[17]

$$(11)\quad \tau(p,x,t) \equiv g^t(p,x) - g^{t-1}(p,x)\ ; \qquad t = 1,2, ...$$

Thus $\tau(p,x,t)$ measures the change in the real income per unit primary input produced by the market sector at the reference real output prices p and reference deflated input quantities used by the market sector x where the first term in the right hand side of (11) uses the period t technology represented by g^t and the second term in (11) uses the period t−1 technology g^{t-1}. Thus each choice of reference vectors p and x will generate a possibly different measure of the shift in technology going from period t−1 to period t. Note that we are using the chain system to measure the shift in technology.

It is natural to choose special reference vectors for the measure of technical progress defined by (11): a *Laspeyres type measure* τ_L^t that chooses the period t−1 reference

[16] If producers in the market sector of the economy are solving the profit maximization problem that is associated with $g^t(P,X)$, which uses the original output prices P and the original primary input vector X, then they will also solve the profit maximization problem that uses the deflated output prices $p \equiv P/P\cdot\mu$ and the deflated primary input vector $x \equiv X/X\cdot\theta$; i.e., they will also solve the revenue maximization problem defined by $g^t(p,x)$.

[17] The corresponding ratio type measure, $\tau(p,x,t) \equiv g^t(p,x)/g^{t-1}(p,x)$ is due to Diewert and Morrison (1986; 662). A special case of it was defined earlier by Diewert (1983; 1063).

vectors p^{t-1} and x^{t-1} and a *Paasche type measure* $\tau_P{}^t$ that chooses the period t reference vectors p^t and x^t:[18]

$$(12)\ \tau_L{}^t \equiv \tau(p^{t-1},x^{t-1},t) = g^t(p^{t-1},x^{t-1}) - g^{t-1}(p^{t-1},x^{t-1})\ ; \qquad t = 1,2, \ldots ;$$
$$(13)\ \tau_P{}^t \equiv \tau(p^t,x^t,t) \quad = g^t(p^t,x^t) - g^{t-1}(p^t,x^t)\ ; \qquad t = 1,2, \ldots .$$

Since both measures of technical progress are equally valid, it is natural to average them to obtain an overall measure of technical change. If we want to treat the two measures in a symmetric manner and we want the measure to satisfy the time reversal property from the difference approach to index number theory[19] (so that the estimate going backwards is equal to the negative of the estimate going forwards), then the arithmetic mean will be the best simple average to take in this context. Thus we define the arithmetic mean of (12) and (13) as follows:

$$(14)\ \tau^t \equiv (1/2)[\tau_L{}^t + \tau_P{}^t]\ ; \qquad t = 1,2, \ldots .$$

At this point, it is not clear how we will obtain empirical estimates for the theoretical productivity growth indexes defined by (12)-(14). One obvious way would be to assume a functional form for the nominal income function $g^t(P,X)$, collect data on output and input prices and quantities for the market sector for a number of years, add error terms to equations (2) and (3) and use econometric techniques to estimate the unknown parameters in the assumed functional form. However, econometric techniques are generally not completely straightforward: different econometricians will make different stochastic specifications and will choose different functional forms.[20] Moreover, as the number of outputs and inputs grows, it will be impossible to estimate a flexible functional form. Thus in the following section, we will suggest methods for estimating productivity change measures like (14) that are based on exact index number techniques.

We turn now to the problem of defining theoretical indexes for the effects on real income per unit primary input due to changes in real output prices. Define a family of *period t real output price change factors* $\alpha(p^{t-1},p^t,x,s)$:[21]

$$(15)\ \alpha(p^{t-1},p^t,x,s) \equiv g^s(p^t,x) - g^s(p^{t-1},x)\ ; \qquad s = 1,2, \ldots .$$

[18] Diewert and Morrison (1986; 662-663) introduced the ratio counterparts to (12) and (13) in the nominal GDP context

[19] Diewert (2005; 366) developed the axiomatic approach to index number theory using differences and introduced this time reversal test, which is the counterpart to the usual time reversal test that can be found in Fisher (1922; 64). Balk (2003; 29) also emphasized the importance of a symmetric treatment of time. Balk (2007) further developed the axiomatic approach using differences.

[20] "The estimation of GDP functions such as (19) can be controversial, however, since it raises issues such as estimation technique and stochastic specification. ... We therefore prefer to opt for a more straightforward index number approach." Ulrich Kohli (2004a; 344).

[21] This measure of real output price change is the difference version of the usual ratio concept due to Fisher and Shell (1972; 56-58), Samuelson and Swamy (1974; 588-592), Archibald (1977; 60-61), Diewert (1980; 460-461) (1983; 1055) and Balk (1998; 83-89).

Thus $\alpha(p^{t-1},p^t,x,s)$ measures the difference in the real income per unit primary input produced by the market sector that is induced by the change in real output prices going from period t−1 to t, using the technology that is available during period s and using the reference input quantities x. Thus each choice of the reference technology s and the reference input vector x will generate a possibly different measure of the effect on real income per unit primary input of a change in real output prices going from period t−1 to period t.

Again, it is natural to choose special reference vectors for the measures defined by (15): a *Laspeyres type measure* $\alpha_L{}^t$ that chooses the period t−1 reference technology and reference input vector x^{t-1} and a *Paasche type measure* $\alpha_P{}^t$ that chooses the period t reference technology and reference input vector x^t:

(16) $\alpha_L{}^t \equiv \alpha(p^{t-1},p^t,x^{t-1},t-1) = g^{t-1}(p^t,x^{t-1}) - g^{t-1}(p^{t-1},x^{t-1})$; t = 1,2, ... ;
(17) $\alpha_P{}^t \equiv \alpha(p^{t-1},p^t,x^t,t) = g^t(p^t,x^t) - g^t(p^{t-1},x^t)$; t = 1,2, ...

Since both measures of real output price change are equally valid, it is natural to average them to obtain an overall measure of the effects on real income per unit of primary input of the change in real output prices:

(18) $\alpha^t \equiv (1/2)[\alpha_L{}^t + \alpha_P{}^t]$; t = 1,2, ...

Finally, we look at the problem of defining theoretical indexes for the effects on real income per unit of primary input due to growth in relative input quantities going from period t−1 to t. Define a family of *period t relative input quantity growth factors* $\beta(x^{t-1},x^t,p,s)$:

(19) $\beta(x^{t-1},x^t,p,s) \equiv g^s(p,x^t) - g^s(p,x^{t-1})$; s = 1,2, ...

Thus $\beta(x^{t-1},x^t,p,s)$ measures the difference in the real income per unit primary input produced by the market sector that is induced by the change in input quantities relative to the index of primary inputs used by the market sector going from period t−1 to t, using the technology that is available during period s and using the reference real output prices p. Thus each choice of the reference technology s and the reference real output price vector p will generate a possibly different measure of the effect on real income of a change in relative input quantities going from period t−1 to period t.

Again, it is natural to choose special reference vectors for the measures defined by (19): a *Laspeyres type measure* $\beta_L{}^t$ that chooses the period t−1 reference technology and reference real output price vector p^{t-1} and a *Paasche type measure* $\beta_P{}^t$ that chooses the period t reference technology and reference real output price vector p^t:

(20) $\beta_L{}^t \equiv \beta(x^{t-1},x^t,p^{t-1},t-1) = g^{t-1}(p^{t-1},x^t) - g^{t-1}(p^{t-1},x^{t-1})$; t = 1,2, ... ;
(21) $\beta_P{}^t \equiv \beta(x^{t-1},x^t,p^t,t) = g^t(p^t,x^t) - g^t(p^t,x^{t-1})$; t = 1,2, ...

Since both measures of (relative) input quantity change are equally valid, it is natural to average them to obtain an overall measure of the effects of relative input change on real income:

$$(22)\ \beta^t \equiv (1/2)[\beta_L{}^t + \beta_P{}^t] ; \qquad t = 1,2, ...$$

Recall that market sector real income for period t was defined by (5) as r^t which is equal to nominal period t factor payments $W^t \cdot X^t$ deflated by an index of fixed weight consumption prices, $P^t \cdot \mu$. Recall also that r^t was further deflated by the fixed weight index of primary inputs, $X^t \cdot \theta$, in order to obtain the period t real income per unit of primary input measure ρ^t defined by (7). Recall also that using the linear homogeneity properties of the income function $g^t(P,X)$ in P and X, we showed that ρ^t is equal to $g^t(p^t,x^t)$. It is convenient to define γ^t as the *absolute amount of growth in real income per unit primary input* going from period t−1 to t:

$$(23)\ \gamma^t \equiv \rho^t - \rho^{t-1} ; \qquad t = 1,2, ...$$

In the following section, we will show that under certain functional form assumptions on the income functions g^t, γ^t is exactly equal to the sum of τ^t, α^t and β^t defined above by (14), (18) and (22) respectively.

4. The Normalized Quadratic Income Function and Bennet Indicators of Price, Quantity and Productivity Change

Suppose that the period t nominal net revenue or income function g^t has the following *normalized quadratic functional form*:[22]

$$(24)\ g^t(P,X) \equiv a^t \cdot P\theta \cdot X + c^t \cdot X\mu \cdot P + (1/2)\ P \cdot AP\ [\theta \cdot X/\mu \cdot P] + P \cdot BX + (1/2)\ X \cdot CX\ [\mu \cdot P/\theta \cdot X] ; \qquad t = 0,1,2,...$$

where a^t and c^t are M and N dimensional vectors of unknown parameters which can be different for each time period t, $A = [a_{mk}]$ is an M by M positive semidefinite symmetric matrix of unknown parameters, $B = [b_{mn}]$ is an M by N matrix of unknown parameters, $C = [c_{ni}]$ is an N by N negative semidefinite symmetric matrix of unknown parameters and μ and θ are the same known vectors of parameters that appeared in definitions (6) and (8) above. Note that with our curvature restrictions on the matrices A and C, $g^t(P,X)$ will be convex (and linearly homogeneous) in the components of P and concave (and linearly homogeneous) in the components of X. Note that by allowing the parameter vectors a^t and c^t to change arbitrarily with time, we are allowing for very general forms of technical progress. However, the theory to be developed below does require that the parameter

[22] This functional form is a generalization to many primary inputs of the normalized quadratic unit profit function introduced by Diewert and Wales (1992; 707) which in turn is an adaptation of the normalized quadratic functional form used by Diewert and Wales (1987) (1988a) (1988b) in a variety of contexts. It is also a generalization of the normalized quadratic profit function introduced by Diewert and Ostensoe (1988; 44).

matrices A, B and C to be fixed over time. In Appendix 1 below, we show that the g^t defined by (24) is a flexible functional form.

Now evaluate (24) at the data for period t, P^t, X^t. Dividing both sides of the resulting equation by $\theta\cdot X^t$ times $\mu\cdot P^t$ gives us the following equation for period t real income per unit primary input, ρ^t:

$$(25)\ \rho^t = g^t(p^t,x^t) = a^t\cdot p^t + c^t\cdot x^t + (1/2)\, p^t\cdot Ap^t + p^t\cdot Bx^t + (1/2)\, x^t\cdot Cx^t\ ; \qquad t = 0,1,2,...$$

where $p^t \equiv P^t/\mu\cdot P^t$ and $x^t \equiv X^t/\theta\cdot P^t$.

Differentiating (24) with respect to the components of P and evaluating the resulting derivatives at the data pertaining to period t leads to the following equations using (2):

$$(26)\ Y^t = \nabla_P g^t(P^t,X^t) = a^t\theta\cdot X^t + c^t\cdot X^t\mu + AP^t[\theta\cdot X^t/\mu\cdot P^t] - (1/2)P^t\cdot AP^t\theta\cdot X^t[\mu\cdot P^t]^{-2}\mu + BX^t + (1/2)[\theta\cdot X^t]^{-1}X^t\cdot CX^t\,\mu\ ; \qquad t = 0,1,2,....$$

Now divide both sides of (26) by $\theta\cdot X^t$, define $p^t \equiv P^t/\mu\cdot P^t$, $x^t \equiv X^t/\theta\cdot X^t$ and $y^t \equiv Y^t/\theta\cdot X^t$ and equations (26) become the following equations:

$$(27)\ y^t = \nabla_P g^t(p^t,x^t) = a^t + c^t\cdot x^t\mu + Ap^t - (1/2)p^t\cdot Ap^t\mu + Bx^t + (1/2)x^t\cdot Cx^t\mu\ ; \qquad t = 0,1,2,....$$

Now premultiply both sides of equation t in (27) by the transpose of p^t. Using $p^t\cdot\mu = P^t\cdot\mu/\mu\cdot P^t = 1$, the resulting equations become:

$$(28)\ p^t\cdot y^t = a^t\cdot p^t + c^t\cdot x^t + (1/2)\, p^t\cdot Ap^t + p^t\cdot Bx^t + (1/2)\, x^t\cdot Cx^t = \rho^t = g^t(p^t,x^t)\ ; \qquad t = 0,1,2,....$$

Recall definition (16) for the Laspeyres period t real output price change factor, $\alpha_L{}^t$. Using the g^t functions defined by (25), we have:

$$(29)\ \alpha_L{}^t \equiv g^{t-1}(p^t,x^{t-1}) - g^{t-1}(p^{t-1},x^{t-1}) \qquad t = 1,2, ...$$
$$= [a^{t-1}\cdot p^t + c^{t-1}\cdot x^{t-1} + (1/2)\, p^t\cdot Ap^t + p^t\cdot Bx^{t-1} + (1/2)\, x^{t-1}\cdot Cx^{t-1}] - p^{t-1}\cdot y^{t-1}$$

where we have used definition (24) in order to evaluate $g^{t-1}(p^t,x^{t-1})$ and we have also used equation t−1 in (28) to obtain $g^{t-1}(p^{t-1},x^{t-1})$ equal to $p^{t-1}\cdot y^{t-1}$.

Similarly, using definition (17) for the Paasche period t real output price change factor, $\alpha_P{}^t$, we have:

$$(30)\ \alpha_P{}^t \equiv g^t(p^t,x^t) - g^t(p^{t-1},x^t) \qquad t = 1,2, ...$$
$$= p^t\cdot y^t - [a^t\cdot p^{t-1} + c^t\cdot x^t + (1/2)\, p^{t-1}\cdot Ap^{t-1} + p^{t-1}\cdot Bx^t + (1/2)\, x^t\cdot Cx^t].$$

Using (29) and (30), it can be seen that the sum of the above two real output price change factors is equal to the following expression:

$$(31)\ 2\alpha^t = \alpha_L{}^t + \alpha_P{}^t = p^t\cdot y^t - p^{t-1}\cdot y^{t-1} \qquad t = 1,2,\ldots$$
$$+ [a^{t-1}\cdot p^t + c^{t-1}\cdot x^{t-1} + (1/2)\, p^t\cdot Ap^t + p^t\cdot Bx^{t-1} + (1/2)\, x^{t-1}\cdot Cx^{t-1}]$$
$$- [a^t\cdot p^{t-1} + c^t\cdot x^t + (1/2)\, p^{t-1}\cdot Ap^{t-1} + p^{t-1}\cdot Bx^t + (1/2)\, x^t\cdot Cx^t].$$

Unfortunately, the expressions on the right hand sides of (29)-(31) are not observable without a knowledge of the unknown parameters in the g^t functions defined by (24). However, the *Bennet* (1920)[23] *indicator of real output price change*, $P^B(p^{t-1},p^t,y^{t-1},y^t)$, defined by (32) is empirically observable:

$$(32)\ P^B(p^{t-1},p^t,y^{t-1},y^t) \equiv (1/2)[y^{t-1} + y^t]\cdot[p^t - p^{t-1}]\ ; \qquad t = 1,2,\ldots$$

where the observable output quantity vectors (deflated by the index of primary inputs) $y^t \equiv Y^t/X^t\cdot\theta$ were defined earlier by (8) and the deflated output price vectors $p^t \equiv P^t/P^t\cdot\mu$ were defined earlier by (6).

We now show that if there is competitive net revenue maximizing behavior in the market sector in each period t and the market sector income functions g^t are defined by (24) above, then the Bennet indicator of real price change, P^B defined by (32), is exactly equal to α^t defined by (18), the arithmetic average of the Laspeyres and Paasche real output price change factors. Using definition (32), we have:

$$(33)\ 2P^B(p^{t-1},p^t,y^{t-1},y^t) = [y^{t-1} + y^t]\cdot[p^t - p^{t-1}]\ ; \qquad t = 1,2,\ldots$$
$$= p^t\cdot y^t - p^{t-1}\cdot y^{t-1} + p^t\cdot y^{t-1} - p^{t-1}\cdot y^t$$
$$= p^t\cdot y^t - p^{t-1}\cdot y^{t-1}$$
$$+ p^t\cdot[a^{t-1} + c^{t-1}\cdot x^{t-1}\mu + Ap^{t-1} - (1/2)p^{t-1}\cdot Ap^{t-1}\mu + Bx^{t-1} + (1/2)x^{t-1}\cdot Cx^{t-1}\mu]$$
$$- p^{t-1}\cdot[a^t + c^t\cdot x^t\mu + Ap^t - (1/2)p^t\cdot Ap^t\mu + Bx^t + (1/2)x^t\cdot Cx^t\mu] \qquad \text{using (27)}$$
$$= p^t\cdot y^t - p^{t-1}\cdot y^{t-1}$$
$$+ [p^t\cdot a^{t-1} + c^{t-1}\cdot x^{t-1} + p^t\cdot Ap^{t-1} - (1/2)p^{t-1}\cdot Ap^{t-1} + p^t\cdot Bx^{t-1} + (1/2)x^{t-1}\cdot Cx^{t-1}]$$
$$- [p^{t-1}\cdot a^t + c^t\cdot x^t + p^{t-1}\cdot Ap^t - (1/2)p^t\cdot Ap^t + p^{t-1}\cdot Bx^t + (1/2)x^t\cdot Cx^t] \qquad \text{using } p^t\cdot\mu = 1$$
$$= p^t\cdot y^t - p^{t-1}\cdot y^{t-1} + [a^{t-1}\cdot p^t + c^{t-1}\cdot x^{t-1} + (1/2)\, p^t\cdot Ap^t + p^t\cdot Bx^{t-1} + (1/2)\, x^{t-1}\cdot Cx^{t-1}]$$
$$- [a^t\cdot p^{t-1} + c^t\cdot x^t + (1/2)\, p^{t-1}\cdot Ap^{t-1} + p^{t-1}\cdot Bx^t + (1/2)\, x^t\cdot Cx^t] \qquad \text{simplifying}$$
$$= \alpha_L{}^t + \alpha_P{}^t \qquad \text{using (31)}$$
$$= 2\alpha^t \qquad \text{using definition (18).}$$

Thus under our assumptions on technology, the theoretical measure of period t per unit primary input market sector real income change due to change in real output prices, α^t

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[23] Bennet noticed that the value aggregate difference $p^t\cdot y^t - p^{t-1}\cdot y^{t-1}$ is exactly equal to the sum of the price change term, $P^B(p^{t-1},p^t,y^{t-1},y^t)$ defined by (32) and the corresponding quantity change term, $Q^B(p^{t-1},p^t,y^{t-1},y^t)$ defined as $(1/2)[p^{t-1} + p^t]\cdot[y^t - y^{t-1}]$. Diewert (1992) termed P^B and Q^B the Bennet *indicators* of price and quantity change for the value aggregate; i.e., he introduced the term *indicator* as the difference counterpart to the price and quantity *index* concepts in traditional ratio type index number theory. Diewert (2005) developed the axiomatic or test approach to price and quantity indicators and showed that the Bennet indicators were the difference counterparts to the Fisher price and quantity indexes in terms of their axiomatic properties. Balk (2007) also looked at the axiomatic properties of the Bennet indicators.

defined by (18), is exactly equal to the observable Bennet indicator of real price change, P^B defined by (32).[24]

The above analysis can be modified to give us an observable estimator for the theoretical measure of the effects of relative input quantity change on market sector real income per unit of primary input, β^t defined by (22) above. However, it is first necessary to differentiate the normalized quadratic functions $g^t(P,X)$ defined above by (24) with respect to the components of X and then use Samuelson's Lemma (3) in order to obtain expressions for the period t input price vectors W^t.

Differentiating (24) with respect to the components of X and evaluating the resulting derivatives at the data pertaining to period t leads to the following equations using (3):

$$(34)\ W^t = \nabla_X g^t(P^t,X^t) \qquad t = 0,1,2,...$$
$$= a^t\cdot P^t\theta + \mu\cdot P^t c^t + (1/2)[\mu\cdot P^t]^{-1}P^t\cdot AP^t\theta$$
$$+ B^T P^t + CX^t\,[\mu\cdot P^t/\theta\cdot X^t] - (1/2)[\theta\cdot X^t]^{-2}X^t\cdot CX^t\mu\cdot P^t\theta.$$

Now divide both sides of (34) by $\mu\cdot P^t$, define $p^t \equiv P^t/\mu\cdot P^t$, $x^t \equiv X^t/\theta\cdot X^t$ and $w^t \equiv W^t/\mu\cdot P^t$ and equations (34) become the following equations:

$$(35)\ w^t = a^t\cdot p^t\theta + c^t + (1/2)p^t\cdot Ap^t\theta + B^T p^t + Cx^t - (1/2)x^t\cdot Cx^t\theta\,; \qquad t = 0,1,2,...\,.$$

Now premultiply both sides of equation t in (35) by the transpose of x^t. Using $x^t\cdot\theta = X^t\cdot\theta/\theta\cdot X^t = 1$, the resulting equations become:

$$(36)\ w^t\cdot x^t = a^t\cdot p^t + c^t\cdot x^t + (1/2)\,p^t\cdot Ap^t + p^t\cdot Bx^t + (1/2)\,x^t\cdot Cx^t\,; \qquad t = 0,1,2,...$$
$$= g^t(p^t,x^t) \qquad \text{using (28).}$$

Now recall definition (20) for the Laspeyres period t relative input quantity growth factor, $\beta_L{}^t$. Using the g^t functions defined by (24), we have:

$$(37)\ \beta_L{}^t \equiv g^{t-1}(p^{t-1},x^t) - g^{t-1}(p^{t-1},x^{t-1}) \qquad t = 1,2,...$$
$$= [a^{t-1}\cdot p^{t-1} + c^{t-1}\cdot x^t + (1/2)\,p^{t-1}\cdot Ap^{t-1} + p^{t-1}\cdot Bx^t + (1/2)\,x^t\cdot Cx^t] - p^{t-1}\cdot y^{t-1}.$$

Similarly, using definition (21) for the Paasche period t relative input quantity growth factor, $\beta_P{}^t$, we have, using (25) and (28):

$$(38)\ \beta_P{}^t \equiv g^t(p^t,x^t) - g^t(p^t,x^{t-1}) \qquad t = 1,2,...$$
$$= p^t\cdot y^t - [a^t\cdot p^t + c^t\cdot x^{t-1} + (1/2)\,p^t\cdot Ap^t + p^t\cdot Bx^{t-1} + (1/2)\,x^{t-1}\cdot Cx^{t-1}].$$

[24] Equations (33) show that the theoretical measure of change in g^t due to changes in real output prices p^t, α^t defined by (18), is equal to $\sum_{m=1}^{M} (1/2)[y_m{}^{t-1} + y_m{}^t][p_m{}^t - p_m{}^{t-1}]$. In Appendix B below, we show that each term in this summation can be interpreted as an approximate theoretical measure of the change in g^t due to the change in a single real price $p_m{}^t$.

Thus using (37) and (38), it can be seen that the sum of the above two relative input quantity growth factors is equal to the following expression:

$$(39)\ 2\beta^t = \beta_L{}^t + \beta_P{}^t \qquad t = 1,2,...$$
$$= p^t\cdot y^t - p^{t-1}\cdot y^{t-1} + [a^{t-1}\cdot p^{t-1} + c^{t-1}\cdot x^t + (1/2)\ p^{t-1}\cdot Ap^{t-1} + p^{t-1}\cdot Bx^t + (1/2)\ x^t\cdot Cx^t]$$
$$-\ [a^t\cdot p^t + c^t\cdot x^{t-1} + (1/2)\ p^t\cdot Ap^t + p^t\cdot Bx^{t-1} + (1/2)\ x^{t-1}\cdot Cx^{t-1}].$$

Unfortunately, the expressions on the right hand sides of (37)-(39) are not observable without a knowledge of the unknown parameters in the g^t functions defined by (24). However, the *Bennet* (1920) *indicator of relative input quantity change*, $Q^B(w^{t-1},w^t,x^{t-1},x^t)$, defined by (40) is empirically observable:

$$(40)\ Q^B(w^{t-1},w^t,x^{t-1},x^t) \equiv (1/2)[w^{t-1} + w^t]\cdot[x^t - x^{t-1}]\ ; \qquad t = 1,2,...$$

where the observable real input price vectors $w^t \equiv W^t/\ P^t\cdot\mu$ were defined earlier by (6) and the input quantity vectors (deflated by the index of primary inputs) $x^t \equiv X^t/X^t\cdot\theta$ were defined earlier by (8).

We now show that if there is competitive profit maximizing behavior in the market sector in each period t and the market sector income functions g^t are defined by (24) above, then the Bennet indicator of relative input quantity change, Q^B defined by (40), is exactly equal to β^t defined by (22), the arithmetic average of the Laspeyres and Paasche relative input quantity growth factors. Using definition (40), we have:

$$(41)\ 2Q^B(w^{t-1},w^t,x^{t-1},x^t) = [w^{t-1} + w^t]\cdot[x^t - x^{t-1}]\ ; \qquad t = 1,2,...$$
$$= w^t\cdot x^t - w^{t-1}\cdot x^{t-1} + w^{t-1}\cdot x^t - w^t\cdot x^{t-1}$$
$$= p^t\cdot y^t - p^{t-1}\cdot y^{t-1} + w^{t-1}\cdot x^t - w^t\cdot x^{t-1} \qquad \text{using (28) and (36)}$$
$$= p^t\cdot y^t - p^{t-1}\cdot y^{t-1}$$
$$+\ x^t\cdot[a^{t-1}\cdot p^{t-1}\theta + c^{t-1} + (1/2)p^{t-1}\cdot Ap^{t-1}\theta + B^Tp^{t-1} + Cx^{t-1} - (1/2)x^{t-1}\cdot Cx^{t-1}\theta]$$
$$-\ x^{t-1}\cdot[\ a^t\cdot p^t\theta + c^t + (1/2)p^t\cdot Ap^t\theta + B^Tp^t + Cx^t - (1/2)x^t\cdot Cx^t\theta] \qquad \text{using (35)}$$
$$= p^t\cdot y^t - p^{t-1}\cdot y^{t-1}$$
$$+\ [a^{t-1}\cdot p^{t-1} + c^{t-1}\cdot x^t + (1/2)p^{t-1}\cdot Ap^{t-1} + p^{t-1}Bx^t + x^t\cdot Cx^{t-1} - (1/2)x^{t-1}\cdot Cx^{t-1}]$$
$$-\ [a^t\cdot p^t + c^t\cdot x^{t-1} + (1/2)p^t\cdot Ap^t + p^t\cdot Bx^{t-1} + x^{t-1}\cdot Cx^t - (1/2)x^t\cdot Cx^t] \quad \text{using } x^{t-1}\cdot\theta = 1$$
$$= p^t\cdot y^t - p^{t-1}\cdot y^{t-1}$$
$$+\ [a^{t-1}\cdot p^{t-1} + c^{t-1}\cdot x^t + (1/2)p^{t-1}\cdot Ap^{t-1} + p^{t-1}Bx^t - (1/2)x^{t-1}\cdot Cx^{t-1}]$$
$$-\ [a^t\cdot p^t + c^t\cdot x^{t-1} + (1/2)p^t\cdot Ap^t + p^t\cdot Bx^{t-1} - (1/2)x^t\cdot Cx^t] \qquad \text{using } x^t\cdot Cx^{t-1} = x^{t-1}\cdot Cx^t$$
$$= (1/2)[\beta_L{}^t + \beta_P{}^t] \qquad \text{using (39)}$$
$$= \beta^t \qquad \text{using definition (22).}$$

Thus under our assumptions on technology, the theoretical measure of period t per unit primary input market sector real income change due to change in relative input quantities,

β^t defined by (22), is exactly equal to the observable Bennet indicator of relative input quantity change, Q^B defined by (40).[25]

We now turn our attention to developing an observable measure of technical progress. Recall the (unobservable) theoretical measures of technical progress $\tau_L{}^t$, $\tau_P{}^t$ and τ^t defined by (12)-(14) respectively. If the income functions g^t are defined by (24), then by substituting these definitions for the g^t into definitions (12)-(14), we obtain the following (unobservable) expressions for these technical progress measures:

$$(42)\ \tau_L{}^t \equiv g^t(p^{t-1},x^{t-1}) - g^{t-1}(p^{t-1},x^{t-1})\ ; \qquad t = 1,2, ...$$
$$= [a^t - a^{t-1}]\cdot p^{t-1} + [c^t - c^{t-1}]\cdot x^{t-1}\ ;$$
$$(43)\ \tau_P{}^t \equiv g^t(p^t,x^t) - g^{t-1}(p^t,x^t)\ ; \qquad t = 1,2, ...$$
$$= [a^t - a^{t-1}]\cdot p^t + [c^t - c^{t-1}]\cdot x^t\ ;$$
$$(44)\ \tau^t \equiv (1/2)[\tau_L{}^t + \tau_P{}^t]\ ; \qquad t = 1,2, ...$$
$$= (1/2)[p^{t-1} + p^t]\cdot[a^t - a^{t-1}] + (1/2)[x^{t-1} + x^t]\cdot[c^t - c^{t-1}]\ .$$

Now look at the period t change in real income per unit primary input over the previous period, $\rho^t - \rho^{t-1}$ equal to $p^t\cdot y^t - p^{t-1}\cdot y^{t-1}$. Subtract the Bennet indicator of real price change $P^B(p^{t-1},p^t,y^{t-1},y^t)$ defined by (32) and subtract the Bennet indicator of relative input quantity change $Q^B(w^{t-1},w^t,x^{t-1},x^t)$ defined by (40) from this income difference and evaluate the resulting expression using the g^t defined by (24). We obtain the following identity:

$$(45)\ p^t\cdot y^t - p^{t-1}\cdot y^{t-1} - (1/2)[y^{t-1} + y^t]\cdot[p^t - p^{t-1}] - (1/2)[w^{t-1} + w^t]\cdot[x^t - x^{t-1}] \qquad t = 1,2, ...$$
$$= -(1/2)[a^{t-1}\cdot p^t + c^{t-1}\cdot x^{t-1} + (1/2)\, p^t\cdot Ap^t + p^t\cdot Bx^{t-1} + (1/2)\, x^{t-1}\cdot Cx^{t-1}]$$
$$+ (1/2)[a^t\cdot p^{t-1} + c^t\cdot x^t + (1/2)\, p^{t-1}\cdot Ap^{t-1} + p^{t-1}\cdot Bx^t + (1/2)\, x^t\cdot Cx^t]$$
$$- (1/2)[a^{t-1}\cdot p^{t-1} + c^{t-1}\cdot x^t + (1/2)p^{t-1}\cdot Ap^{t-1} + p^{t-1}Bx^t - (1/2)x^{t-1}\cdot Cx^{t-1}]$$
$$+ (1/2)[a^t\cdot p^t + c^t\cdot x^{t-1} + (1/2)p^t\cdot Ap^t + p^t\cdot Bx^{t-1} - (1/2)x^t\cdot Cx^t] \qquad \text{using (33) and (41)}$$
$$= (1/2)[p^{t-1} + p^t]\cdot[a^t - a^{t-1}] + (1/2)[x^{t-1} + x^t]\cdot[c^t - c^{t-1}] \qquad \text{canceling terms}$$
$$= \tau^t \qquad \text{using (44).}$$

Thus the first line in (45) give us an observable exact estimator for the theoretical technical progress measure τ^t. Using (32), (40) and (45), it can be seen that under the assumption that the income functions g^t are defined by (24), we have the following exact decomposition for the change in real income per unit primary input, $\rho^t - \rho^{t-1}$:

$$(46)\ \gamma^t \equiv p^t\cdot y^t - p^{t-1}\cdot y^{t-1} = \rho^t - \rho^{t-1} = \alpha^t + \beta^t + \tau^t\ ; \qquad t = 1,2, ...\ .$$

The above equation says the change in real income per unit primary input is equal to the sum of a change in real output prices factor α^t plus a change in relative primary input

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[25] Equations (41) show that the theoretical measure of change in g^t due to changes in relative input quantities x^t, β^t defined by (22), is equal to $\sum_{n=1}^N (1/2)[w_n{}^{t-1} + w_n{}^t][x_n{}^t - x_n{}^{t-1}]$. In Appendix B below, we show that each term in this summation can be interpreted as an approximation to a theoretical measure of the change in g^t due to the change in a single deflated input quantity $x_n{}^t$.

quantities factor β^t plus a change in technical efficiency term τ^t where all three explanatory factors can be estimated using observable price and quantity data pertaining to periods t and t−1.[26]

Rather than look at explanatory factors for the difference in real income per unit of primary input between the adjacent periods, it is sometimes convenient to express the difference in real income between the current period t and the reference year 0 in terms of the difference in the indicator of the technology level T^t, of the change in the level of real output prices in period t, A^t, and of the difference in the level of primary input quantities in period t, B^t. Thus, we use the growth factors τ^t, α^t, and β^t as follows to define the

(47) $T^0 \equiv 0;\ T^t = T^{t-1} + \tau^t;$ $t = 1,2, \ldots$
(48) $A^0 \equiv 0;\ A^t = A^{t-1} + \alpha^t;$ $t = 1,2, \ldots$
(49) $B^0 \equiv 0;\ B^t = B^{t-1} + \beta^t;$ $t = 1,2, \ldots .$

Using the chain links that appear in (47)-(49), we can establish the following exact relationship for the *cumulative change in real income per unit of primary input* going from period 0 to period t:

(50) $\rho^t - \rho^0 = A^t + B^t + T^t;$ $t = 1,2, \ldots .$

Instead of using the first line in (45) to define the technical progress term that turns out to be equal to τ^t, we can obtain some alternative expressions for technical progress (or the change in Total Factor Productivity) using Bennet's (1920) identity for the decomposition of the change of a value aggregate into price and quantity components; i.e., Bennet showed that the following exact identity holds:

(51) $p^t \cdot y^t - p^{t-1} \cdot y^{t-1} = (1/2)[y^{t-1} + y^t] \cdot [p^t - p^{t-1}] + (1/2)[p^{t-1} + p^t] \cdot [y^t - y^{t-1}]$.

Substituting (51) into (45) leads to the following alternative exact expression for the technical progress term τ^t:

(52) $\tau^t = (1/2)[p^{t-1} + p^t] \cdot [y^t - y^{t-1}] - (1/2)[w^{t-1} + w^t] \cdot [x^t - x^{t-1}]$; $t = 1,2, \ldots .$

Thus the absolute changes in the (deflated) output quantities, $y^t - y^{t-1}$, are weighted by the average of the real output prices for periods t−1 and t, $(1/2)[p^{t-1} + p^t]$, and then we subtract the absolute changes in the (deflated) primary input quantities, $x^t - x^{t-1}$, weighted by the average of the real input prices for periods t−1 and t, $(1/2)[w^{t-1} + w^t]$. We call the right hand side of (52) the *primal Bennet measure of technical progress*.[27] It is a

[26] The decomposition (46) is a difference counterpart to the ratio decomposition of nominal income growth obtained by Diewert and Morrison (1986; 663-665) and Kohli (1990).
[27] Balk (2003; 29) (2007), Diewert (2005; 353) and Diewert and Fox (2005; 8) all suggested variants of this Bennet indicator of real profit change as a measure of efficiency improvement.

difference counterpart to the following *primal Fisher index of productivity growth* or index of technical progress:[28]

(53) $T^t \equiv [p^t \cdot y^t \; p^{t-1} \cdot y^t / p^t \cdot y^{t-1} \; p^{t-1} \cdot y^{t-1}]^{1/2} / [w^t \cdot x^t \; w^{t-1} \cdot x^t / w^t \cdot x^{t-1} \; w^{t-1} \cdot x^{t-1}]^{1/2}$.

However, we can obtain a third expression for τ^t, which is also instructive. Under our assumptions (4), it can be seen that $p^t \cdot y^t$ is equal to $w^t \cdot x^t$ for each t. Thus we have:

$$(54)\; p^t \cdot y^t - p^{t-1} \cdot y^{t-1} = w^t \cdot x^t - w^{t-1} \cdot x^{t-1}$$
$$= (1/2)[x^{t-1} + x^t] \cdot [w^t - w^{t-1}] + (1/2)[w^{t-1} + w^t] \cdot [x^t - x^{t-1}]$$

where we have applied the Bennet value difference decomposition to obtain the second equality in (54). Substituting (54) into the first line of (45) leads to the following exact expression for τ^t:

(55) $\tau^t = (1/2)[x^{t-1} + x^t] \cdot [w^t - w^{t-1}] - (1/2)[y^{t-1} + y^t] \cdot [p^t - p^{t-1}]$.

Thus if real input prices increase faster than real output prices, there will be positive technical progress. We call the right hand side of (55) the *dual Bennet measure of technical progress*. It is a difference counterpart to the following *dual Fisher index of productivity growth* or index of technical progress:[29]

(56) $\tau_F{}^t \equiv [w^t \cdot x^t \; w^t \cdot x^{t-1} / w^{t-1} \cdot x^t \; w^{t-1} \cdot x^{t-1}]^{1/2} / [p^t \cdot y^t \; p^t \cdot y^{t-1} / p^{t-1} \cdot y^t \; p^{t-1} \cdot y^{t-1}]^{1/2}$.

The above difference approach can be converted into a more traditional growth rate approach. The period t rate of growth of real income per unit primary input is equal to the period t change in real income generated by the market sector, $\rho^t - \rho^{t-1}$, divided by last period's real income, ρ^{t-1}. Using (40), we have:

(57) $[\rho^t - \rho^{t-1}]/\rho^{t-1} = [\alpha^t + \beta^t + \tau^t]/\rho^{t-1}$; $t = 1,2, \ldots$.

Thus the rate of growth of market sector real income per unit primary input is explained by a sum of three additional explanatory factors, α^t/ρ^{t-1} (the contribution of real output price change), plus β^t/ρ^{t-1} (the contribution of input quantity growth relative to the average growth of primary inputs), plus τ^t/ρ^{t-1} (the contribution of technical change).[30]

[28] See Diewert and Nakamura (2003) for material on the Fisher (1922) index and its use in the traditional ratio approach to productivity measurement.

[29] The fact that primal indexes of Total Factor Productivity Growth (an index of output quantity growth divided by an index of input quantity growth) could be also written in dual form (an index of input prices divided by an index of output prices) dates back to the pioneering contributions of Jorgenson and Griliches (1967).

[30] If there are only two primary inputs, labour and capital, and the primary input weighting vector θ is the unit vector (1,0) so that the input aggregate collapses down to labour input, then the term β^t/ρ^{t-1} is the contribution of capital deepening; i.e., of the growth of capital input relative to the growth of labour input.

Using the results derived in Appendix B, the overall period t real price change term α^t is equal to the sum of M individual real price change terms, $\sum_{m=1}^{M} \alpha_m^t$, and the overall relative input quantity change term β^t is equal to the sum of N individual relative input quantity change terms, $\sum_{n=1}^{N} \beta_n^t$. Moreover, these individual price and quantity terms can be calculated empirically without econometric estimation. Using these results from Appendix B, (57) can be rewritten as follows:

$$(58)\ [\rho^t - \rho^{t-1}]/\rho^{t-1} = [\textstyle\sum_{m=1}^{M} \alpha_m^t + \sum_{n=1}^{N} \beta_n^t + \tau^t]/\rho^{t-1}\ ; \qquad t = 1,2, \ldots .$$

The decomposition of market sector real income growth (per unit primary input) given by (58) is comparable to the decomposition of real income growth that was derived by Diewert, Mizobuchi and Nomura (2005) and Diewert and Lawrence (2006) using the Translog methodology that was originally developed by Diewert and Morrison (1986) and Kohli (1990). However, the present normalized quadratic methodological approach has an advantage over the earlier Translog approach in that *the present approach allows individual prices and quantities to be zero*, whereas the Translog approach fails if any (exogenous) price or quantity becomes zero. Since a great deal of R&D effort is devoted to the development of *new* goods and services, it is useful to have a methodology that is able to deal with the creation of new products. A second advantage of the present approach is that when we specialize the fixed weight input index to be labour input, a "better" decomposition of labour productivity into explanatory factors is obtained; i.e., in our methodological approach, real output is replaced by real income and hence the effects on real income per unit of labour input of changes in the terms of trade can be modeled using our present approach.

5. An Application to the Japanese Economy for 1955-2004

5.1. The Japanese Data

We apply our methodology to a modified version of the Japanese productivity database developed by Diewert, Mizobuchi, and Nomura (2008). This database consists of the quantity, price, and value series for eleven main classes of outputs and inputs for the Japanese market sector, with some further detailed breakdowns for the investment outputs and capital service inputs; see Appendix C for a detailed listing of the data. We briefly mention the data construction procedure of Diewert, Mizobuchi, and Nomura (2008). We followed the conventions introduced by Jorgenson and Griliches on the treatment of taxes; i.e., we adjusted prices for tax wedges whenever possible so that the adjusted prices reflect the prices that producers face.[31] We also included the services of

[31] Thus our suggested treatment of indirect commodity taxes in an accounting framework that is suitable for productivity analysis follows the example set by Jorgenson and Griliches who advocated the following treatment of indirect taxes: "In our original estimates, we used gross product at market prices; we now employ gross product from the producers' point of view, which includes indirect taxes levied on factor outlay, but excludes indirect taxes levied on output." Dale W. Jorgenson and Zvi Griliches (1972; 85). All other taxes such as taxes on financial assets and poll taxes which do not affect the producers' behaviour are ignored in this study.

inventories and land as additional capital inputs. A listing of the outputs and inputs follows.

The 7 main classes of net outputs are:

- C; Domestic final consumption expenditure of households (excluding imputed rent for owner-occupied houses);
- N; Final consumption expenditure of private non-profit institutions serving households (NPISHs);
- G; Net sales of goods and services by the market sector to the general government sector; i.e., the value aggregate is equal to minus government sales of goods and services to the market production sector plus purchases of intermediate inputs from the market sector;
- X; Exports of goods and services (excluding direct purchases in the domestic market by non-resident households) and
- M; Imports of goods and services (excluding direct purchases abroad by resident households);
- I; Investment which consists of the following 12 subaggregates: I1: Animals and plants; I2: Construction; I3: Textile products; I4: Wood products; I5: Furniture and fixtures; I6: Metallic products; I7: General machinery; I8: Electric machinery; I9: Automobiles; I10: Other transportation; I11: Precision machinery; I12: Other investment products;
- IV; Change in Inventories which consists of the following 4 subaggregates: IV1: Finished goods inventory change; IV2: Work in progress inventory change; IV3: Work in progress inventory change for cultivated assets; IV4: Change in materials inventory.

The 4 main classes of primary inputs are:

- K; Capital service which consists of the following 12 subaggregates: K1: Animals and plants; K2: Construction; K3: Textile products; K4: Wood products; K5: Furniture and fixtures; K6: Metallic products; K7: General machinery; K8: Electric machinery; K9: Automobiles; K10: Other transportation; K11: Precision machinery; K12: Other investment products;
- KIV; Inventory service which consists of the following 4 components: KIV1: Finished good inventory services; KIV2: Work in progress inventory services; KIV3: Work in progress inventory services for cultivated assets; KIV4: Materials inventory services;
- LD; Land service which consists of the following 4 subaggregates: LD1: Agricultural land services; LD2: Industrial land services; LD3: Commercial land services; LD4: Residential land services for renters;
- LB; Labour input which consists of the following 3 subaggregates: LB1: Labour input of the self-employed; LB2: Labour input of family workers; LB3: Labour input of employees in the market sector;

Prices and quantities for the net output aggregates Y_C, Y_N, Y_G, Y_X, Y_M have been constructed using data in the national accounts based on 1968 SNA and 1993 SNA[32] and National Income Statistics which was the national accounting system prior to the introduction of 1968 SNA. We took the numbers constructed on the basis of 1993 JSNA as our standard. We extended the data series backwards by using data from 1968 JSNA and the earlier national income statistics.

The capital stocks are stocks at the beginning of the year. Estimates for the reproducible capital stocks during 1955-2006 have been constructed by applying the perpetual inventory method to the initial stocks in 1955 and using the investment data and asset specific depreciation rates. The initial capital stocks, the investment data and the change in inventory data have been taken from capital and investment data in the KEO database. This is a comprehensive productivity database for the Japanese economy provided at Keio University. The detailed procedures used to construct these capital data are explained in Nomura (2004). It should be noted that the KEO data base of price and quantity data for investments and the corresponding capital stocks for 95 classes of asset. This paper has aggregated these 95 asset classes into 12 classes for reproducible capital. Estimates of the quantities of labour services X_{LB} are based on hours of work. There are three different types of workers; the self employed, family workers and employees. Hours of works for each type of worker are aggregated into the quantity of aggregate labour input by applying a Fisher (1922) price index.

Price and quantity data for market sector net outputs and primary inputs are listed in Tables C1 (prices) and C2 (quantities) below. The detailed data on investments, changes in inventory, capital stocks, inventory stocks and land are listed in Tables C3-C9 (prices) and C10-C16 (quantities).

5.2. The Decomposition of Real Income Growth into Explanatory Factors

Substituting the estimates defined in (32)(41)(45) into equation (52), we can decompose the growth rate of real income per unit of labour $(\rho^t - \rho^{t-1})/\rho^{t-1} = \gamma^t/\rho^{t-1}$ into the contribution of technical progress τ^t/ρ^{t-1} changes in output prices $\alpha_C{}^t/\rho^{t-1}$ (domestic final consumption), $\alpha_N{}^t/\rho^{t-1}$ (non-profit institution final consumption), $\alpha_G{}^t/\rho^{t-1}$ (net government purchases from the market sector), $\alpha_X{}^t/\rho^{t-1}$ (exports), $\alpha_M{}^t/\rho^{t-1}$ (imports), $\alpha_I{}^t/\rho^{t-1}$ (investments in reproducible capital) and $\alpha_{IV}{}^t/\rho^{t-1}$ (inventory changes)[33] and growth in relative input quantities $\beta_K{}^t/\rho^{t-1}$ (capital services), $\beta_{KIV}{}^t/\rho^{t-1}$ (inventory services), $\beta_{LD}{}^t/\rho^{t-1}$ (land services) and $\beta_{LB}{}^t/\rho^{t-1}$ (labour input).[34] The chain link information on period by period changes in real income per unit of labour input that corresponds to (52) is given in Table 1. The effect of changes in the terms of trade is $\alpha_{XM}{}^t/\rho^{t-1}$ and is simply the sum of the contributions of real price changes in exports and imports $\alpha_X{}^t/\rho^{t-1}$ and $\alpha_M{}^t/\rho^{t-1}$.

[32] We call Japanese national accounts based on 1968 (1993) SNA simply 1968 (1993) JSNA, hereafter.

[33] Since we divided market sector nominal income by the price of consumption, $\alpha_C{}^t/\rho^{t-1}$ will be identically equal to zero and hence it is not listed.

[34] Since we divided market sector nominal income by the quantity of labour input, $\beta_{LB}{}^t/\rho^{t-1}$ will be identically equal to zero and hence it is not listed.

Table 1: Decomposition of Growth Rate of Real Income Per Unit of Labour (%)

	γ^t/ρ^{t-1}	τ^t/ρ^{t-1}	α_N^t/ρ^{t-1}	α_G^t/ρ^{t-1}	α_X^t/ρ^{t-1}	α_M^t/ρ^{t-1}	α_{XM}^t/ρ^{t-1}	α_I^t/ρ^{t-1}	α_{IV}^t/ρ^{t-1}	β_K^t/ρ^{t-1}	β_{KIV}^t/ρ^{t-1}	β_{LD}^t/ρ^{t-1}
1956	4.44867	6.16182	0.06632	0.11226	0.41197	-0.77573	-0.36376	1.89131	0.31518	-2.17691	-0.04565	-1.51192
1957	2.87446	4.57677	0.03315	0.03240	-0.33890	-0.07741	-0.41631	0.74566	-0.14922	-0.81207	0.04929	-1.18521
1958	2.00107	1.40635	0.00374	-0.12786	-0.82820	2.09237	1.26417	-0.84716	-0.04260	0.74947	0.13084	-0.53588
1959	9.40710	10.00776	0.00620	-0.08480	-0.13447	0.50801	0.37354	-1.00889	-0.02934	0.67686	-0.03759	-0.49664
1960	11.39564	12.25692	0.02537	-0.08627	-0.02266	0.34772	0.32506	-0.26007	-0.16189	0.45406	0.05441	-1.21196
1961	12.02089	9.87093	0.01186	-0.11711	-0.74467	0.61915	-0.12553	0.45644	-0.42145	2.43849	0.18021	-0.27296
1962	0.65867	1.02717	-0.00037	-0.19034	-1.00636	0.97419	-0.03217	-1.61067	-0.27202	2.01110	0.27188	-0.54592
1963	6.00309	5.81813	-0.00565	-0.24375	-0.53866	0.62781	0.08915	-2.37507	-0.18648	2.60020	0.12646	0.18011
1964	10.38152	8.83887	0.05422	-0.11601	-0.18209	0.15700	-0.02509	-0.39846	-0.04880	2.08023	0.15132	-0.15475
1965	0.26884	0.14921	-0.01467	-0.18490	-0.92682	0.91727	-0.00955	-1.81352	-0.11846	2.08152	0.15052	0.02870
1966	8.30842	6.67815	0.02531	-0.06157	-0.52277	0.38307	-0.13970	-0.15269	-0.07812	1.36975	0.03927	0.62803
1967	10.90512	7.79238	0.03298	-0.01017	-0.37574	0.56626	0.19052	0.29416	-0.11398	1.74651	0.10816	0.86455
1968	11.20952	8.93386	0.00975	-0.10101	-0.59204	0.59657	0.00453	-0.73902	-0.10851	2.62481	0.27313	0.31197
1969	12.66910	8.31488	0.03252	-0.08557	-0.28794	0.07037	-0.21756	0.18416	0.01995	2.89693	0.25461	1.26919
1970	6.81164	4.69959	0.06019	-0.12126	-0.45890	0.54747	0.08857	-0.88636	-0.14864	2.72774	0.20001	0.19178
1971	0.80235	-1.13843	0.04221	-0.19201	-0.55358	1.10380	0.55021	-1.68052	-0.08855	2.84638	0.16764	0.29540
1972	7.04619	4.57045	0.05415	-0.18580	-0.81746	1.09228	0.27482	-0.49672	0.02079	2.34018	0.01274	0.45557
1973	8.21062	5.16147	0.04033	-0.06564	-0.19834	-0.80638	-1.00472	1.71536	0.13194	1.61309	-0.03921	0.65799
1974	-2.10599	-3.30126	0.02096	0.52592	0.78894	-3.55585	-2.76691	-0.55100	-0.20618	2.79465	0.16055	1.21729
1975	-0.80101	-0.05185	-0.00123	-0.78528	-0.92131	0.26167	-0.65964	-2.72388	-0.05333	2.24790	0.14923	1.07707
1976	-0.91568	1.70858	0.01237	-0.09118	-1.09543	0.53152	-0.56391	-1.86509	-0.00081	0.26413	-0.09106	-0.28872
1977	-0.04042	0.28968	0.00604	-0.11593	-1.65183	1.48929	-0.16254	-0.97772	-0.00704	0.68083	-0.00065	0.24692
1978	4.61772	3.07762	-0.00407	-0.19174	-1.53785	2.47558	0.93773	-0.40146	0.01203	0.98133	-0.02657	0.23283
1979	6.24388	5.79355	0.02299	0.04859	0.62635	-2.77360	-2.14725	1.32970	0.03592	1.13219	-0.04378	0.07197
1980	-2.46584	-0.23679	-0.00283	0.18431	0.19778	-3.56576	-3.36798	-0.06985	-0.08939	0.97508	0.09021	0.05138
1981	-0.47804	-0.75036	-0.01721	0.03119	-0.20801	0.62034	0.41233	-0.96505	-0.03501	0.90058	0.04771	-0.10223
1982	0.91102	0.50508	-0.00083	-0.10075	0.20521	-0.02997	0.17524	-0.67400	-0.00256	0.94956	0.00435	0.05493
1983	-1.76589	-1.32994	-0.00522	-0.04176	-0.83239	1.12161	0.28922	-0.72845	0.00494	0.30348	-0.02194	-0.23621
1984	3.82543	3.21652	-0.00034	-0.06847	-0.12652	0.71447	0.58795	-0.70254	-0.00412	0.89396	-0.02571	-0.07181
1985	3.34933	2.14525	0.00298	0.13122	-0.83450	1.04168	0.20719	-0.81838	-0.02988	1.58949	0.00965	0.11181
1986	2.04506	0.05804	0.00398	-0.25710	-2.03370	3.88014	1.84644	-0.79428	0.00312	1.32266	0.00234	-0.14014
1987	2.73782	1.58341	0.00412	-0.13556	-0.54988	0.74751	0.19763	-0.21337	-0.00167	1.37929	-0.02728	-0.04875
1988	4.22853	3.25248	0.00553	-0.10609	-0.24517	0.28680	0.04163	0.20014	0.00079	0.99066	-0.01045	-0.14616
1989	4.86549	3.54667	0.00518	0.01110	0.18092	-0.45688	-0.27596	0.45343	-0.00443	1.25601	-0.00436	-0.12214
1990	3.88612	2.57180	0.02425	0.02607	-0.08930	-0.44836	-0.53766	0.13151	-0.01065	1.61561	0.02519	0.04000
1991	1.54626	0.35054	0.00875	-0.06525	-0.54072	0.67575	0.13503	-0.27589	-0.00826	1.48972	0.01028	-0.09865
1992	0.97480	-0.83497	-0.00305	-0.05479	-0.46585	0.63862	0.17277	-0.41528	0.00096	1.83890	0.01108	0.25918
1993	-0.80061	-1.92132	-0.00987	-0.04899	-0.87092	0.79541	-0.07552	-0.30133	0.00918	1.28832	0.00345	0.25547
1994	0.53597	0.11707	0.00443	-0.08087	-0.37287	0.39526	0.02239	-0.39280	0.00852	0.71471	-0.01116	0.15367
1995	0.62974	0.52496	0.00554	-0.04283	-0.19063	0.15194	-0.03869	-0.11381	-0.00090	0.30076	-0.01266	0.00738
1996	1.51755	1.53595	0.00386	-0.03194	0.39919	-0.63814	-0.23895	-0.46359	-0.00133	0.59592	0.01089	0.10674
1997	0.93056	0.54779	0.00482	0.01910	0.09015	-0.51950	-0.42934	-0.40024	-0.00896	1.01264	0.01890	0.16585
1998	-2.32846	-3.52030	0.00747	-0.00915	0.23108	0.24313	0.47422	-0.49102	-0.00100	0.99101	0.01754	0.20277
1999	-0.50530	-0.55889	-0.00740	-0.03375	-1.12645	0.86508	-0.26138	-0.52810	-0.00355	0.77766	0.00263	0.10747
2000	-0.01529	0.69245	-0.00019	0.00444	-0.46803	-0.23583	-0.70386	0.08725	-0.00511	-0.00730	-0.02959	-0.05339
2001	0.42896	-0.40427	0.00900	0.00880	0.44108	-0.36885	0.07223	-0.56832	0.01433	1.09798	0.00850	0.19071
2002	1.62364	0.78536	-0.01750	0.00953	0.06361	-0.05021	0.01340	-0.22275	-0.00316	0.95067	-0.01162	0.11970
2003	0.59777	0.53901	-0.00713	-0.02316	-0.37656	0.02356	-0.35300	-0.09819	-0.00613	0.45800	-0.00841	0.09678
2004	2.61774	2.66161	0.00727	0.01929	0.03238	-0.57112	-0.53874	0.26765	-0.00546	0.28555	-0.01758	-0.06185
2005	2.69552	2.91441	0.02956	0.05311	0.46561	-1.43705	-0.97143	0.18762	0.00639	0.48674	-0.01571	0.00481
2006	2.76460	2.28553	0.03999	0.08351	1.01583	-1.59688	-0.58104	0.48778	0.02672	0.38950	0.02927	0.00336

Average	γ^t/ρ^{t-1}	τ^t/ρ^{t-1}	α_N^t/ρ^{t-1}	α_G^t/ρ^{t-1}	α_X^t/ρ^{t-1}	α_M^t/ρ^{t-1}	α_{XM}^t/ρ^{t-1}	α_I^t/ρ^{t-1}	α_{IV}^t/ρ^{t-1}	β_K^t/ρ^{t-1}	β_{KIV}^t/ρ^{t-1}	β_{LD}^t/ρ^{t-1}
1956-2006	3.27008	2.60627	0.01235	-0.05800	-0.35132	0.18932	-0.16200	-0.38418	-0.03620	1.20032	0.04493	0.04659
1956-1973	6.96794	5.84035	0.02653	-0.10163	-0.45098	0.49688	0.04590	-0.38789	-0.08223	1.57046	0.11378	-0.05733
1974-1979	1.16642	1.25272	0.00951	-0.10160	-0.63185	-0.26190	-0.89375	-0.86491	-0.03657	1.35017	0.02462	0.42623
1980-1990	1.92173	1.32383	0.00178	-0.02962	-0.39414	0.35560	-0.03854	-0.38008	-0.01535	1.10694	0.00816	-0.05539
1991-2001	0.26493	-0.31555	0.00212	-0.03047	-0.26127	0.18208	-0.07919	-0.35119	0.00035	0.91821	0.00271	0.11793
2002-2006	2.05985	1.83719	0.01044	0.02846	0.24018	-0.72634	-0.48616	0.12442	0.00367	0.51409	-0.00481	0.03256

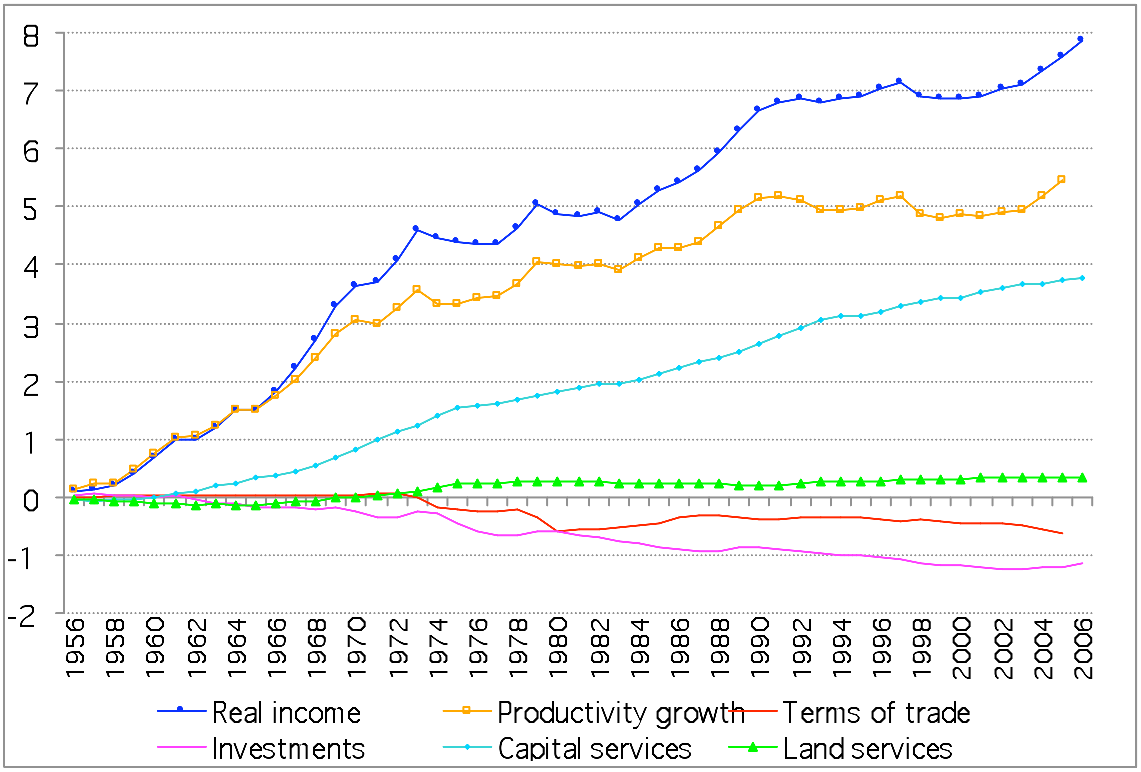
Looking at Table 1, it can be seen that there are five different periods: the rapid economic growth for 1955-1973, the slowdown of economic growth between two oil shocks for 1974-1979, the revival of steady economic growth for 1980-1990, the long recession for 1991-2001, and the modest economic recovery for 2002-2006. Over the 52 period, real income per unit of labour ρ^t grew on average by 3.27008% annually. Productivity growth τ^t contributed the most to the overall annual growth in real income per unit of labour (2.60627%), the growth in the capital services $\beta_K{}^t$ contributed the second largest amount to the overall annual growth in real income per unit of labour (1.20032%), declining real import prices $\alpha_M{}^t$ contributed 0.18932% per year and the changes in land services β_{LD}^t contributed 0.04659% per year. The largest negative contributing factor to the growth of real income per unit of labour over the sample period was the fall in real investment prices α_I^t (−0.38418% per year) while falls in real export prices α_X^t contributed −0.35132% per year. The remaining contributions were very small. Thus, the effect of changes in the terms of real income per unit of labour α_{XM}^t was very small for Japan on average over the entire period 1955-2006: an overall negative contribution of −0.162% per year. We can see that the importance of the contribution of capital services growth relative to the contribution of productivity growth increases over time. The average contribution after 1973 of capital services growth β_K^t (1.1964% per year) was larger than that of productivity growth τ^t (0.8368% per year). However, this is not the end of story. During the period of economic recovery 2002-2006, the average contribution of productivity growth τ^t of 1.889% became bigger than the average contribution of capital services growth β_K^t, which was 0.504%. Thus we can observe that the recent increase of real income per unit of labour was mostly boosted by productivity growth τ^t rather than by capital deepening β_K^t.

The annual change information in the previous table can be converted into cumulative changes using equations (47)-(49). The difference between the current level of net real income and its level in 1955, $\rho^t - \rho^0$ is decomposed into the sum of the level of productivity factor T^t, the levels of several real output price factors A_C^t, A_N^t, A_G^t, A_X^t, A_M^t, A_I^t, and A_{IV}^t, and the levels of several input quantity factors B_K^t, B_{KIV}^t, B_{LD}^t, and B_{LB}^t. The cumulative effect of changes in the terms of trade is A_T^t and is simply the product of the levels of real export and import prices A_X^t and A_M^t. Following Table 2 and Figure 1 give this cumulative growth information.

Table 2: Decomposition of the Cumulative Change in Real Income Per Unit of Labour (thousand yen at 1955 prices)

	$\rho^t - \rho^{1955}$	T^t	$A_N{}^t$	$A_G{}^t$	$A_X{}^t$	$A_M{}^t$	$(A_4{}^t + A_5{}^t)$	$A_I{}^t$	$A_{IV}{}^t$	$B_K{}^t$	$B_{KIV}{}^t$	$B_{LD}{}^t$
1956	0.08816	0.12211	0.00131	0.00222	0.00816	-0.01537	-0.00721	0.03748	0.00625	-0.04314	-0.00090	-0.02996
1957	0.14765	0.21684	0.00200	0.00290	0.00115	-0.01697	-0.01583	0.05291	0.00316	-0.05995	0.00012	-0.05449
1958	0.19026	0.24678	0.00208	0.00017	-0.01649	0.02758	0.01109	0.03487	0.00225	-0.04399	0.00290	-0.06590
1959	0.39458	0.46414	0.00221	-0.00167	-0.01941	0.03861	0.01921	0.01296	0.00161	-0.02929	0.00209	-0.07669
1960	0.66536	0.75539	0.00282	-0.00372	-0.01995	0.04687	0.02693	0.00678	-0.00223	-0.01850	0.00338	-0.10549
1961	0.98356	1.01668	0.00313	-0.00682	-0.03966	0.06326	0.02361	0.01886	-0.01339	0.04605	0.00815	-0.11271
1962	1.00309	1.04714	0.00312	-0.01246	-0.06950	0.09215	0.02265	-0.02890	-0.02146	0.10568	0.01621	-0.12890
1963	1.18226	1.22079	0.00295	-0.01974	-0.08557	0.11089	0.02531	-0.09978	-0.02702	0.18329	0.01998	-0.12353
1964	1.51073	1.50045	0.00467	-0.02341	-0.09134	0.11586	0.02452	-0.11239	-0.02857	0.24911	0.02477	-0.12842
1965	1.52012	1.50566	0.00415	-0.02987	-0.12370	0.14789	0.02419	-0.17573	-0.03270	0.32180	0.03003	-0.12742
1966	1.81106	1.73951	0.00504	-0.03202	-0.14201	0.16130	0.01929	-0.18107	-0.03544	0.36977	0.03140	-0.10543
1967	2.22466	2.03505	0.00629	-0.03241	-0.15626	0.18278	0.02652	-0.16992	-0.03976	0.43601	0.03551	-0.07264
1968	2.69616	2.41084	0.00670	-0.03666	-0.18116	0.20788	0.02671	-0.20100	-0.04433	0.54642	0.04700	-0.05952
1969	3.28880	2.79979	0.00822	-0.04066	-0.19463	0.21117	0.01653	-0.19239	-0.04339	0.68193	0.05891	-0.00014
1970	3.64780	3.04748	0.01140	-0.04705	-0.21882	0.24002	0.02120	-0.23910	-0.05123	0.82569	0.06945	0.00996
1971	3.69297	2.98340	0.01377	-0.05786	-0.24998	0.30216	0.05218	-0.33371	-0.05621	0.98593	0.07888	0.02659
1972	4.09282	3.24275	0.01685	-0.06840	-0.29637	0.36414	0.06777	-0.36189	-0.05503	1.11872	0.07961	0.05244
1973	4.59157	3.55628	0.01929	-0.07239	-0.30842	0.31516	0.00674	-0.25769	-0.04702	1.21671	0.07723	0.09241
1974	4.45314	3.33928	0.02067	-0.03782	-0.25656	0.08142	-0.17514	-0.29391	-0.06057	1.40041	0.08778	0.17243
1975	4.40159	3.33595	0.02059	-0.08835	-0.31584	0.09826	-0.21758	-0.46919	-0.06400	1.54506	0.09738	0.24174
1976	4.34314	3.44501	0.02138	-0.09417	-0.38577	0.13219	-0.25358	-0.58824	-0.06405	1.56192	0.09157	0.22331
1977	4.34059	3.46333	0.02176	-0.10150	-0.49024	0.22639	-0.26386	-0.65008	-0.06450	1.60498	0.09153	0.23892
1978	4.63253	3.65791	0.02151	-0.11362	-0.58747	0.38290	-0.20457	-0.67546	-0.06374	1.66702	0.08985	0.25364
1979	5.04551	4.04110	0.02303	-0.11041	-0.54604	0.19945	-0.34659	-0.58751	-0.06136	1.74191	0.08695	0.25840
1980	4.87223	4.02446	0.02283	-0.09746	-0.53214	-0.05113	-0.58327	-0.59242	-0.06764	1.81043	0.09329	0.26201
1981	4.83947	3.97303	0.02165	-0.09532	-0.54640	-0.00861	-0.55501	-0.65857	-0.07004	1.87215	0.09656	0.25501
1982	4.90161	4.00749	0.02159	-0.10219	-0.53240	-0.01065	-0.54305	-0.70454	-0.07022	1.93692	0.09686	0.25875
1983	4.78006	3.91594	0.02123	-0.10507	-0.58970	0.06655	-0.52315	-0.75468	-0.06988	1.95781	0.09535	0.24250
1984	5.03872	4.13344	0.02121	-0.10970	-0.59825	0.11486	-0.48339	-0.80219	-0.07015	2.01826	0.09361	0.23764
1985	5.27386	4.28404	0.02142	-0.10049	-0.65684	0.18799	-0.46885	-0.85964	-0.07225	2.12985	0.09429	0.24549
1986	5.42224	4.28825	0.02171	-0.11914	-0.80439	0.46952	-0.33488	-0.91727	-0.07203	2.22581	0.09446	0.23532
1987	5.62495	4.40548	0.02201	-0.12918	-0.84511	0.52486	-0.32024	-0.93307	-0.07215	2.32793	0.09244	0.23171
1988	5.94659	4.65289	0.02243	-0.13725	-0.86375	0.54668	-0.31708	-0.91784	-0.07209	2.40329	0.09164	0.22059
1989	6.33234	4.93408	0.02284	-0.13637	-0.84941	0.51045	-0.33896	-0.88189	-0.07244	2.50287	0.09130	0.21091
1990	6.65543	5.14790	0.02486	-0.13420	-0.85684	0.47318	-0.38366	-0.87096	-0.07333	2.63719	0.09339	0.21424
1991	6.78898	5.17817	0.02562	-0.13984	-0.90354	0.53154	-0.37200	-0.89479	-0.07404	2.76586	0.09428	0.20572
1992	6.87448	5.10494	0.02535	-0.14464	-0.94440	0.58755	-0.35684	-0.93121	-0.07396	2.92714	0.09525	0.22845
1993	6.80358	4.93478	0.02448	-0.14898	-1.02153	0.65800	-0.36353	-0.95790	-0.07314	3.04124	0.09556	0.25107
1994	6.85066	4.94507	0.02486	-0.15608	-1.05428	0.69272	-0.36156	-0.99240	-0.07239	3.10403	0.09458	0.26457
1995	6.90628	4.99144	0.02535	-0.15987	-1.07112	0.70614	-0.36498	-1.00246	-0.07247	3.13059	0.09346	0.26522
1996	7.04116	5.12795	0.02570	-0.16271	-1.03564	0.64942	-0.38622	-1.04366	-0.07259	3.18355	0.09443	0.27471
1997	7.12513	5.17738	0.02613	-0.16098	-1.02751	0.60255	-0.42496	-1.07977	-0.07340	3.27492	0.09613	0.28968
1998	6.91308	4.85679	0.02681	-0.16182	-1.00646	0.62469	-0.38177	-1.12449	-0.07349	3.36517	0.09773	0.30814
1999	6.86813	4.80708	0.02615	-0.16482	-1.10666	0.70164	-0.40502	-1.17146	-0.07381	3.43434	0.09796	0.31770
2000	6.86678	4.86836	0.02614	-0.16442	-1.14808	0.68077	-0.46731	-1.16374	-0.07426	3.43370	0.09535	0.31298
2001	6.90474	4.83259	0.02693	-0.16364	-1.10905	0.64813	-0.46092	-1.21403	-0.07299	3.53085	0.09610	0.32985
2002	7.04902	4.90238	0.02538	-0.16280	-1.10340	0.64367	-0.45973	-1.23382	-0.07327	3.61533	0.09506	0.34049
2003	7.10300	4.95105	0.02474	-0.16489	-1.13740	0.64579	-0.49161	-1.24269	-0.07383	3.65669	0.09431	0.34923
2004	7.34081	5.19285	0.02540	-0.16314	-1.13446	0.59391	-0.54055	-1.21837	-0.07432	3.68263	0.09271	0.34361
2005	7.59210	5.46455	0.02815	-0.15819	-1.09105	0.45994	-0.63111	-1.20088	-0.07373	3.72801	0.09124	0.34406
2006	7.85678	5.68336	0.03198	-0.15019	-0.99380	0.30706	-0.68674	-1.15419	-0.07117	3.76530	0.09405	0.34438

Figure 1: Real Income Change Per Unit of Labour (in thousands of 1955 yen)



Over the 52 year period, real income per unit of labour grew by 7.85678 yen at the prices of 1955. From Table 2, it can be seen that productivity growth contributed the most to the overall growth in real income per unit of labour (T^{2006}=5.68336 yen, 72.34 % of the overall change in real income per unit of labour), the growth in capital services made the second largest contribution (B_K^{2006}=3.7653 yen, 47.92 % of the overall change in real income per unit of labour), the change in real investment prices made the third largest contribution in magnitude, (A_I^{2006}=−1.15419 yen, −14.69 % of the overall change in real income per unit of labour), the change in terms of trade made the forth largest contribution (A_{XM}^{2006}=−0.68674 yen, −8.74 % of the overall change in real income per unit of labour) followed closely by the growth in land services (B_{LD}^{2006}=0.34438 yen, 4.38 % of the overall change in real income per unit of labour). The change in the real price of the consumption of NPISHs and the growth in inventory services has very small impact on the growth in the real income per unit of labour (less than one percent of the overall change in real income per unit of labour). Figure 1 plots real income per unit of labour and main factors contributing to its growth.

5.3. The Decomposition of Net Real Income Growth into Explanatory Factors

In the previous subsection 5.2, we focus on the real income per unit of labor. Deflating the income of the market sector by the price of household consumption, we obtain the (gross) real income. Real income captures how much consumption people can purchase for their income. Since economic welfare comes from consumption, while real GDP is the measure of output, real income is the measure of welfare. However, it is well known that real net income is the better measure of welfare, because it captures the sustainable level of welfare. Net income is the gross income net of the value of depreciated assets in the production period. By deducting depreciation from the income, we come closer to a measure of income that could be consumed in the present period without impairing production possibilities in future period. Deflating the income of the market sector by the price of household consumption, we obtain the (gross) real income. This subsection analyses the real net income produced by per unit of labour. By applying our methodology to the Japanese economy, we can decompose the change in net real income per unit of labour into explanatory factors.

In this section, we consider the production model based on net output concept. We illustrate the theory by considering a very simple two output, two input model of the market sector. One of the outputs is output in year t, Y^t and the other output is an investment good, I^t. One of the inputs is the flow of noncapital primary input X^t and the other input is K^t, capital services. Suppose that the average prices during period t of a unit of Y^t, X^t and I^t are P_Y^t, P_X^t and P_I^t respectively. Suppose further that the interest rate prevailing at the beginning of period t is r^t. The value of the beginning of period t capital stock is assumed to be P_I^t, the investment price for period t. The user cost of capital is calculated such as $u^t = (r^t + \delta^t + \tau^t)P_I^t/(1+r^t)$. As usual, it represents price of capital services input. Thus, the period t profit of the market sector is expressed as follows:

$$(59)\ \prod{}^t = P_Y{}^t\, Y^t + P_I{}^t\, I^t - P_X{}^t\, X^t - [(r^{t*} + \delta)P_I{}^t/(1+r^{t*})]K^t$$

Under the assumption of constant returns to scale, a zero profit condition should be satisfied such as $\prod^t = 0$. Using this condition, we obtain the following value of output equals value of input equation:

$$(60)\ P_Y{}^t\, Y^t + P_I{}^t\, I^t = P_X{}^t\, X^t + [(r^{t*} + \delta)P_I{}^t/(1+r^{t*})]K^t.$$

Equation (60) is essentially the closed economy counterpart to the (gross) value of outputs equals (gross) value of primary inputs equation (4), $P^t \cdot Y^t = W^t \cdot X^t$. The (gross) payment to primary inputs that is defined by the right hand side of (60) is not income, in the sense of Hicks. Net income in this paper is the same concept as Hicks' third concept of income: "Income No. 3 must be defined as the maximum amount of money which the individual can spend this week, and still be able to expect to spend this week, and still be able to expect to spend the same amount in real terms in each ensuing week." (Hicks, 1946).

The owner of a unit of capital cannot spend the entire period t gross rental income $(r^{t*} + \delta)P_I{}^t/(1+r^{t*})$ on consumption during period t because the depreciation portion of the rental, $\delta P_I{}^t/(1+r^{t*})$, is required in order to keep his or her capital intact. Thus the owner of a new unit of capital at the beginning of period t loans the unit to the market sector and gets the gross return $(r^{t*} + \delta)P_I{}^t$ at the end of the period plus the depreciated unit of the initial capital, which is worth only $(1 - \delta)P_I{}^t$. Thus $\delta P_I{}^t$ of this gross return must be set aside in order to restore the lender of the capital services to his or her original wealth position at the beginning of period t. This means that *period t Hicksian market sector income* is not the value of payments to primary inputs, $P_X{}^t\, X^t + [(r^{t*} + \delta)P_I{}^t/(1+r^{t*})]K^t$;

instead it is the value of payments to labour $P_X^t X^t$ plus the reward for waiting, $[r^{t*}P_I^t/(1+r^{t*})]K^t$. Using this definition of market sector net income, we can rearrange equation (60) as follows:

(61) Hicksian market sector income $\equiv P_X^t X^t + [r^{t*}P_I^t/(1+r^{t*})]K^t$
$= P_Y^t Y^t + P_I^t I^t - [\delta P_I^t/(1+r^{t*})]K^t$
= Value of consumption + value of gross investment − value of depreciation.

Thus in this net income framework, our new output concept is equal to our old output concept less the value of depreciation. Hence the overstatement of income problem that is implicit in the approaches used in previous subsections can readily be remedied: all we need to do is to take the user cost formula for an asset and decompose it into two parts:

- One part that represents depreciation and foreseen obsolescence, $[\delta P_I^t/(1+r^{t*})]$ and
- The remaining part that is the reward for postponing consumption, $[r^{t*}P_I^t/(1+r^{t*})]$.

Thus, in this subsection, we split up each user cost times the beginning of the period stock K^t into the depreciation component $\delta^t P_I^t K^t/(1+r^{t*})$ and the remaining term $r^{t*}P_I^t K^t/(1+r^{t*})$. The first term is considered as an intermediate input cost for the market sector and is an offset to gross investment made by the market sector during the period under consideration. We regard the second term as a genuine income component and call it *waiting capital services*. We take the price of depreciation P_{DEP}^t to be the corresponding investment price and the quantity of depreciation Y_{DEP}^t is taken to be the depreciation rate times the beginning of the period stock such as $P_{DEP}^t = P_I^t/(1+r^{t*})$ and $Y_{DEP}^t = \delta_n^t K^t$. We take the price of waiting capital services W_{KW}^t to be the corresponding investment price times the real rate of return and the quantity of waiting capital services X_{KW}^t is taken to be the beginning of the period stock such as $W_{KW}^t = r^{t*}P_I^t/(1+r^{t*})$, and $X_{KW}^t = K^t$.[35]

Thus in the net production model of this subsection, we add depreciations to the original list of net outputs and use waiting capital services instead of capital services among the original list of primary inputs. Substituting the estimates defined in (32)(41)(45) into equation (52), we can decompose the growth rate of real income per unit of labour $(\rho^t - \rho^{t-1})/\rho^{t-1} = \gamma^t/\rho^{t-1}$ into the contribution of technical progress τ^t/ρ^{t-1} changes in output prices α_C^t/ρ^{t-1}, α_N^t/ρ^{t-1}, α_G^t/ρ^{t-1}, α_X^t/ρ^{t-1}, α_M^t/ρ^{t-1}, α_I^t/ρ^{t-1}, $\alpha_{DEP}^t/\rho^{t-1}$, and α_{IV}^t/ρ^{t-1},[36] and growth in relative input quantity changes β_{KW}^t/ρ^{t-1}, β_{KIV}^t/ρ^{t-1}, β_{LD}^t/ρ^{t-1}, β_{LB}^t/ρ^{t-1}.[37] The chain link information on period by period changes in net real income per unit of labour input that corresponds to (52) is given in Table 3.

Table 3: Decomposition of Growth Rate of Net Real Income Per Unit of Labour (%)

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[35] These data have already been constructed in Diewert, Nomura, and Mizobuchi (2009).
[36] Since we divided market sector nominal income by the price of consumption, α_C^t/ρ^{t-1} will be identically equal to zero and hence it is not listed.
[37] Since we divided market sector nominal income by the quantity of labour services, β_{LB}^t/ρ^{t-1} will be identically equal to zero and hence it is not listed.

	$\gamma^{\tau}/\rho^{\tau-1}$	τ^{t}/ρ^{t-1}	α_N^{t}/ρ^{t-1}	α_G^{t}/ρ^{t-1}	α_X^{t}/ρ^{t-1}	α_M^{t}/ρ^{t-1}	$\alpha_{XM}^{t}/\rho^{t-1}$	α_I^{t}/ρ^{t-1}	$\alpha_{DEP}^{t}/\rho^{t-1}$	$\alpha_{IV}^{t}/\rho^{t-1}$	$\beta_{KW}^{t}/\rho^{t-1}$	$\beta_{KIV}^{t}/\rho^{t-1}$	$\beta_{LD}^{t}/\rho^{t-1}$
1956	5.16234	7.09905	0.07684	0.13006	0.47728	-0.89871	-0.42143	2.19117	-1.34163	0.36516	-1.13236	-0.05289	-1.75163
1957	3.42361	5.31072	0.03815	0.03728	-0.38997	-0.08907	-0.47904	0.85802	-0.31773	-0.17171	-0.54500	0.05672	-1.36380
1958	2.15837	1.71104	0.00428	-0.14635	-0.94794	2.39487	1.44694	-0.96964	0.38901	-0.04875	0.23544	0.14976	-0.61336
1959	10.90201	11.44312	0.00709	-0.09691	-0.15368	0.58056	0.42688	-1.15297	0.66740	-0.03353	0.25145	-0.04296	-0.56757
1960	12.89222	13.95568	0.02860	-0.09726	-0.02555	0.39202	0.36647	-0.29321	0.33740	-0.18251	0.08207	0.06134	-1.36637
1961	12.41574	11.56860	0.01319	-0.13028	-0.82842	0.68878	-0.13964	0.50777	0.18408	-0.46884	0.98403	0.20048	-0.30365
1962	-0.05697	1.06168	-0.00041	-0.21100	-1.11560	1.07994	-0.03566	-1.78551	0.50639	-0.30155	1.01286	0.30139	-0.60518
1963	6.07559	6.47743	-0.00631	-0.27214	-0.60141	0.70094	0.09953	-2.65175	0.90162	-0.20820	1.39313	0.14119	0.20109
1964	10.59010	9.83337	0.06050	-0.12943	-0.20317	0.17517	-0.02799	-0.44457	0.22028	-0.05445	1.13624	0.16883	-0.17266
1965	-0.29847	0.13606	-0.01634	-0.20590	-1.03213	1.02149	-0.01064	-2.01958	0.62043	-0.13192	1.12983	0.16762	0.03196
1966	8.80586	7.47759	0.02834	-0.06895	-0.58548	0.42902	-0.15646	-0.17101	0.29033	-0.08750	0.74617	0.04399	0.70337
1967	11.19742	8.56801	0.03677	-0.01134	-0.41889	0.63129	0.21240	0.32795	0.07765	-0.12706	1.02863	0.12058	0.96383
1968	11.35070	9.90224	0.01085	-0.11232	-0.65829	0.66333	0.00504	-0.82172	0.34066	-0.12065	1.49602	0.30370	0.34688
1969	12.40148	9.17208	0.03611	-0.09503	-0.31975	0.07815	-0.24160	0.20451	-0.03339	0.02215	1.64448	0.28275	1.40943
1970	6.28300	5.16961	0.06701	-0.13498	-0.51082	0.60941	0.09860	-0.98664	0.28873	-0.16545	1.51001	0.22264	0.21348
1971	-0.44911	-1.37424	0.04722	-0.21479	-0.61928	1.23479	0.61551	-1.87996	0.44914	-0.09906	1.48908	0.18754	0.33046
1972	6.83319	5.11441	0.06134	-0.21046	-0.92597	1.23728	0.31130	-0.56265	0.36379	0.02355	1.20144	0.01443	0.51604
1973	8.03421	5.79734	0.04577	-0.07450	-0.22512	-0.91524	-1.14036	1.94693	-0.17785	0.14975	0.78480	-0.04450	0.74682
1974	-4.04094	-3.71652	0.02382	0.59790	0.89691	-4.04248	-3.14557	-0.62641	0.17587	-0.23439	1.31797	0.18253	1.38388
1975	-1.38038	-0.10221	-0.00142	-0.91075	-1.06851	0.30348	-0.76504	-3.15909	1.11363	-0.06186	1.08412	0.17308	1.24916
1976	-0.14292	1.96380	0.01443	-0.10636	-1.27792	0.62007	-0.65785	-2.17580	1.07874	-0.00094	0.18410	-0.10623	-0.33681
1977	-0.05550	0.30838	0.00699	-0.13420	-1.91210	1.72395	-0.18815	-1.13178	0.44191	-0.00814	0.36441	-0.00075	0.28582
1978	5.07852	3.52999	-0.00471	-0.22198	-1.78042	2.86607	1.08565	-0.46479	0.40251	0.01393	0.49912	-0.03076	0.26956
1979	6.22923	6.69302	0.02650	0.05600	0.72197	-3.19702	-2.47505	1.53269	-0.21115	0.04141	0.53332	-0.05046	0.08295
1980	-3.28620	-0.29272	-0.00327	0.21248	0.22800	-4.11068	-3.88267	-0.08052	0.22556	-0.10305	0.47476	0.10400	0.05923
1981	-0.76339	-0.89638	-0.02001	0.03627	-0.24183	0.72121	0.47938	-1.12197	0.44163	-0.04070	0.42176	0.05547	-0.11885
1982	0.58638	0.56313	-0.00097	-0.11747	0.23926	-0.03494	0.20432	-0.78584	0.24192	-0.00298	0.41516	0.00508	0.06404
1983	-2.05034	-1.61699	-0.00610	-0.04885	-0.97365	1.31195	0.33830	-0.85207	0.32618	0.00578	0.10538	-0.02567	-0.27630
1984	4.24524	3.74457	-0.00040	-0.08033	-0.14842	0.83815	0.68972	-0.82416	0.45662	-0.00484	0.37844	-0.03016	-0.08424
1985	3.13115	2.52050	0.00348	0.15332	-0.97501	1.21708	0.24207	-0.95617	0.42739	-0.03491	0.63356	0.01127	0.13064
1986	1.71035	0.03128	0.00466	-0.30103	-2.38115	4.54306	2.16191	-0.92998	0.42064	0.00365	0.48056	0.00274	-0.16408
1987	2.27648	1.81145	0.00484	-0.15924	-0.64595	0.87810	0.23215	-0.25064	0.16284	-0.00196	0.56634	-0.03204	-0.05727
1988	4.15239	3.84301	0.00653	-0.12519	-0.28930	0.33842	0.04912	0.23617	-0.02917	0.00093	0.35578	-0.01234	-0.17247
1989	4.59268	4.16851	0.00611	0.01310	0.21364	-0.53952	-0.32587	0.53544	-0.08498	-0.00523	0.43499	-0.00515	-0.14423
1990	3.28436	3.02015	0.02871	0.03086	-0.10573	-0.53083	-0.63656	0.15570	0.03319	-0.01261	0.58774	0.02982	0.04735
1991	0.76672	0.38625	0.01042	-0.07770	-0.64391	0.80471	0.16080	-0.32854	0.16323	-0.00984	0.56734	0.01224	-0.11747
1992	0.06414	-0.94522	-0.00366	-0.06576	-0.55905	0.76638	0.20733	-0.49836	0.34787	0.00115	0.69645	0.01330	0.31103
1993	-1.89217	-2.39426	-0.01195	-0.05932	-1.05467	0.96322	-0.09145	-0.36490	0.17705	0.01111	0.52801	0.00418	0.30937
1994	0.46194	0.16668	0.00542	-0.09902	-0.45656	0.48398	0.02742	-0.48096	0.33154	0.01044	0.32593	-0.01367	0.18816
1995	0.74731	0.64555	0.00679	-0.05249	-0.23359	0.18617	-0.04741	-0.13945	0.16285	-0.00110	0.17906	-0.01552	0.00904
1996	1.77005	1.85391	0.00473	-0.03909	0.48858	-0.78103	-0.29245	-0.56740	0.39290	-0.00163	0.27511	0.01333	0.13065
1997	0.51581	0.70358	0.00588	0.02332	0.11007	-0.63425	-0.52418	-0.48864	0.22306	-0.01094	0.35816	0.02308	0.20248
1998	-3.77675	-4.38280	0.00916	-0.01121	0.28329	0.29806	0.58135	-0.60195	0.01909	-0.00122	0.34076	0.02150	0.24858
1999	-0.91622	-0.70282	-0.00920	-0.04199	-1.40173	1.07648	-0.32525	-0.65716	0.43183	-0.00441	0.25578	0.00327	0.13373
2000	0.05146	0.91206	-0.00024	0.00555	-0.58482	-0.29467	-0.87950	0.10902	0.07476	-0.00639	-0.06015	-0.03697	-0.06671
2001	-0.11205	-0.54598	0.01124	0.01099	0.55078	-0.46058	0.09020	-0.70966	0.41341	0.01789	0.35110	0.01061	0.23814
2002	1.42882	0.97646	-0.02197	0.01196	0.07986	-0.06304	0.01682	-0.27966	0.27758	-0.00396	0.31590	-0.01459	0.15028
2003	0.61393	0.73102	-0.00897	-0.02914	-0.47366	0.02963	-0.44403	-0.12351	0.23225	-0.00772	0.15285	-0.01058	0.12174
2004	2.87675	3.39291	0.00914	0.02427	0.04073	-0.71828	-0.67756	0.33661	-0.16542	-0.00686	0.06357	-0.02211	-0.07779
2005	2.78109	3.69571	0.03708	0.06663	0.58412	-1.80279	-1.21867	0.23537	-0.18021	0.00802	0.15082	-0.01970	0.00603
2006	2.59345	2.98263	0.05012	0.10467	1.27331	-2.00163	-0.72832	0.61141	-0.51884	0.03349	0.01738	0.03669	0.00421

Average	$\gamma^{\tau}/\rho^{\tau-1}$	τ^{t}/ρ^{t-1}	α_N^{t}/ρ^{t-1}	α_G^{t}/ρ^{t-1}	α_X^{t}/ρ^{t-1}	α_M^{t}/ρ^{t-1}	$\alpha_{XM}^{t}/\rho^{t-1}$	α_I^{t}/ρ^{t-1}	$\alpha_{DEP}^{t}/\rho^{t-1}$	$\alpha_{IV}^{t}/\rho^{t-1}$	$\beta_{KW}^{t}/\rho^{t-1}$	$\beta_{KIV}^{t}/\rho^{t-1}$	$\beta_{LD}^{t}/\rho^{t-1}$
1956-2006	3.12280	2.97005	0.01416	-0.06624	-0.40356	0.21122	-0.19234	-0.44215	0.23084	-0.04036	0.54517	0.05016	0.05351
1956-1973	7.09563	6.57910	0.02994	-0.11357	-0.50468	0.55633	0.05166	-0.42794	0.20924	-0.09114	0.80269	0.12681	-0.07116
1974-1979	0.94800	1.44608	0.01094	-0.11990	-0.73668	-0.28766	-1.02434	-1.00420	0.50025	-0.04167	0.66384	0.02790	0.48909
1980-1990	1.62537	1.53605	0.00214	-0.03510	-0.46183	0.42109	-0.04074	-0.44310	0.23835	-0.01781	0.44132	0.00937	-0.06511
1991-2001	-0.21089	-0.39119	0.00260	-0.03697	-0.31833	0.21895	-0.09938	-0.42982	0.24887	0.00046	0.34705	0.00321	0.14427
2002-2006	2.05881	2.35575	0.01308	0.03568	0.30087	-0.91122	-0.61035	0.15604	-0.07093	0.00459	0.14010	-0.00606	0.04090

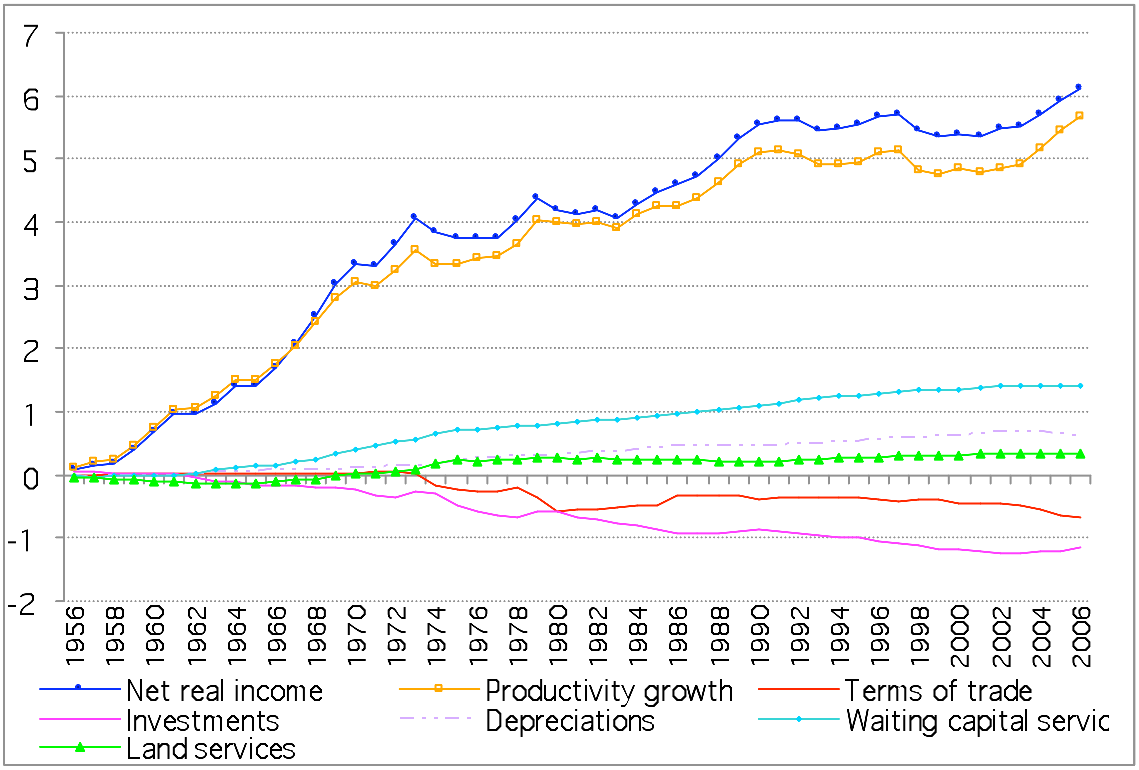
The new results are quite interesting. While the average growth rate of real income ρ^t was 3.27008% per year in the gross output model, the average growth rate of net real income ρ^t has now decreased by 0.14718 percent points per year to 3.1228%. More importantly, there are some big shifts in the explanatory factors. Productivity growth τ^t now accounts for 2.97005% of the overall annual growth in net real income per unit of labour compared to 2.60627% of the overall annual growth in real income per unit of labour, an increase of 0.36378 percentage points per year. The growth in capital services $\beta_K{}^t$ accounted for 1.20032% of the overall annual growth in real income per unit of labour while the growth in waiting capital services $\beta_{KW}{}^t$ accounted for 0.54517% of the overall annual growth in net real income per unit of labour, a decrease of 0.65515 percentage points per year. The average contributions of changes in real export prices $\alpha_X{}^t$ and real import prices $\alpha_M{}^t$ remain quite similar estimates in the previous gross output model. Thus, as we stated in the previous analysis, the effect of changes in the terms of trade on living standards α_T^t was negligible for Japan on average over the entire periods 1955-2006: an overall negative contribution of -0.19234%. The negative contribution of the change in real investment prices α_I^t equal to −0.44215% was offset by the positive contribution of the change in real depreciation prices α_{DEP}^t equal to 0.23084%. Finally, we note that the productivity recovery in the period 2002-2006 is quite striking. Using the previous gross output model, the average contribution of productivity growth during this period was 1.83719% per year and using the current net output model, its average contribution increases to a very respectable 2.35575% per year.

The annual change information in the previous table can be converted into cumulative changes using equations (47)-(49). The difference between the current level of net real income and its level in 1955, $\rho^t - \rho^0$ is decomposed into the sum of the level of productivity factor T^t, the levels of several real output price factors A_C^t, A_N^t, A_G^t, A_X^t, A_M^t, A_I^t, A_{DEP}^t, and A_{IV}^t, and the levels of several input quantity factors B_{KW}^t, B_{KIV}^t, B_{LD}^t, and B_{LB}^t. Following Table 4 and Figure 2 give this cumulative growth information.

Table 4: Decomposition of the Cumulative Change in Net Real Income Per Unit of Labour (thousand yen at 1955 prices)

	$\rho^t - \rho^{1955}$	T^t	A_N^t	A_G^t	A_X^t	A_M^t	A_{XM}^t	A_I^t	A_{DEP}^t	A_{IV}^t	B_{KW}^t	B_{KIV}^t	B_{LD}^t
1956	0.08830	0.12143	0.00131	0.00222	0.00816	-0.01537	-0.00721	0.03748	-0.02295	0.00625	-0.01937	-0.00090	-0.02996
1957	0.14988	0.21695	0.00200	0.00290	0.00115	-0.01697	-0.01583	0.05291	-0.02866	0.00316	-0.02917	0.00012	-0.05449
1958	0.19004	0.24879	0.00208	0.00017	-0.01649	0.02758	0.01109	0.03487	-0.02143	0.00225	-0.02479	0.00290	-0.06590
1959	0.39723	0.46626	0.00221	-0.00167	-0.01941	0.03861	0.01921	0.01296	-0.00874	0.00161	-0.02001	0.00209	-0.07669
1960	0.66896	0.76041	0.00282	-0.00372	-0.01995	0.04687	0.02693	0.00678	-0.00163	-0.00223	-0.01828	0.00338	-0.10549
1961	0.96438	1.03567	0.00313	-0.00682	-0.03966	0.06326	0.02361	0.01886	0.00275	-0.01339	0.00513	0.00815	-0.11271
1962	0.96286	1.06407	0.00312	-0.01246	-0.06950	0.09215	0.02265	-0.02890	0.01629	-0.02146	0.03222	0.01621	-0.12890
1963	1.12528	1.23723	0.00295	-0.01974	-0.08557	0.11089	0.02531	-0.09978	0.04040	-0.02702	0.06947	0.01998	-0.12353
1964	1.42559	1.51608	0.00467	-0.02341	-0.09134	0.11586	0.02452	-0.11239	0.04664	-0.02857	0.10169	0.02477	-0.12842
1965	1.41622	1.52035	0.00415	-0.02987	-0.12370	0.14789	0.02419	-0.17573	0.06610	-0.03270	0.13712	0.03003	-0.12742
1966	1.69156	1.75415	0.00504	-0.03202	-0.14201	0.16130	0.01929	-0.18107	0.07518	-0.03544	0.16045	0.03140	-0.10543
1967	2.07250	2.04564	0.00629	-0.03241	-0.15626	0.18278	0.02652	-0.16992	0.07782	-0.03976	0.19544	0.03551	-0.07264
1968	2.50189	2.42023	0.00670	-0.03666	-0.18116	0.20788	0.02671	-0.20100	0.09071	-0.04433	0.25204	0.04700	-0.05952
1969	3.02428	2.80659	0.00822	-0.04066	-0.19463	0.21117	0.01653	-0.19239	0.08930	-0.04339	0.32131	0.05891	-0.00014
1970	3.32177	3.05136	0.01140	-0.04705	-0.21882	0.24002	0.02120	-0.23910	0.10297	-0.05123	0.39280	0.06945	0.00996
1971	3.29917	2.98221	0.01377	-0.05786	-0.24998	0.30216	0.05218	-0.33371	0.12557	-0.05621	0.46774	0.07888	0.02659
1972	3.64148	3.23842	0.01685	-0.06840	-0.29637	0.36414	0.06777	-0.36189	0.14380	-0.05503	0.52793	0.07961	0.05244
1973	4.07147	3.54869	0.01929	-0.07239	-0.30842	0.31516	0.00674	-0.25769	0.13428	-0.04702	0.56993	0.07723	0.09241
1974	3.83783	3.33380	0.02067	-0.03782	-0.25656	0.08142	-0.17514	-0.29391	0.14445	-0.06057	0.64613	0.08778	0.17243
1975	3.76124	3.32813	0.02059	-0.08835	-0.31584	0.09826	-0.21758	-0.46919	0.20623	-0.06400	0.70628	0.09738	0.24174
1976	3.75342	3.43559	0.02138	-0.09417	-0.38577	0.13219	-0.25358	-0.58824	0.26526	-0.06405	0.71636	0.09157	0.22331
1977	3.75039	3.45244	0.02176	-0.10150	-0.49024	0.22639	-0.26386	-0.65008	0.28941	-0.06450	0.73627	0.09153	0.23892
1978	4.02772	3.64520	0.02151	-0.11362	-0.58747	0.38290	-0.20457	-0.67546	0.31139	-0.06374	0.76352	0.08985	0.25364
1979	4.38516	4.02926	0.02303	-0.11041	-0.54604	0.19945	-0.34659	-0.58751	0.29927	-0.06136	0.79413	0.08695	0.25840
1980	4.18485	4.01142	0.02283	-0.09746	-0.53214	-0.05113	-0.58327	-0.59242	0.31302	-0.06764	0.82307	0.09329	0.26201
1981	4.13984	3.95857	0.02165	-0.09532	-0.54640	-0.00861	-0.55501	-0.65857	0.33906	-0.07004	0.84793	0.09656	0.25501
1982	4.17415	3.99152	0.02159	-0.10219	-0.53240	-0.01065	-0.54305	-0.70454	0.35321	-0.07022	0.87222	0.09686	0.25875
1983	4.05349	3.89637	0.02123	-0.10507	-0.58970	0.06655	-0.52315	-0.75468	0.37240	-0.06988	0.87842	0.09535	0.24250
1984	4.29819	4.11220	0.02121	-0.10970	-0.59825	0.11486	-0.48339	-0.80219	0.39872	-0.07015	0.90023	0.09361	0.23764
1985	4.48633	4.26365	0.02142	-0.10049	-0.65684	0.18799	-0.46885	-0.85964	0.42440	-0.07225	0.93830	0.09429	0.24549
1986	4.59231	4.26559	0.02171	-0.11914	-0.80439	0.46952	-0.33488	-0.91727	0.45047	-0.07203	0.96808	0.09446	0.23532
1987	4.73580	4.37976	0.02201	-0.12918	-0.84511	0.52486	-0.32024	-0.93307	0.46073	-0.07215	1.00378	0.09244	0.23171
1988	5.00347	4.62749	0.02243	-0.13725	-0.86375	0.54668	-0.31708	-0.91784	0.45885	-0.07209	1.02671	0.09164	0.22059
1989	5.31182	4.90736	0.02284	-0.13637	-0.84941	0.51045	-0.33896	-0.88189	0.45315	-0.07244	1.05592	0.09130	0.21091
1990	5.54246	5.11944	0.02486	-0.13420	-0.85684	0.47318	-0.38366	-0.87096	0.45548	-0.07333	1.09719	0.09339	0.21424
1991	5.59807	5.14746	0.02562	-0.13984	-0.90354	0.53154	-0.37200	-0.89479	0.46732	-0.07404	1.13834	0.09428	0.20572
1992	5.60275	5.07838	0.02535	-0.14464	-0.94440	0.58755	-0.35684	-0.93121	0.49274	-0.07396	1.18924	0.09525	0.22845
1993	5.46438	4.90328	0.02448	-0.14898	-1.02153	0.65800	-0.36353	-0.95790	0.50569	-0.07314	1.22785	0.09556	0.25107
1994	5.49752	4.91524	0.02486	-0.15608	-1.05428	0.69272	-0.36156	-0.99240	0.52948	-0.07239	1.25124	0.09458	0.26457
1995	5.55138	4.96177	0.02535	-0.15987	-1.07112	0.70614	-0.36498	-1.00246	0.54121	-0.07247	1.26414	0.09346	0.26522
1996	5.67992	5.09640	0.02570	-0.16271	-1.03564	0.64942	-0.38622	-1.04366	0.56975	-0.07259	1.28412	0.09443	0.27471
1997	5.71804	5.14840	0.02613	-0.16098	-1.02751	0.60255	-0.42496	-1.07977	0.58623	-0.07340	1.31059	0.09613	0.28968
1998	5.43749	4.82282	0.02681	-0.16182	-1.00646	0.62469	-0.38177	-1.12449	0.58765	-0.07349	1.33590	0.09773	0.30814
1999	5.37200	4.77258	0.02615	-0.16482	-1.10666	0.70164	-0.40502	-1.17146	0.61852	-0.07381	1.35419	0.09796	0.31770
2000	5.37564	4.83718	0.02614	-0.16442	-1.14808	0.68077	-0.46731	-1.16374	0.62381	-0.07426	1.34993	0.09535	0.31298
2001	5.36770	4.79849	0.02693	-0.16364	-1.10905	0.64813	-0.46092	-1.21403	0.65311	-0.07299	1.37481	0.09610	0.32985
2002	5.46883	4.86760	0.02538	-0.16280	-1.10340	0.64367	-0.45973	-1.23382	0.67275	-0.07327	1.39717	0.09506	0.34049
2003	5.51291	4.92009	0.02474	-0.16489	-1.13740	0.64579	-0.49161	-1.24269	0.68943	-0.07383	1.40814	0.09431	0.34923
2004	5.72071	5.16517	0.02540	-0.16314	-1.13446	0.59391	-0.54055	-1.21837	0.67748	-0.07432	1.41273	0.09271	0.34361
2005	5.92738	5.43980	0.02815	-0.15819	-1.09105	0.45994	-0.63111	-1.20088	0.66409	-0.07373	1.42394	0.09124	0.34406
2006	6.12546	5.66761	0.03198	-0.15019	-0.99380	0.30706	-0.68674	-1.15419	0.62446	-0.07117	1.42527	0.09405	0.34438

Figure 2: Net Real Income Change Per Unit of Labour (in thousands of 1955 yen)



Over the 52 year period, net real income grew by about 6.12546 yen at the price of 1955 ($\rho^{2006}-\rho^{0}$=6.12546). From the above Table 7, it can be seen that productivity growth contributed the most to the overall growth in net real income per unit of labour (T^{2006}=5.66761, 92.53% of the overall growth in net real income per unit of labour) and the growth in waiting capital services made the next largest contribution ($B_{KW}{}^{2006}$=1.42527, 23.27% of the overall growth in net real income per unit of labour) followed by the change in real investment price in magnitude ($A_I{}^{2006}$ =-1.15419, -18.84% of the overall growth in net real income per unit of labour). There were smaller effects due to the changes in real output prices such as the contributions of the changes in real depreciation prices ($A_{DEP}{}^{2006}$=0.62446, 10.19% of the overall growth in net real income per unit of labour), real export prices ($A_X{}^{2006}$=–0.9938, –16.22% of the overall growth in net real income per unit of labour), and real import prices ($A_M{}^{2006}$ =0.30706, 5.01% of the overall growth in net real income per unit of labour). The change in the real price of the consumption of NPISHs and the growth in inventory services had very small impact on the growth in net real income per unit of labour (less than one percent of the overall change in real income per unit of labour). Figure 2 plots net real income per unit of labour and main factors contributing to its growth.

6. Conclusion

In this paper, we derived a decomposition for changes in real income per unit of primary input into explanatory factors that is exact for a flexible functional form; i.e., we showed that the change in real income generated by the market sector per unit of primary input is equal to the sum of a productivity growth term, plus terms due to changes in real output prices, plus terms due to changes in relative primary input quantities where all three sets of explanatory factors can be calculated using observable price and quantity data. The above difference approach can be converted into a growth rate approach. However, the present approach has an advantage over earlier exact Translog approaches in that the present approach allows individual prices and quantities to be zero. Our present approach also allows value subaggregates (such as inventory change or net exports) to change sign from period to period. If prices or quantities are zero or a value aggregate changes sign, the Translog approach fails, so our present approach offers some clear advantages in these situations.

We applied our methodology to analyze changes in the amount of real income per unit of labour generated by the market sector of the Japanese economy for the years 1955-2006. The main findings emerging from this application is that, taken over the entire time period of 52 years, productivity growth and the growth of reproducible capital stocks and their resulting services are the two main contributors to the growth of real income per unit of labour. We also observed that changes in the terms of trade had very small effects on real income per unit of labour on average. We moved to our theoretically preferred measure of net real income per unit of labour. We applied our methodology to analyze changes in the amount of net real income per unit of labour generated by the market sector of the Japanese economy. We observed that in the net approach, productivity growth was still the largest contributor (and was an even more important factor than before). However, the contribution of capital services was greatly reduced.

Appendix A: On the Flexibility of the Normalized Quadratic Income Function

Recall that the period t normalized quadratic nominal net revenue or income function $g^t(P,X)$ was defined by (24) above. In this Appendix, we will establish the flexibility of a special case of this functional form where there is no technical progress so that the parameter vectors a^t and c^t which appeared in (24) are simply the constant vectors a and c respectively. Thus in this Appendix, we consider the following functional form for g(P,X):

$$\text{(A1)}\ g(P,X) \equiv a{\cdot}P\theta{\cdot}X + c{\cdot}X\mu{\cdot}P + (1/2)\, P{\cdot}AP\, [\theta{\cdot}X/\mu{\cdot}P] + P{\cdot}BX + (1/2)\, X{\cdot}CX\, [\mu{\cdot}P/\theta{\cdot}X].$$

We will show that the above functional form is flexible at the arbitrary positive vector of net output prices, $P^* \gg 0_M$, and primary input quantities, $X^* \gg 0_N$. The nonnegative, nonzero weighting vectors $\mu > 0_M$ and $\theta > 0_N$ are assumed to be known and we scale these vectors so that the following restrictions are satisfied:

$$\text{(A2)}\ \mu{\cdot}P^* = \mu^TP^* = 1\ ;\ \theta{\cdot}X^* = \theta^TX^* = 1.$$

The M by M parameter matrix A and the N by N parameter matrix C are assumed to be symmetric;[38] i.e.:

[38] We assume that A is positive semidefinite and that C is negative semidefinite. With these definiteness restrictions, $g^t(P,X)$ will be globally convex in P and globally concave in X. These curvature properties can

(A3) $A = A^T$; $C = C^T$.

Thus the normalized quadratic function g(P,X) defined by (A1) has M unknown a_m parameters in the a vector, N unknown c_n parameters in the c vector, M(M+1)/2 unknown a_{mj} parameters in the A matrix, MN unknown b_{mn} parameters in the B matrix and N(N+1)/2 unknown c_{nk} parameters in the C matrix.

Let $g^*(P,X)$ be an arbitrary twice continuously differentiable income function (at the point P^*,X^*) which satisfies the appropriate regularity conditions for an income function.[39] In order for g(P,X) defined by (A1) to be a flexible functional form[40] at the point (P^*,X^*), the following equations must be satisfied:

(A4) $g(P^*,X^*) = g^*(P^*,X^*)$;
(A5) $\nabla_P\, g(P^*,X^*) = \nabla_P\, g^*(P^*,X^*)$;
(A6) $\nabla_X\, g(P^*,X^*) = \nabla_X\, g^*(P^*,X^*)$;
(A7) $\nabla_{PP}^2\, g(P^*,X^*) = \nabla_{PP}^2\, g^*(P^*,X^*)$;
(A8) $\nabla_{XX}^2\, g(P^*,X^*) = \nabla_{XX}^2\, g^*(P^*,X^*)$;
(A9) $\nabla_{PX}^2\, g(P^*,X^*) = \nabla_{PX}^2\, g^*(P^*,X^*)$.

The linear homogeneity of $g^*(P,X)$ in P and Euler's Theorem on homogeneous functions will imply the following restrictions on the level and first and second order derivatives of g^* evaluated at P^*,X^*:

(A10) $g^*(P^*,X^*) = P^{*T}\nabla_P\, g^*(P^*,X^*)$;
(A11) $\nabla_{PP}^2\, g^*(P^*,X^*)P^* = 0_M$;
(A12) $P^{*T}\nabla_{PX}^2\, g^*(P^*,X^*) = \nabla_X\, g^*(P^*,X^*)^T$.

The linear homogeneity of $g^*(P,X)$ in X and Euler's Theorem on homogeneous functions will also imply the following restrictions on the level and first and second order derivatives of g^* evaluated at P^*,X^*:

(A13) $g^*(P^*,X^*) = X^{*T}\nabla_P\, g^*(P^*,X^*)$;
(A14) $\nabla_{XX}^2\, g^*(P^*,X^*)X^* = 0_N$;
(A15) $\nabla_{PX}^2\, g^*(P^*,X^*)X^* = \nabla_P\, g^*(P^*,X^*)$.

Since g(P,X) is also linearly homogeneous in P and X separately, g will also satisfy the restrictions (A10)-(A15) with g replacing g^*.

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be imposed econometrically without destroying the flexibility of the functional form using the techniques explained in Diewert and Wales (1987) (1992).

[39] See Diewert (1973) (1974; 136) for a listing of these regularity conditions. The important properties for our purposes are that g(P,X) is linearly homogeneous and convex in the components of P and linearly homogeneous and concave in the components of X. See also Samuelson (1953; 20) and Gorman (1968).

[40] Diewert (1974; 113) introduced the concept of a flexible functional form. Diewert (1974; 137-139) also gave some examples of flexible functional forms for income functions (or variable profit functions using his terminology), including the Translog functional form.

The restrictions (A10)-A(15) mean that we do not require all of the parameters in the a and c vectors and in the A, B and C matrices in order for the normalized quadratic g(P,X) to be a flexible functional form. Thus we impose the following linear restrictions on the unknown parameters of g:

(A16) $c^TX^* = 0$;
(A17) $AP^* = 0_M$;
(A18) $CX^* = 0_N$;
(A19) $P^{*T}B = 0_N{}^T$;
(A20) $BX^* = 0_M$.

The N plus M linear restrictions are not all independent; any N+M−1 of these restrictions will imply the remaining restriction.[41]

Using (A1)-(A3) along with the restrictions (A16)-(A20) leads to the following expressions for the first and second derivatives of the normalized quadratic income function g evaluated at (P^*,X^*), which we set equal to the corresponding derivatives of g^*:

(A21) $\nabla_P\, g(P^*,X^*) = a \qquad = \nabla_P\, g^*(P^*,X^*)$;
(A22) $\nabla_X\, g(P^*,X^*) = c + a^TP^*\theta \qquad = \nabla_X\, g^*(P^*,X^*)$;
(A23) $\nabla_{PP}{}^2\, g(P^*,X^*) = A \qquad = \nabla_{PP}{}^2\, g^*(P^*,X^*)$;
(A24) $\nabla_{XX}{}^2\, g(P^*,X^*) = C \qquad = \nabla_{XX}{}^2\, g^*(P^*,X^*)$;
(A25) $\nabla_{PX}{}^2\, g(P^*,X^*) = B + a\theta^T + \mu c^T = \nabla_{PX}{}^2\, g^*(P^*,X^*)$.

It is immediately evident that we can set the parameter vector a equal to $\nabla_P\, g^*(P^*,X^*)$ and we can set the parameter matrices A and C equal to $\nabla_{PP}{}^2\, g^*(P^*,X^*)$ and $\nabla_{XX}{}^2\, g^*(P^*,X^*)$ respectively. The restrictions (A11) and (A14) on the derivatives of g^* imply that A and C satisfy the restrictions (A17) and (A18) respectively. Given that the parameter vector a is determined by (A21), we can use (A22) in order to determine the parameter vector c:

(A26) $c \equiv \nabla_X\, g^*(P^*,X^*) - a^TP^*\theta$
$= \nabla_X\, g^*(P^*,X^*) - \nabla_P\, g^*(P^*,X^*)^TP^*\theta$ using (A21)
$= \nabla_X\, g^*(P^*,X^*) - g^*(P^*,X^*)\theta$ using (A10).

We need to check whether the c defined by (A26) satisfies the restriction (A16):

(A27) $X^{*T}c = X^{*T}[\nabla_X\, g^*(P^*,X^*) - g^*(P^*,X^*)\theta]$ using (A26)
$= g^*(P^*,X^*) - g^*(P^*,X^*)$ using (A13) and (A2)
$= 0$.

[41] The restrictions (A16)-(A20) imply that the normalized quadratic functional form defined by (A1) has M + N − 1 + (M−1)(N−1) + (M(M−1)/2) + (N(N−1)/2) linearly independent free parameters, which is the minimal number required for a flexible functional form in this context. Thus the normalized quadratic functional form with the restrictions (A16)-(A20) imposed is a *parsimonious flexible functional form*.

Thus the restriction (A16) is satisfied by c.

Now that a and c have been defined, use (A25) to define the matrix B:

$$(A28)\ B \equiv \nabla_{PX}^2 g^*(P^*,X^*) - \nabla_P g^*(P^*,X^*)\theta^T - \mu[\nabla_X g^*(P^*,X^*) - g^*(P^*,X^*)\theta]^T \quad \text{using (A21) and (A26)}$$
$$= \nabla_{PX}^2 g^*(P^*,X^*) - \nabla_P g^*(P^*,X^*)\theta^T - \mu\nabla_X g^*(P^*,X^*)^T + g^*(P^*,X^*)\mu\theta^T.$$

Now check whether the B defined by (A28) satisfies the restrictions (A19):

$$(A29)\ P^{*T}B = P^{*T}[\nabla_{PX}^2 g^*(P^*,X^*) - \nabla_P g^*(P^*,X^*)\theta^T - \mu\nabla_X g^*(P^*,X^*)^T + g^*(P^*,X^*)\mu\theta^T]$$
$$= \nabla_X g^*(P^*,X^*)^T - g^*(P^*,X^*)\theta^T - \nabla_X g^*(P^*,X^*)^T + g^*(P^*,X^*)\theta^T \quad \text{using (A12), (A10) and (A2)}$$
$$= 0_N^T .$$

Finally, check whether the B defined by (A28) satisfies the restrictions (A20):

$$(A30)\ BX^* = [\nabla_{PX}^2 g^*(P^*,X^*) - \nabla_P g^*(P^*,X^*)\theta^T - \mu\nabla_X g^*(P^*,X^*)^T + g^*(P^*,X^*)\mu\theta^T]X^*$$
$$= \nabla_P g^*(P^*,X^*) - \nabla_P g^*(P^*,X^*) - \mu\, g^*(P^*,X^*) + g^*(P^*,X^*)\mu \quad \text{using (A15), (A2) and (A13)}$$
$$= 0_M .$$

Thus the normalized quadratic income function is a parsimonious flexible functional form.

Appendix B: Measures for the Effects of Individual Price and Quantity Changes

For many purposes, it is convenient to decompose the aggregate period t contribution factor due to changes in all deflated output prices α^t into separate effects for a change in each output price. Similarly, it can sometimes be useful to decompose the aggregate period t contribution factor due to changes in all deflated market sector primary input quantities β^t into separate effects for a change in each input quantity. In this Appendix, we indicate how this can be done.

We first model the effects of a change on per unit primary input real income of a single real output price, say p_m, going from period t−1 to t. Recall the definitions of the overall theoretical Laspeyres and Paasche type price indexes defined by (16) and (17). We adapt these definitions to the case where only a single real output price changes. Thus the mth *Laspeyres measure of real output price change* α_{Lm}^t chooses the period t−1 reference technology and holds constant other real output prices at their period t−1 levels and holds deflated inputs constant at their period t−1 levels x^{t-1} and the mth *Paasche measure of real output price change* α_{Pm}^t chooses the period t reference technology and reference deflated input vector x^t and holds constant other real output prices at their period t levels:

$$(B1)\ \alpha_{Lm}^t \equiv g^{t-1}(p_1^{t-1},...,p_{m-1}^{t-1},p_m^t,p_{m+1}^{t-1},..., p_M^{t-1},x^{t-1}) - g^{t-1}(p^{t-1},x^{t-1}) ; \quad m = 1,...,M;$$

(B2) $\alpha_{Pm}{}^{t} \equiv g^{t}(p^{t},x^{t}) - g^{t}(p_1{}^{t},...,p_{m-1}{}^{t},p_m{}^{t-1},p_{m+1}{}^{t},..., p_M{}^{t},x^{t})$; m = 1,...,M.

Since both measures of real output price change are equally valid, it is natural to average them to obtain an *overall measure of the effects on deflated real income of the change in the real price of output m*:

(B3) $\alpha_m{}^{t} \equiv (1/2)[\alpha_{Lm}{}^{t} + \alpha_{Pm}{}^{t}]$; m = 1,...,M ; t = 1,2, ...

We are not able to obtain observable exact measures for the theoretical measures defined by (B1)-(B3) but we are able to obtain observable first order approximations to these theoretical measures. Note that $g^{t}(p^{t},x^{t})$ which appears in (B2) is equal to $p^{t}\cdot y^{t}$ which is observable and $g^{t-1}(p^{t-1},x^{t-1})$ which appears in (B1) is equal to $p^{t-1}\cdot y^{t-1}$ which is also observable. Using Hotelling's Lemma (9), it is straightforward to obtain the following first order approximations to the unobservable terms in (B1) and (B2):

(B4) $g^{t-1}(p_1{}^{t-1},...,p_{m-1}{}^{t-1},p_m{}^{t},p_{m+1}{}^{t-1},..., p_M{}^{t-1},x^{t-1})$ m = 1,...,M ; t = 1,2, ...
$\approx g^{t-1}(p^{t-1},x^{t-1}) + [\partial g^{t-1}(p^{t-1},x^{t-1})/\partial p_m][p_m{}^{t} - p_m{}^{t-1}]$
$= p^{t-1}\cdot y^{t-1} + y_m{}^{t-1}[p_m{}^{t} - p_m{}^{t-1}]$ using (9);
$= [p_1{}^{t-1},...,p_{m-1}{}^{t-1},p_m{}^{t},p_{m+1}{}^{t-1},..., p_M{}^{t-1}]\cdot y^{t-1}$ rearranging terms.

(B5) $g^{t}(p_1{}^{t},...,p_{m-1}{}^{t},p_m{}^{t-1},p_{m+1}{}^{t},..., p_M{}^{t},x^{t})$ m = 1,...,M ; t = 1,2, ...
$\approx g^{t}(p^{t},x^{t}) + [\partial g^{t}(p^{t},x^{t})/\partial p_m][p_m{}^{t-1} - p_m{}^{t}]$
$= p^{t}\cdot y^{t} + y_m{}^{t}[p_m{}^{t-1} - p_m{}^{t}]$ using (9)
$= [p_1{}^{t},...,p_{m-1}{}^{t},p_m{}^{t-1},p_{m+1}{}^{t},..., p_M{}^{t}]\cdot y^{t}$.

Substituting (B4) and (B5) into (B1) and (B2) leads to the following first order approximations to the theoretical price change measures $\alpha_{Lm}{}^{t}$ and $\alpha_{Pm}{}^{t}$:

(B6) $\alpha_{Lm}{}^{t} \approx y_m{}^{t-1}[p_m{}^{t} - p_m{}^{t-1}]$ m = 1,...,M ; t = 1,2, ...
$\equiv a_{Lm}{}^{t}$;
(B7) $\alpha_{Pm}{}^{t} \approx y_m{}^{t}[p_m{}^{t-1} - p_m{}^{t}]$ m = 1,...,M ; t = 1,2, ...
$\equiv a_{Pm}{}^{t}$

where we have defined the mth observable *Laspeyres and Paasche partial indicators of real output price change*, $a_{Lm}{}^{t}$ and $a_{Pm}{}^{t}$, in (B6) and (B7) respectively. These partial indicators are first order approximations to the theoretical measures of price change defined by (B1) and (B2).

Note that y^{t-1} is a feasible solution to the revenue maximization problem defined by $g^{t-1}(p_1{}^{t-1},...,p_{m-1}{}^{t-1},p_m{}^{t},p_{m+1}{}^{t-1},..., p_M{}^{t-1},x^{t-1})$ and thus the following inequality will hold:

(B8) $g^{t-1}(p_1{}^{t-1},...,p_{m-1}{}^{t-1},p_m{}^{t},p_{m+1}{}^{t-1},...,p_M{}^{t-1},x^{t-1}) \geq [p_1{}^{t-1},...,p_{m-1}{}^{t-1},p_m{}^{t},p_{m+1}{}^{t-1},..., p_M{}^{t-1}]\cdot y^{t-1}$.

Thus using (B1), (B4), (B6) and (B8), we have

(B9) $\alpha_{Lm}{}^{t} \geq a_{Lm}{}^{t}$; $\quad m = 1,...,M ; t = 1,2, ... ;$

i.e., the observable first order approximation indicator of real price change $a_{Lm}{}^{t}$ will always be equal to or less than its theoretical counterpart $\alpha_{Lm}{}^{t}$. The difference between $\alpha_{Lm}{}^{t}$ and $a_{Lm}{}^{t}$ is thus due to substitution bias.

In a similar fashion, we can show that y^t is a feasible solution to the revenue maximization problem defined by $g^t(p_1{}^t ,...,p_{m-1}{}^t,p_m{}^{t-1},p_{m+1}{}^t,...,p_M{}^t,x^t)$ and thus the following inequality will hold:

(B10) $g^t(p_1{}^t,...,p_{m-1}{}^t,p_m{}^{t-1},p_{m+1}{}^t,...,p_M{}^t,x^t) \geq [p_1{}^t,...,p_{m-1}{}^t,p_m{}^{t-1},p_{m+1}{}^t,..., p_M{}^t]\cdot y^t$.

Thus using (B2), (B5), (B7) and (B10), we have

(B11) $\alpha_{Pm}{}^{t} \leq a_{Pm}{}^{t}$; $\quad m = 1,...,M ; t = 1,2, ... ;$

i.e., the observable first order approximation indicator of real price change $a_{Pm}{}^{t}$ will always be equal to or greater than its theoretical counterpart $\alpha_{Pm}{}^{t}$.

Since the substitution bias for our observable partial price indicators goes in opposite directions, this suggests that taking an average of these two indicators should lead to a closer approximation to the average of the underlying theoretical partial indicators. Thus define the period t *Bennet* (1920) *mth partial indicator of real price change for output price p_m*, $P_m{}^B(p^{t-1},p^t,y^{t-1},y^t)$, as the arithmetic average of the Laspeyres and Paasche partial indicators:

$$
\begin{aligned}
\text{(B12) } P_m{}^B(p_m{}^{t-1},p_m{}^t,y_m{}^{t-1},y_m{}^t) &\equiv (1/2)[y_m{}^{t-1} + y_m{}^t][p_m{}^t - p_m{}^{t-1}] ; \quad m =1,...,M ; t = 1,2, ... \\
&= (1/2)\, a_{Lm}{}^t + (1/2)\, a_{Pm}{}^t \\
&\approx (1/2)\, \alpha_{Lm}{}^t + (1/2)\, \alpha_{Pm}{}^t
\end{aligned}
$$

where $y_m{}^t$ is the mth component of the observable period t output quantity vector (deflated by the index of primary inputs) $y^t \equiv Y^t/X^t\cdot\theta$ which was defined by (8) and $p_m{}^t$ is the mth component of the period t deflated output price vector $p^t \equiv P^t/P^t\cdot\mu$ which was defined earlier by (6). Note that the sum over m of the individual price change Bennet indicators, $\sum_{m=1}^{M} P_m{}^B(p_m{}^{t-1},p_m{}^t,y_m{}^{t-1},y_m{}^t)$, is equal to the overall Bennet indicator of real price change, $P^B(p^{t-1},p^t,y^{t-1},y^t)$, defined in section 4 by (32).

Thus the Bennet partial indicators of real price change, $P_m{}^B(p_m{}^{t-1},p_m{}^t,y_m{}^{t-1},y_m{}^t)$, will be at least a first order approximations to the theoretical measures of real price change, $(1/2)\ \alpha_{Lm}{}^t + (1/2)\ \alpha_{Pm}{}^t$, and we would normally expect this approximation to be better than a first order approximation.

The approximations derived thus far are completely nonparametric. However, if we assume that the functions g^t are of the normalized quadratic type defined by (24) where the parameters satisfy counterparts to the restrictions (A20)-(A24) in Appendix A, then we can obtain an explicit expression for the bias in the approximations given by (B12); i.e., it can be shown that

(B13) $P_m^B(p_m^{t-1},p_m^t,y_m^{t-1},y_m^t) = (1/2)\ \alpha_{Lm}^t + (1/2)\ \alpha_{Pm}^t + \text{Bias}_m^t \qquad m = 1,...,M\ ;\ t = 1,2,...$

where the Bias between the mth Bennet partial indicator for period t is equal to:

(B14) $\text{Bias}_m^t = (1/2)\mu_m[p_m^t - p_m^{t-1}]\{c^{t-1}\cdot x^{t-1} + c^t\cdot x^t - (1/2)[p^{t-1}\cdot Ap^{t-1} + p^t\cdot Ap^t] + (1/2)[x^{t-1}\cdot Cx^{t-1} + x^t\cdot Cx^t]\}.$

The normalized prices p_m^t weighted by the μ_m sum to unity; i.e., we have:

(B15) $\sum_{m=1}^M \mu_m p_m^t = \sum_{m=1}^M \mu_m p_m^{t-1}\ ; \qquad t = 1,2,\ldots.$

Using (B15), it can be seen that

(B16) $\sum_{m=1}^M \text{Bias}_m^t = 0\ ; \qquad t = 1,2,\ldots.$

Recall the normalizations (A20)-(A24) in Appendix A. If the price vectors P^t are all proportional to the reference price vector P^* and if the primary input vectors X^t are all proportional to X^*, then it can be seen that all of the terms in the curly brackets in (B14) will be equal to zero and hence all of the bias terms Bias_m^t will also be equal to zero. This observation indicates that if the price and quantity variations in our data set are not too far from being proportional and the technology functions g^t have the normalized quadratic functional form, then the bias terms will be small.[42]

It is also useful to have a decomposition of the overall contribution of deflated input growth to the growth of real income into separate contributions for each deflated primary input that is used by the market sector. Recall definitions (20) and (21) for the overall theoretical Laspeyres and Paasche type measures of quantity change. We now want to adapt these definitions to the case where only a single deflated quantity changes going from one period to the next. Thus the nth *Laspeyres measure of deflated input quantity change* β_{Ln}^t chooses the period t−1 reference technology and holds constant other deflated input quantities at their period t−1 levels and holds real output prices at their period t−1 levels p^{t-1} and the nth *Paasche measure of deflated input quantity change* β_{Pn}^t chooses the period t reference technology and reference real output price vector p^t and holds constant other deflated input quantities at their period t levels:

(B17) $\beta_{Ln}^t \equiv g^{t-1}(p^{t-1},x_1^{t-1},...,x_{n-1}^{t-1},x_n^t,x_{n+1}^{t-1},..., x_N^{t-1}) - g^{t-1}(p^{t-1},x^{t-1})\ ; \qquad n = 1,...,N;$
(B18) $\beta_{Pn}^t \equiv g^t(p^t,x^t) - g^t(p^t,x_1^t,...,x_{n-1}^t,x_n^{t-1},x_{n+1}^t,..., p_N^t)\ ; \qquad n = 1,...,N.$

[42] Our results can be regarded as approximate difference counterparts to the ratio type results obtained by Diewert and Morrison (1986; 672) for the Translog functional form. As noted earlier, an advantage of the present approach is that it is well defined even if some prices are zero whereas the Diewert-Morrison approach breaks down as any price approaches zero.

Since both measures of input change are equally valid, as usual, we average them to obtain *an overall measure of the effects on real income (per unit of input) of the change in the quantity of deflated input n*:

(B19) $\beta_n^t \equiv (1/2)[\beta_{Pn}^t + \beta_{Pn}^t]$; $n = 1,...,N$; $t = 1,2, ...$.

As was the case for the partial price change measures, we are not able to obtain observable exact measures for the theoretical measures defined by (B17)-(B19) but we are able to obtain observable first order approximations to these theoretical measures. Note that $g^t(p^t,x^t)$ which appears in (B18) is equal to $p^t \cdot y^t$ (which is equal to $w^t \cdot x^t$) which is observable and $g^{t-1}(p^{t-1},x^{t-1})$ which appears in (B17) is equal to $p^{t-1} \cdot y^{t-1}$ (which is equal to $w^{t-1} \cdot x^{t-1}$) which is also observable. Using Samuelson's Lemma (10), it is straightforward to obtain the following first order approximations to the unobservable terms in (B17) and (B18):

$$\text{(B20)}\; g^{t-1}(p^{t-1},x_1^{t-1},...,x_{n-1}^{t-1},x_n^t,x_{n+1}^{t-1},..., x_N^{t-1}) \qquad n = 1,...,N ; t = 1,2, ...$$
$$\approx g^{t-1}(p^{t-1},x^{t-1}) + [\partial g^{t-1}(p^{t-1},x^{t-1})/\partial x_n][x_n^t - x_n^{t-1}]$$
$$= w^{t-1} \cdot x^{t-1} + w_n^{t-1}[x_n^t - x_n^{t-1}] \qquad \text{using (7) and (10)}$$
$$= [x_1^{t-1},...,x_{n-1}^{t-1},x_n^t,x_{n+1}^{t-1},..., x_N^{t-1}] \cdot w^{t-1} \qquad \text{rearranging terms.}$$

$$\text{(B21)}\; g^t(p^t,x_1^t,...,x_{n-1}^t,x_n^{t-1},x_{n+1}^t,..., p_N^t) \qquad n = 1,...,N ; t = 1,2, ...$$
$$\approx g^t(p^t,x^t) + [\partial g^t(p^t,x^t)/\partial x_n][x_n^{t-1} - x_n^t]$$
$$= w^t \cdot x^t + w_n^t[x_n^{t-1} - x_n^t] \qquad \text{using (7) and (10)}$$
$$= [x_1^t,...,x_{n-1}^t,x_n^{t-1},x_{n+1}^t,..., x_N^t] \cdot w^t \qquad \text{rearranging terms.}$$

Substituting (B20) and (B21) into (B17) and (B18) leads to the following first order approximations to the theoretical quantity change measures β_{Ln}^t and β_{Pn}^t:

$$\text{(B22)}\; \beta_{Ln}^t \approx w_n^{t-1}[x_n^t - x_n^{t-1}] \equiv b_{Ln}^t \; ; \qquad n = 1,...,N ; t = 1,2, ...$$
$$\text{(B23)}\; \beta_{Pn}^t \approx w_n^t[x_n^{t-1} - x_n^t] \equiv b_{Pn}^t \qquad n = 1,...,N ; t = 1,2, ...$$

where we have defined the mth observable *Laspeyres and Paasche partial indicators of deflated input quantity change*, b_{Ln}^t and b_{Pn}^t, in (B22) and (B23) respectively. These partial indicators are first order approximations to the theoretical measures of quantity change defined by (B17) and (B18).

Using the fact that each function $g^t(p,x)$ is concave in x, it can be seen that the first line in (B20) is equal to or less than the third line and the first line in (B21) is equal to or less than the third line. Using these inequalities and (B17)-(B23), it can be seen that the following inequalities will be satisfied:

$$\text{(B24)}\; \beta_{Ln}^t \le b_{Ln}^t \; ; \qquad n = 1,...,N ; t = 1,2, ... ;$$
$$\text{(B25)}\; \beta_{Pn}^t \ge b_{Pn}^t \; ; \qquad n = 1,...,N ; t = 1,2, ... ;$$

i.e., the observable Laspeyres partial indicator of deflated quantity change $b_{Ln}{}^t$ will always be equal to or greater than its theoretical counterpart $\beta_{Ln}{}^t$ while these inequalities will be reversed for the Paasche measures. These differences between $\beta_{Ln}{}^t$ and $b_{Ln}{}^t$ and $\beta_{Ln}{}^t$ and $b_{Ln}{}^t$ are due to substitution bias.

Since the substitution bias for our observable partial quantity indicators goes in opposite directions, this suggests that taking an average of these two indicators should lead to a closer approximation to the average of the underlying theoretical partial indicators. Thus define the period t *Bennet* (1920) *indicator of relative input quantity change for deflated input n*, $Q_n{}^B(w^{t-1},w^t,x^{t-1},x^t)$, as follows:

(B26) $Q_n{}^B(w_n{}^{t-1},w_n{}^t,x_n{}^{t-1},x_n{}^t) \equiv (1/2)[w_n{}^{t-1} + w_n{}^t][x_n{}^t - x_n{}^{t-1}]$; $n = 1,...,N$; $t = 1,2, ...$

where $w_n{}^t$ is the nth component of the observable real input price vector $w^t \equiv W^t/P^t{\cdot}\mu$ defined earlier by (6) and $x_n{}^t$ is the nth component of the input quantity vector (deflated by the index of primary inputs) $x^t \equiv X^t/X^t{\cdot}\theta$ defined earlier by (8).

The Bennet partial indicators of deflated input quantity change, $Q_n{}^B(w_n{}^{t-1},w_n{}^t,x_n{}^{t-1},x_n{}^t)$, will be at least a first order approximations to the corresponding theoretical measures of input quantity change, $(1/2)\beta_{Lm}{}^t + (1/2)\beta_{Pm}{}^t$, and we would normally expect these approximations to be better than a first order approximation. Note that these approximations are nonparametric.[43]

Note that the sum over n of the individual input quantity change Bennet indicators, $\sum_{n=1}^N Q_n{}^B(w_n{}^{t-1},w_n{}^t,x_n{}^{t-1},x_n{}^t)$, is equal to the overall Bennet indicator of relative input quantity change , $Q^B(w^{t-1},w^t,x^{t-1},x^t)$, defined in section 4 by (40).

[43] These are difference counterparts to the ratio type results obtained by Diewert and Morrison (1986; 672) for the Translog functional form. As noted earlier, an advantage of the present approach is that it is well defined even if some inputs are zero whereas the Diewert-Morrison approach breaks down as any input quantity approaches zero..

Appendix C: Japanese Price and Quantity Data

Table C1: Prices of Main Aggregates

	P_C^t	P_N^t	P_G^t	P_X^t	P_M^t	P_I^t	P_{IV}^t	W_K^t	W_{KIV}^t	W_{LD}^t	W_{LB}^t	P_{DEP}^t	W_{KW}^t
1955	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1956	0.997030	1.046150	1.033880	1.029650	1.053760	1.077990	1.047120	1.081910	1.052520	1.162470	1.051630	1.084910	1.079060
1957	1.027380	1.104460	1.078260	1.034260	1.091090	1.141070	1.052240	1.156770	1.192040	1.361140	1.106040	1.141810	1.170910
1958	1.026250	1.106340	1.026590	0.968780	0.941460	1.106830	1.038450	1.081550	1.090750	1.409410	1.171500	1.110370	1.053760
1959	1.058940	1.146820	1.026600	0.989370	0.933430	1.104060	1.063200	1.152270	1.283080	1.840230	1.276470	1.093170	1.210480
1960	1.090840	1.203860	1.021790	1.017380	0.936580	1.128280	1.061450	1.272050	1.541940	2.538670	1.385160	1.096620	1.446280
1961	1.154690	1.285830	1.027970	1.010180	0.947610	1.210020	1.050610	1.434600	1.877510	3.192780	1.569720	1.142800	1.727580
1962	1.236120	1.376100	1.005770	0.981420	0.937460	1.238500	1.063230	1.416650	1.737270	3.301640	1.762160	1.169640	1.661880
1963	1.324490	1.467960	0.964130	0.999170	0.954060	1.244100	1.078100	1.447480	1.828620	3.733480	2.005080	1.162510	1.734530
1964	1.359500	1.567040	0.935380	1.008560	0.967250	1.263460	1.092800	1.507670	1.930700	4.281610	2.270120	1.171690	1.852010
1965	1.458860	1.664440	0.903830	0.997200	0.959750	1.287310	1.129580	1.485270	1.808260	4.274190	2.548200	1.192280	1.779180
1966	1.516950	1.760620	0.907170	0.994030	0.965610	1.332890	1.142600	1.568190	2.008820	4.874150	2.831780	1.210740	1.937200
1967	1.567860	1.864450	0.931760	0.995450	0.950970	1.388610	1.144880	1.647800	2.227060	5.716390	3.174850	1.243300	2.071540
1968	1.649960	1.977760	0.918190	0.995730	0.950360	1.433910	1.175620	1.729260	2.422450	6.820760	3.655630	1.271050	2.217000
1969	1.710270	2.109430	0.900760	1.008100	0.978950	1.493070	1.224460	1.777010	2.473840	7.761170	4.256680	1.321240	2.258340
1970	1.827940	2.400920	0.890240	1.038050	0.996770	1.561730	1.256900	1.809760	2.397440	8.440790	4.993850	1.378520	2.255240
1971	1.949920	2.700550	0.841400	1.060330	0.961500	1.597710	1.286390	1.693960	1.959910	7.871140	5.799770	1.418850	1.930930
1972	2.059210	3.056210	0.795480	1.052060	0.915090	1.666860	1.411500	1.686480	1.923710	8.744840	6.654600	1.459570	1.855120
1973	2.293510	3.598060	0.851860	1.152710	1.096240	1.933810	1.663190	1.884260	2.241680	10.207760	8.061310	1.646310	2.050080
1974	2.832640	4.587410	1.450220	1.512360	1.756610	2.355840	1.909370	2.165680	2.276260	9.169440	10.334070	2.008760	2.185050
1975	3.160420	5.108800	1.024940	1.586770	1.927220	2.451120	2.008040	2.122670	1.942790	7.836040	12.225790	2.092580	1.959380
1976	3.465260	5.709340	1.061470	1.617900	2.038490	2.556210	2.111660	2.289860	2.290330	8.911060	13.168570	2.146540	2.276390
1977	3.715340	6.174370	1.056740	1.557680	1.966810	2.667580	2.153110	2.361340	2.214140	8.857750	14.391590	2.237830	2.312000
1978	3.873910	6.403510	0.977400	1.458740	1.681340	2.751380	2.180310	2.501630	2.444300	10.079840	15.412520	2.275030	2.589080
1979	3.997830	6.807150	1.039560	1.577670	2.150860	2.941630	2.390120	2.725840	2.855770	11.720960	16.353280	2.379530	2.966870
1980	4.324130	7.337000	1.256800	1.730550	2.937710	3.175590	2.487100	2.822050	2.741050	11.660220	17.353530	2.536570	2.963810
1981	4.521580	7.520150	1.336600	1.786490	2.958080	3.233280	2.531950	2.793550	2.509730	11.548060	18.445300	2.581790	2.828850
1982	4.628340	7.690270	1.299130	1.851010	3.033520	3.247360	2.575510	2.761550	2.369060	11.493750	19.267130	2.605050	2.717670
1983	4.717860	7.791750	1.296640	1.792800	2.869880	3.239120	2.579670	2.706850	2.182940	11.147980	19.589510	2.604860	2.586560
1984	4.828580	7.971620	1.284070	1.821300	2.796780	3.246000	2.559260	2.809070	2.396290	12.596350	20.454430	2.597070	2.845270
1985	4.918420	8.146100	1.396140	1.767550	2.641360	3.225550	2.466960	2.856330	2.455580	13.682780	21.099150	2.580600	2.985830
1986	4.945450	8.225600	1.236200	1.551800	1.847600	3.166480	2.327550	2.796780	2.279880	14.049020	21.674870	2.532860	2.914060
1987	4.947370	8.264210	1.155100	1.486870	1.696490	3.147750	2.266760	2.753690	2.160910	14.759250	22.097420	2.510940	2.841630
1988	4.961360	8.335650	1.095340	1.460980	1.647590	3.174480	2.279210	2.796000	2.238680	16.225280	22.782910	2.522040	2.930240
1989	5.048840	8.529050	1.121160	1.509190	1.756420	3.269890	2.298600	2.872150	2.302370	18.225000	23.940750	2.578240	3.030920
1990	5.171530	8.962910	1.163740	1.534660	1.875460	3.360790	2.323190	2.879450	2.193120	18.891880	25.645820	2.636300	2.950530
1991	5.283030	9.239720	1.152410	1.498570	1.796120	3.408810	2.338150	2.837150	1.989660	17.662450	27.319490	2.670850	2.779400
1992	5.358280	9.343340	1.140510	1.460650	1.703170	3.419440	2.741140	2.800310	1.896910	16.157210	28.206060	2.663460	2.695070
1993	5.402760	9.334670	1.125730	1.362610	1.565730	3.419160	2.976970	2.700410	1.618010	13.767730	28.854180	2.663610	2.429470
1994	5.417350	9.396900	1.091400	1.320330	1.497590	3.390380	2.920080	2.663570	1.558300	12.943610	29.342760	2.631810	2.388270
1995	5.396620	9.405670	1.068760	1.292550	1.466560	3.366170	2.894290	2.655530	1.546450	12.567620	29.507420	2.602870	2.418170
1996	5.395910	9.434970	1.055420	1.338230	1.560540	3.320700	2.886630	2.673330	1.650710	12.773090	29.696380	2.557340	2.545400
1997	5.455230	9.577770	1.075030	1.362740	1.650650	3.318420	2.877270	2.657580	1.607780	12.041860	30.492160	2.559810	2.499340
1998	5.408500	9.553270	1.062110	1.375150	1.601710	3.241150	2.837860	2.501010	1.285240	9.664080	30.131830	2.535780	2.125720
1999	5.374790	9.443580	1.043110	1.253040	1.470210	3.168390	3.178060	2.451880	1.276670	9.168170	29.642000	2.476100	2.101710
2000	5.338730	9.378940	1.037690	1.201000	1.491250	3.155730	3.071270	2.475800	1.370380	9.257340	29.195500	2.452120	2.209440
2001	5.281180	9.339730	1.029590	1.227830	1.520400	3.065950	2.992370	2.370120	1.173960	8.139020	29.358540	2.386180	2.044290
2002	5.192820	9.070420	1.015560	1.212870	1.500880	2.992920	2.959490	2.366130	1.270450	8.230890	28.803100	2.321190	2.153630
2003	5.144860	8.943230	0.998580	1.171080	1.484310	2.955340	2.972450	2.366580	1.368130	8.134240	28.443330	2.279190	2.233090
2004	5.072680	8.859370	0.990770	1.157000	1.525140	2.940310	2.944410	2.464030	1.616600	8.750360	28.024610	2.261540	2.533910
2005	5.009530	8.913120	0.996010	1.173800	1.653540	2.922030	2.956980	2.489540	1.684710	8.827350	28.286970	2.249050	2.627870
2006	4.928420	8.986470	1.009670	1.218630	1.783590	2.921970	2.968640	2.519310	1.737280	8.972510	28.400300	2.257610	2.694240

Table C2: Quantities of Main Aggregates

	Y_C^t	Y_N^t	Y_G^t	Y_X^t	Y_M^t	Y_I^t	Y_{IV}^t	X_K^t	X_{KIV}^t	X_{LD}^t	X_{LB}^t	Y_{DEP}^t	X_{KW}^t
1955	5021.93	99.7	246.2	887.1	-931.19	1618.55	467.26	2070.78	250.35	1349.35	3739.08	-1013.98	1056.79
1956	5530.33	108.3	221.3	1062.31	-1181.97	1988.94	501.74	2082.05	268.12	1352.06	4057.71	-1010.72	1071.35
1957	6012.13	115.17	226.2	1201.54	-1452.99	2412.77	492.66	2153.18	290.27	1358.11	4332.02	-1048.35	1104.88
1958	6386.92	117.23	261.74	1276.55	-1256.36	2535.86	157.46	2283.23	311.15	1365.44	4476.21	-1122.85	1160.28
1959	6887.8	130.53	281.12	1465.38	-1543.03	3008.9	529.22	2417.85	317.86	1377.29	4620.40	-1201.87	1216.19
1960	7717.12	147.44	313.67	1647.27	-1892.68	4052.44	653.59	2584.59	340.40	1384.91	4878.13	-1298.45	1288.70
1961	8572.65	164.8	347.87	1730.88	-2394.61	5029.95	1122.73	2873.57	366.67	1412.86	5027.46	-1474.96	1410.54
1962	9191.56	173.97	409.64	2062.21	-2366.16	5640.77	501.09	3267.51	410.20	1441.78	5241.33	-1725.00	1571.28
1963	10044.54	187.81	486.24	2225.54	-2830.42	6310.78	767.75	3679.77	429.94	1470.95	5309.69	-1985.28	1740.23
1964	11177.28	229.16	504.18	2733.79	-3217.16	7337.75	874.4	4109.11	460.32	1502.12	5454.66	-2255.18	1918.35
1965	11716.05	254.02	539.26	3415.77	-3396.70	7589.15	652.63	4615.61	495.59	1546.55	5609.24	-2574.29	2128.03
1966	12983.96	260.36	607.93	4004.31	-3805.25	8685.01	855.72	5071.31	521.74	1650.33	5834.14	-2855.77	2320.45
1967	14445.61	258.09	613.57	4273.06	-4671.89	10288.48	1521.19	5612.72	554.12	1753.33	5995.64	-3200.82	2543.56
1968	15681.11	263.03	743.96	5297.63	-5244.24	12292.33	1597.88	6327.71	609.57	1796.40	6072.94	-3669.37	2831.89
1969	17369.93	272.4	897.58	6425.65	-5950.82	14428.67	1738.5	7218.94	667.30	1902.65	6147.65	-4263.00	3186.26
1970	18768.86	219.54	1107.79	7530.68	-7299.40	16754.68	1696.7	8287.84	730.14	1958.63	6283.77	-4976.28	3608.98
1971	19777.23	200.85	1357.62	8841.4	-7766.17	17458.28	339.73	9543.67	792.13	2010.11	6370.20	-5821.68	4094.67
1972	21619.87	195.7	1630.08	9225.33	-8530.08	19078.74	251.81	10678.56	804.77	2070.15	6433.29	-6557.78	4548.30
1973	23557.21	182.35	1874.96	9733.5	-10560.73	21349.75	1720.37	11840.44	813.85	2197.03	6637.98	-7317.38	5006.56
1974	23359.72	174.96	1363.1	12014.83	-11045.24	20170.17	1765.38	13105.60	874.89	2288.91	6507.01	-8134.31	5511.41
1975	24406.43	164.74	2084.9	11903.76	-9894.33	19936.55	-486.32	14071.44	937.80	2382.35	6336.13	-8722.92	5928.25
1976	25034.78	164.12	2107.73	13889.81	-10533.70	20780.55	565.45	14889.30	920.42	2443.24	6626.77	-9209.92	6292.83
1977	25928.14	195.11	2384.03	15520.87	-10906.27	21330.58	-238.05	15693.36	940.76	2541.07	6776.30	-9690.84	6648.35
1978	27211.57	199.64	2760.08	15504.03	-11461.00	23135.89	-400.64	16534.23	932.26	2609.20	6850.16	-10218.00	6997.19
1979	29010	222.93	2985.7	16156.37	-12917.35	24388.68	2330.44	17572.47	917.67	2664.22	6963.09	-10897.68	7404.59
1980	29175.73	247.44	2961.08	18909.38	-11991.39	24266.08	1632.92	18658.69	1001.93	2724.27	7096.29	-11595.97	7841.87
1981	29574.38	255.82	3221.61	21126.97	-12220.92	24825.15	499.1	19745.21	1061.03	2757.40	7232.80	-12297.31	8275.47
1982	31045.18	257.46	3490.64	21105.75	-12401.61	24952.99	331.17	20819.31	1079.45	2805.51	7330.42	-13006.19	8688.31
1983	32055.76	277.14	3832.22	21747.47	-12004.07	24683.58	-482.75	21837.91	1091.80	2847.83	7573.77	-13686.58	9070.12
1984	32799.64	294.98	4078.99	24565.89	-13270.73	25793.75	664.29	22836.71	1073.80	2858.20	7641.00	-14372.98	9426.97
1985	33816.99	309.17	3823.26	26018.54	-13399.11	28001.84	471.72	24609.99	1098.48	2918.38	7741.62	-15621.28	10041.42
1986	34804.25	331.6	4761.21	24414.83	-13447.72	29879.8	-494.43	26285.34	1115.88	2928.86	7844.75	-16865.70	10568.07
1987	36189.58	342.67	4864.93	24188.97	-15084.68	32170.76	354.18	28206.74	1097.86	2965.80	7970.15	-18221.64	11232.70
1988	37910.76	358.7	5392.22	25437.14	-17470.04	36216.42	432.53	29998.10	1110.60	3010.66	8168.81	-19513.54	11828.58
1989	39642.56	377.64	5872.14	27800	-20373.36	39761.58	1511.75	32068.35	1126.02	3046.20	8328.94	-21067.91	12467.18
1990	41428.75	393.58	6377.27	29635.75	-21940.26	43023.96	1007.8	34502.60	1179.56	3090.27	8428.33	-22885.06	13223.59
1991	42338.79	417.64	7506.15	30911.36	-21754.50	44379.49	543.81	37050.83	1214.98	3112.64	8546.10	-24716.06	14069.82
1992	43274.97	455.31	8017.22	32142.49	-21465.20	43490.55	30.73	39485.17	1234.44	3165.26	8523.37	-26456.44	14880.78
1993	43637.05	475.91	8417.17	32169.17	-21223.42	42076.69	-578.01	41286.14	1235.88	3217.54	8478.52	-27694.61	15526.74
1994	44856.84	490.11	9057.28	33343.86	-22881.54	41384.3	-485.12	42385.26	1208.95	3267.65	8488.29	-28380.94	15996.10
1995	45742.75	516.3	9800.58	34832.96	-25941.84	41473.75	682.7	43217.56	1186.30	3298.99	8563.54	-28871.26	16383.14
1996	46895.81	524.71	10109.66	36801.75	-29815.42	43423.66	941.34	44299.00	1218.30	3344.56	8591.77	-29593.21	16793.51
1997	46975.66	516.52	10034.35	40874.61	-30304.37	43709.9	898.42	45691.61	1262.57	3388.62	8561.06	-30619.22	17224.38
1998	46367.62	574.19	10676.12	39773.36	-28421.29	40835.54	-24.09	47134.23	1305.07	3444.28	8503.13	-31679.72	17664.83
1999	46760.01	620.35	11401.73	40588.1	-29240.97	40555.97	-1246.7	48066.24	1303.77	3465.14	8440.53	-32380.15	17926.38
2000	47416.24	574.99	11257.3	45789.53	-32202.49	40715.85	121.16	48854.51	1236.66	3505.16	8595.55	-32980.39	18141.21
2001	48359.52	584.1	11463.2	42592.6	-32777.73	40621.5	-933.62	49938.82	1243.22	3535.54	8456.40	-33842.68	18395.67
2002	48857.91	617.14	11668.24	45801.94	-33060.49	38912.12	-501.05	50927.95	1192.16	3552.41	8360.61	-34627.69	18631.25
2003	49114.48	657.09	11982.72	49728.8	-34448.27	38698.68	-702.5	51479.05	1164.74	3590.69	8342.31	-35050.16	18782.74
2004	50179.52	690.57	12023.29	56596.35	-37014.35	39369.2	-101.29	52176.34	1126.40	3585.42	8396.39	-35619.17	18946.98
2005	51190.08	723.24	11688.76	60478.87	-39484.71	40884.87	1130.43	52938.69	1089.79	3590.65	8403.60	-36236.69	19138.86
2006	52727.91	742.48	10671.31	66512.12	-41765.17	41476.98	2469.53	54059.57	1172.18	3635.58	8505.20	-37180.62	19394.42

Table C3: Prices of Investment Goods

	P_{I1}^t	P_{I2}^t	P_{I3}^t	P_{I4}^t	P_{I5}^t	P_{I6}^t	P_{I7}^t	P_{I8}^t	P_{I9}^t	P_{I10}^t	P_{I11}^t	P_{I12}^t
1955	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1956	1.013960	1.076640	1.006890	1.025680	1.039070	1.103920	1.111840	0.995050	1.063490	1.196700	1.047690	1.002980
1957	1.080960	1.108720	1.081150	1.126050	1.099140	1.210150	1.334730	1.033280	1.114810	1.313380	1.058980	0.999740
1958	1.039700	1.052270	1.078030	1.067620	1.047520	1.113850	1.366490	1.082550	1.109340	1.192400	1.060670	0.987220
1959	1.061940	1.058460	1.072890	1.117600	1.085140	1.080550	1.325200	1.009920	1.157220	1.189420	1.027890	0.986490
1960	1.170090	1.084550	1.033570	1.176150	1.126280	1.141930	1.387940	1.011260	1.135630	1.177050	1.048870	1.002990
1961	1.271830	1.212600	1.107270	1.374640	1.222520	1.171780	1.419660	1.012840	1.131200	1.166290	1.030430	1.043140
1962	1.283050	1.250300	1.121380	1.376590	1.291040	1.132420	1.472200	1.018640	1.122300	1.137570	1.046120	1.067480
1963	1.392270	1.276650	1.155190	1.422010	1.301660	1.119300	1.440100	0.960180	1.124600	1.119170	1.064250	1.092740
1964	1.394690	1.312550	1.166080	1.426540	1.385530	1.149880	1.443800	0.931880	1.111750	1.121010	1.138170	1.128690
1965	1.523840	1.349260	1.204320	1.414380	1.414300	1.177330	1.448240	0.944150	1.112520	1.119800	1.134280	1.134290
1966	1.822960	1.429770	1.004380	1.524080	1.404440	1.197440	1.446440	0.926330	1.114520	1.124980	1.143920	1.151880
1967	2.171790	1.515940	1.233400	1.664090	1.402840	1.242930	1.482450	0.911510	1.112220	1.192510	1.153410	1.164540
1968	2.551500	1.577280	1.283510	1.754590	1.418160	1.258000	1.550190	0.929670	1.103700	1.193470	1.158150	1.204550
1969	2.826740	1.664830	1.416990	1.825200	1.418540	1.293050	1.603910	0.950400	1.092270	1.268130	1.225660	1.218920
1970	2.956160	1.769140	1.338410	1.909170	1.538020	1.354870	1.648180	0.969220	1.100890	1.344780	1.285960	1.236670
1971	2.906870	1.818690	1.425620	1.918090	1.568150	1.379620	1.688930	0.955250	1.114280	1.414590	1.330410	1.309630
1972	2.922680	1.948420	1.411250	2.203380	1.637650	1.390760	1.718050	0.922610	1.131220	1.425120	1.340760	1.373480
1973	3.379110	2.374400	1.602980	3.107440	2.002810	1.551900	1.889760	0.938260	1.166320	1.824390	1.477850	1.482730
1974	3.680360	2.862390	1.835910	3.425690	2.597900	2.156160	2.410360	1.118460	1.443260	1.884620	1.904720	1.891850
1975	4.186620	2.996910	1.923020	3.378120	2.591000	2.163640	2.453290	1.128530	1.488720	2.261930	1.973130	2.006550
1976	4.586530	3.179360	2.077470	3.582320	2.659050	2.241420	2.469560	1.122230	1.521240	2.305070	2.027470	2.041120
1977	4.906660	3.332960	2.163310	3.609510	2.850350	2.362540	2.558420	1.138570	1.562110	2.595450	2.127180	2.094250
1978	5.036890	3.497150	2.135720	3.357750	2.832040	2.421600	2.580640	1.108140	1.577450	2.577170	2.107920	2.156250
1979	5.149150	3.825380	2.363480	4.088120	3.093030	2.491710	2.662150	1.122710	1.606320	2.536380	2.192150	2.270590
1980	5.372870	4.195140	2.439750	4.354170	3.322060	2.710570	2.789050	1.158250	1.631930	2.879040	2.333480	2.466950
1981	5.717980	4.296200	2.492800	4.031610	3.335350	2.752170	2.829740	1.151250	1.643400	2.938310	2.363120	2.486140
1982	5.947020	4.312120	2.514700	4.303630	3.331890	2.723780	2.831030	1.137130	1.642560	3.248480	2.397090	2.533930
1983	6.094470	4.315810	2.492790	4.305520	3.311860	2.742610	2.811790	1.107430	1.630370	3.419340	2.360120	2.531910
1984	6.287140	4.371940	2.498700	4.440060	3.342740	2.811900	2.810440	1.082000	1.623540	3.015210	2.350210	2.531630
1985	6.392240	4.372990	2.498290	4.583920	3.337870	2.808680	2.808240	1.034230	1.605130	2.985130	2.350940	2.528720
1986	6.368370	4.344430	2.437090	4.261590	3.324890	2.765360	2.773380	0.963970	1.592580	2.816950	2.275550	2.470260
1987	6.146010	4.389660	2.421480	4.540350	3.313510	2.690430	2.725670	0.905460	1.573860	2.737830	2.289130	2.452040
1988	6.154320	4.469690	2.426240	4.375910	3.343560	2.743440	2.738000	0.885840	1.570470	2.721350	2.315690	2.451640
1989	6.305180	4.670140	2.481040	4.592570	3.420720	2.848770	2.802670	0.887970	1.577650	2.760760	2.337570	2.456000
1990	6.420740	4.857770	2.548100	4.666190	3.517260	2.910200	2.870330	0.888020	1.577920	2.810990	2.387530	2.477360
1991	6.190660	4.975180	2.569040	4.584930	3.572650	2.960160	2.906220	0.865850	1.572320	2.916740	2.439920	2.501760
1992	5.948050	5.048310	2.525920	4.556200	3.625700	2.961700	2.900560	0.845750	1.577050	2.872270	2.429750	2.434270
1993	5.666390	5.079810	2.560560	4.984620	3.604260	2.947700	2.880620	0.827700	1.568860	2.999960	2.424740	2.415900
1994	5.423990	5.086130	2.467130	4.782250	3.538880	2.894820	2.850640	0.799820	1.554450	2.879680	2.382140	2.343930
1995	4.805810	5.106470	2.443430	4.614750	3.516910	2.901790	2.825860	0.755750	1.542220	2.866900	2.365060	2.311520
1996	5.068370	5.102960	2.494960	4.768820	3.488730	2.863760	2.800280	0.696560	1.521700	2.842940	2.334700	2.288520
1997	5.238470	5.141800	2.544490	4.887030	3.491090	2.874880	2.815980	0.666090	1.520340	2.894040	2.342460	2.280060
1998	5.219020	5.016590	2.520910	4.511860	3.473840	2.828210	2.772050	0.637920	1.496530	2.915610	2.324650	2.241050
1999	5.497600	4.927600	2.424140	4.535090	3.425900	2.768720	2.714390	0.604340	1.479770	2.774340	2.293180	2.198360
2000	5.579630	4.962480	2.406320	4.553340	3.433250	2.764470	2.706840	0.574070	1.480040	2.719610	2.322340	2.173680
2001	5.540430	4.877160	2.365110	4.489180	3.411480	2.731070	2.664630	0.514400	1.449730	2.606870	2.315480	2.123380
2002	5.477710	4.801400	2.331370	4.411300	3.379950	2.700530	2.616250	0.474780	1.423230	2.651800	2.284410	2.065220
2003	5.698420	4.825420	2.325400	4.425600	3.375280	2.707690	2.587950	0.434590	1.409490	2.751410	2.262830	1.991220
2004	6.055530	4.873480	2.326260	4.467240	3.370950	2.748040	2.573640	0.403960	1.405250	2.841450	2.255820	1.951780
2005	6.037300	4.917170	2.341050	4.434590	3.402800	2.802230	2.561750	0.373860	1.397110	2.890480	2.245130	1.916640
2006	5.908520	4.983170	2.362840	4.552560	3.447370	2.821160	2.549440	0.358830	1.391600	2.903540	2.227290	1.893860

Table C4: Prices of Capital Services

	W_{K1}^{t}	W_{K2}^{t}	W_{K3}^{t}	W_{K4}^{t}	W_{K5}^{t}	W_{K6}^{t}	W_{K7}^{t}	W_{K8}^{t}	W_{K9}^{t}	W_{K10}^{t}	W_{K11}^{t}	W_{K12}^{t}
1955	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1956	1.012050	1.077430	1.005060	1.023650	1.036750	1.101050	1.105140	0.972690	1.033170	1.244750	1.024550	0.953070
1957	1.082300	1.119180	1.081150	1.128260	1.105460	1.221940	1.343230	1.032100	1.081750	1.370320	1.040920	0.922020
1958	1.036350	1.008900	1.075690	1.063690	1.040010	1.103100	1.373200	1.075690	1.075080	1.241840	1.070270	0.872360
1959	1.077010	1.093970	1.083080	1.135560	1.117800	1.126010	1.409080	1.045390	1.148570	1.305500	1.074560	0.895500
1960	1.203150	1.218890	1.058440	1.224930	1.216120	1.267880	1.558900	1.101150	1.164690	1.362870	1.134330	0.949120
1961	1.330910	1.387420	1.146730	1.458200	1.365470	1.359460	1.656420	1.141420	1.186980	1.424550	1.136320	1.021840
1962	1.324890	1.367720	1.150330	1.440740	1.409610	1.278160	1.683790	1.123650	1.154810	1.428090	1.142210	1.002910
1963	1.447030	1.428390	1.189100	1.497860	1.440720	1.285080	1.662370	1.067200	1.162630	1.444680	1.174130	1.036810
1964	1.462730	1.514700	1.207090	1.515950	1.559180	1.346950	1.709260	1.051720	1.165100	1.495720	1.318690	1.094400
1965	1.572900	1.492980	1.233270	1.480210	1.549860	1.336840	1.674740	1.038150	1.152250	1.509730	1.290400	1.056480
1966	1.881800	1.618340	1.031570	1.605060	1.565760	1.384320	1.708460	1.029830	1.180130	1.588210	1.313730	1.091330
1967	2.248610	1.741130	1.271520	1.763020	1.585790	1.457060	1.775520	1.024040	1.189470	1.698130	1.342410	1.120500
1968	2.657290	1.849670	1.328170	1.871460	1.629310	1.499940	1.879140	1.055910	1.200070	1.710980	1.375430	1.176490
1969	2.927180	1.924470	1.458650	1.931520	1.603950	1.515810	1.933460	1.072060	1.185430	1.791720	1.455620	1.172070
1970	3.046350	1.977810	1.366690	1.996570	1.694320	1.548840	1.975060	1.079840	1.178160	1.859480	1.527050	1.157930
1971	2.824270	1.807930	1.423220	1.936800	1.618540	1.455450	1.907550	1.012030	1.149940	1.841510	1.525790	1.097220
1972	2.740030	1.820380	1.395810	2.192560	1.645310	1.417090	1.897240	0.961770	1.166220	1.824920	1.534750	1.093020
1973	3.078940	2.147640	1.575690	3.061930	1.976590	1.543770	2.054100	0.962450	1.209740	2.301960	1.618810	1.148230
1974	3.245840	2.364510	1.775050	3.297970	2.466230	2.040550	2.510100	1.103250	1.485980	2.284500	1.956530	1.373060
1975	3.582610	2.264900	1.831500	3.177460	2.356980	1.934150	2.468460	1.081880	1.518380	2.643720	1.995580	1.335690
1976	3.963840	2.536440	1.996120	3.415180	2.473300	2.070410	2.576560	1.102220	1.623090	2.789440	2.078130	1.406050
1977	4.222730	2.603100	2.073640	3.425670	2.622330	2.153650	2.666290	1.116500	1.690080	3.140220	2.145390	1.416290
1978	4.381570	2.856680	2.064060	3.223960	2.654570	2.264860	2.761330	1.113010	1.751790	3.222250	2.170050	1.480380
1979	4.507120	3.229380	2.296310	3.960250	2.947250	2.385620	2.922340	1.149320	1.814480	3.306750	2.267230	1.593590
1980	4.629820	3.361650	2.348560	4.161370	3.090950	2.516620	3.023000	1.168260	1.818290	3.687790	2.338690	1.687360
1981	4.897570	3.315220	2.387560	3.820410	3.053940	2.497370	3.034880	1.152790	1.804050	3.711510	2.321440	1.662290
1982	5.063120	3.256280	2.401880	4.057290	3.020360	2.434700	3.010630	1.132990	1.788470	4.034760	2.335070	1.663930
1983	5.161810	3.182600	2.370400	4.030210	2.964510	2.406330	2.969630	1.099600	1.779340	4.208090	2.289180	1.631130
1984	5.396870	3.394960	2.396840	4.209080	3.059780	2.540550	3.042230	1.100480	1.817140	3.826880	2.304600	1.669640
1985	5.548270	3.504910	2.413670	4.386810	3.102460	2.586630	3.079140	1.074140	1.808960	3.842860	2.321890	1.680970
1986	5.563380	3.466120	2.356050	4.077840	3.086050	2.538310	3.031630	1.014030	1.803670	3.588160	2.292400	1.628410
1987	5.402230	3.471320	2.341440	4.338350	3.059810	2.449140	2.965490	0.958280	1.782560	3.444300	2.293980	1.602940
1988	5.470500	3.577000	2.352960	4.197370	3.105530	2.516640	2.993940	0.941050	1.784180	3.400520	2.315760	1.611700
1989	5.650760	3.764730	2.406990	4.408540	3.182880	2.619500	3.032410	0.938360	1.769470	3.378120	2.335350	1.610510
1990	5.742780	3.804640	2.458140	4.442400	3.230190	2.625070	3.053280	0.926120	1.749770	3.369270	2.365620	1.596720
1991	5.520460	3.752770	2.470550	4.336430	3.236490	2.614190	3.047600	0.894070	1.710640	3.465960	2.386030	1.576300
1992	5.294250	3.705370	2.425190	4.298630	3.269270	2.599370	3.023300	0.869210	1.705660	3.401700	2.375420	1.516260
1993	4.993690	3.549070	2.440840	4.648790	3.183790	2.508930	2.946570	0.837700	1.669510	3.493200	2.345220	1.474790
1994	4.786800	3.508130	2.350170	4.454260	3.124680	2.454030	2.929910	0.812120	1.664170	3.397830	2.309870	1.437440
1995	4.267640	3.542170	2.328860	4.301660	3.113400	2.464630	2.914430	0.773300	1.664370	3.421450	2.291760	1.431600
1996	4.510910	3.651830	2.386500	4.470490	3.121960	2.468630	2.915450	0.728620	1.649980	3.435220	2.281130	1.432150
1997	4.654280	3.634830	2.437210	4.584900	3.121700	2.473760	2.927520	0.703160	1.630260	3.493230	2.297450	1.419600
1998	4.607190	3.349300	2.413980	4.207790	3.047550	2.358460	2.836220	0.668440	1.579560	3.460420	2.251170	1.359580
1999	4.852360	3.300830	2.323920	4.235000	3.012780	2.313840	2.787150	0.636730	1.569080	3.299830	2.233120	1.334600
2000	4.934550	3.385540	2.311170	4.268620	3.041830	2.340500	2.797650	0.612830	1.573730	3.254070	2.258180	1.333210
2001	4.880130	3.249700	2.265460	4.187620	2.993430	2.277340	2.729250	0.556500	1.533910	3.090160	2.223150	1.287900
2002	4.846020	3.314760	2.239720	4.137190	2.995220	2.289080	2.702340	0.525110	1.508660	3.179660	2.203310	1.268810
2003	5.052020	3.378150	2.236370	4.162620	3.009580	2.320640	2.688440	0.493220	1.492050	3.323880	2.193470	1.234080
2004	5.438420	3.638170	2.259960	4.264880	3.078460	2.441150	2.734320	0.476890	1.497340	3.508700	2.213520	1.245540
2005	5.450470	3.736330	2.284620	4.258030	3.134320	2.516730	2.749200	0.454490	1.493440	3.593580	2.212090	1.235870
2006	5.372410	3.814800	2.321220	4.404170	3.209360	2.562170	2.759480	0.443600	1.488780	3.626360	2.207230	1.238990

Table C5: Prices of Inventory Services, Land Services and Labour Inputs

	W_{KIV1}^t	W_{KIV2}^t	W_{KIV3}^t	W_{KIV4}^t	W_{LD1}^t	W_{LD2}^t	W_{LD3}^t	W_{LD4}^t	W_{LB1}^t	W_{LB2}^t	W_{LB3}^t
1955	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1956	1.043440	1.043440	1.061550	1.043440	1.135470	1.270140	1.244560	1.230670	1.051630	1.051630	1.051630
1957	1.104180	1.104190	1.293860	1.104180	1.288190	1.631390	1.571420	1.583180	1.106040	1.106040	1.106040
1958	1.008100	1.008100	1.187050	1.008100	1.278250	1.871490	1.785990	1.812570	1.171500	1.171500	1.171500
1959	1.196480	1.196480	1.379930	1.196480	1.566800	2.784440	2.656850	2.573610	1.276470	1.276470	1.276470
1960	1.425300	1.425300	1.679750	1.425300	1.931170	4.889860	4.252750	4.101770	1.385160	1.385160	1.385160
1961	1.599870	1.574240	2.327070	1.585280	2.166240	7.303600	5.966420	5.854340	1.569720	1.569720	1.569720
1962	1.522540	1.465500	2.079910	1.486780	2.040700	8.362630	6.661360	6.492260	1.762160	1.762160	1.762160
1963	1.613500	1.530380	2.179840	1.560980	2.160570	10.128740	7.818580	7.725070	2.005080	2.005080	2.005080
1964	1.717140	1.619760	2.270420	1.654510	2.325780	12.016850	9.306510	9.297390	2.270120	2.270120	2.270120
1965	1.647410	1.540450	2.006850	1.580010	2.287510	11.832880	9.412880	9.436570	2.548200	2.548200	2.548200
1966	1.781710	1.658730	2.430620	1.699360	2.630230	12.915870	10.826490	10.761300	2.831780	2.831780	2.831780
1967	1.889920	1.750660	3.091250	1.785290	3.112180	14.536570	12.654850	12.867060	3.174850	3.174850	3.174850
1968	2.045230	1.870940	3.486630	1.891400	3.553510	17.232300	15.372530	15.973710	3.655630	3.655630	3.655630
1969	2.083160	1.898010	3.637120	1.900690	3.738480	19.732820	17.992710	19.083260	4.256680	4.256680	4.256680
1970	2.037990	1.850400	3.439130	1.839060	3.792670	21.853770	19.841280	21.687540	4.993850	4.993850	4.993850
1971	1.689130	1.523130	2.693710	1.501920	3.427970	20.778490	18.427540	20.803210	5.799770	5.799770	5.799770
1972	1.657300	1.526290	2.585020	1.482420	3.871070	23.079710	19.926660	23.835990	6.654600	6.654600	6.654600
1973	1.886700	1.706180	3.338750	1.740610	4.454890	27.259960	22.794220	28.875430	8.061310	8.061310	8.061310
1974	1.907800	1.693230	3.480970	1.778920	4.406940	23.623040	19.854000	25.636530	10.334070	10.334070	10.334070
1975	1.647420	1.469720	2.749260	1.541890	4.002770	19.682780	16.648130	21.702340	12.225790	12.225790	12.225790
1976	1.948290	1.727020	3.233080	1.809120	4.808830	21.819870	18.496890	24.682640	13.168570	13.168570	13.168570
1977	1.909690	1.691140	2.960590	1.727370	4.917040	21.186930	18.063160	24.849900	14.391590	14.391590	14.391590
1978	2.128160	1.858560	3.113320	1.922920	5.762680	23.525680	20.116500	28.776380	15.412520	15.412520	15.412520
1979	2.423090	2.129990	3.896530	2.355190	6.719480	26.715940	22.977900	34.554990	16.353280	16.353280	16.353280
1980	2.263380	2.036210	3.998700	2.345790	6.619480	26.085410	22.493770	35.622920	17.353530	17.353530	17.353530
1981	2.099820	1.895890	3.255290	2.191290	6.514820	25.445460	22.030920	36.129500	18.445300	18.445300	18.445300
1982	1.997340	1.806670	2.958250	2.048130	6.404970	25.134460	21.849260	36.504720	19.267130	19.267130	19.267130
1983	1.859660	1.657540	2.657310	1.858210	6.178410	24.255490	21.157290	35.658400	19.589510	19.589510	19.589510
1984	2.057540	1.798990	2.850640	2.031680	6.962820	27.329720	23.994040	40.212750	20.454430	20.454430	20.454430
1985	2.134220	1.839390	3.035610	1.958820	7.468590	29.542980	26.364940	43.459370	21.099150	21.099150	21.099150
1986	2.018290	1.713900	2.928790	1.651870	7.366850	29.941610	27.711480	44.516390	21.674870	21.674870	21.674870
1987	1.914660	1.615940	2.905170	1.521990	7.100350	31.150430	30.270820	46.693680	22.097420	22.097420	22.097420
1988	1.983820	1.670870	3.026100	1.573340	7.226160	34.306810	34.428140	50.844950	22.782910	22.782910	22.782910
1989	2.028980	1.702280	3.184420	1.646110	7.308000	38.864760	39.915930	57.033460	23.940750	23.940750	23.940750
1990	1.925880	1.607040	3.170560	1.560640	6.903300	40.492950	42.289090	59.310810	25.645820	25.645820	25.645820
1991	1.755410	1.457820	2.866360	1.391640	6.503750	38.182040	39.506590	55.201480	27.319490	27.319490	27.319490
1992	1.688670	1.398690	2.583530	1.310020	6.498810	35.382610	35.194900	50.527510	28.206060	28.206060	28.206060
1993	1.449980	1.194810	2.144730	1.098350	5.791880	30.781750	29.114040	43.851060	28.854180	28.854180	28.854180
1994	1.403520	1.150490	1.997840	1.052880	5.642240	29.634820	26.604400	41.967690	29.342760	29.342760	29.342760
1995	1.398960	1.138530	1.867270	1.065900	5.601670	29.534330	24.922760	42.033650	29.507420	29.507420	29.507420
1996	1.496620	1.204350	1.927880	1.165540	5.797960	30.872150	24.405080	44.063770	29.696380	29.696380	29.696380
1997	1.462750	1.169530	1.835550	1.132920	5.640450	29.694680	22.168570	42.613240	30.492160	30.492160	30.492160
1998	1.179070	0.932520	1.382110	0.892890	4.640500	24.246230	17.113960	35.183780	30.131830	30.131830	30.131830
1999	1.174830	0.916650	1.320130	0.904520	4.590610	23.464550	15.641170	33.935310	29.642000	29.642000	29.642000
2000	1.263410	0.970420	1.374550	1.001190	4.789800	24.058540	15.208670	34.946080	29.195500	29.195500	29.195500
2001	1.121900	0.858110	1.130760	0.623190	4.400390	21.310100	12.830460	31.314010	29.358540	29.358540	29.358540
2002	1.219940	0.929910	1.135470	0.677610	4.640480	21.655060	12.476130	32.190200	28.803100	28.803100	28.803100
2003	1.310760	0.993370	1.278020	0.737270	4.856630	21.156920	11.909550	32.077210	28.443330	28.443330	28.443330
2004	1.548720	1.169200	1.439040	0.909110	5.487130	22.369050	12.506160	34.599480	28.024610	28.024610	28.024610
2005	1.616290	1.214560	1.494870	0.944380	5.753020	21.938540	12.575080	34.622810	28.286970	28.286970	28.286970
2006	1.669090	1.248430	1.536550	0.970710	5.798470	21.336310	13.187400	34.798120	28.400300	28.400300	28.400300

Table C6: Prices of Depreciations

	P_{DEP1}^{t}	P_{DEP2}^{t}	P_{DEP3}^{t}	P_{DEP4}^{t}	P_{DEP5}^{t}	P_{DEP6}^{t}	P_{DEP7}^{t}	P_{DEP8}^{t}	P_{DEP9}^{t}	P_{DEP10}^{t}	P_{DEP11}^{t}	P_{DEP12}^{t}
1955	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1956	1.012540	1.077900	1.005470	1.024220	1.037650	1.102360	1.103620	0.973900	1.065050	1.235480	1.025110	0.948160
1957	1.075650	1.091770	1.075670	1.120340	1.093440	1.204020	1.305920	0.993290	1.111480	1.310730	1.033150	0.900770
1958	1.041520	1.030710	1.079880	1.069450	1.049060	1.115760	1.345570	1.041480	1.114470	1.220350	1.076740	0.865700
1959	1.054040	1.017810	1.064890	1.109270	1.076850	1.072490	1.300630	0.967800	1.152040	1.206240	1.050440	0.836640
1960	1.134610	1.017590	1.012050	1.151670	1.102500	1.118160	1.346710	0.951990	1.115010	1.179570	1.066270	0.810010
1961	1.222830	1.106210	1.074980	1.334550	1.186110	1.137610	1.366590	0.942980	1.101090	1.183690	1.039100	0.815730
1962	1.235310	1.133890	1.090300	1.338440	1.254180	1.101040	1.425320	0.951820	1.092980	1.232970	1.060250	0.817290
1963	1.334030	1.147580	1.117710	1.375880	1.258460	1.082990	1.385300	0.900060	1.089840	1.234250	1.076730	0.815260
1964	1.331800	1.172960	1.124440	1.375590	1.334870	1.108810	1.390880	0.874740	1.073480	1.254650	1.194120	0.830880
1965	1.457870	1.207020	1.163430	1.366360	1.365040	1.137360	1.400230	0.878990	1.076860	1.311960	1.188950	0.826310
1966	1.719320	1.264690	0.964150	1.463040	1.352050	1.149480	1.388890	0.858300	1.072700	1.342830	1.194180	0.818060
1967	2.033800	1.322240	1.180770	1.593090	1.351260	1.189900	1.421410	0.844420	1.068350	1.419850	1.208800	0.816060
1968	2.372390	1.360590	1.222950	1.671800	1.363790	1.198640	1.484920	0.853040	1.056100	1.411420	1.224130	0.826190
1969	2.641530	1.436730	1.353300	1.743150	1.367160	1.234930	1.556670	0.872640	1.048870	1.500390	1.309340	0.833620
1970	2.791870	1.524390	1.281700	1.828270	1.481360	1.297460	1.631160	0.891490	1.061370	1.593860	1.394280	0.846800
1971	2.724000	1.573590	1.381190	1.858320	1.528890	1.336630	1.693200	0.888540	1.088280	1.696460	1.461410	0.866430
1972	2.700430	1.682480	1.373860	2.145010	1.605000	1.353920	1.732220	0.860030	1.110360	1.721860	1.499520	0.883520
1973	3.080060	2.052020	1.565850	3.035460	1.970390	1.515950	1.920580	0.874440	1.149870	2.212220	1.604430	0.952510
1974	3.329620	2.469610	1.794600	3.348610	2.558530	2.107650	2.457810	1.033680	1.424800	2.290580	1.988690	1.146970
1975	3.784020	2.592300	1.890020	3.320150	2.566820	2.126510	2.521840	1.046660	1.480330	2.782010	2.088380	1.167850
1976	4.123850	2.743330	2.036270	3.511280	2.623010	2.196970	2.538620	1.036720	1.509610	2.833170	2.139430	1.178570
1977	4.425620	2.874430	2.126520	3.548130	2.818240	2.322370	2.640110	1.054360	1.555630	3.206470	2.225760	1.195710
1978	4.542370	3.011400	2.098710	3.299570	2.799320	2.379640	2.666090	1.024750	1.570550	3.199620	2.223550	1.189060
1979	4.618700	3.270220	2.313720	4.002050	3.046810	2.439250	2.746910	1.033680	1.593930	3.151150	2.293310	1.247690
1980	4.813760	3.575080	2.391950	4.268870	3.278250	2.657470	2.888340	1.065500	1.620720	3.582660	2.403220	1.355250
1981	5.147220	3.668540	2.452900	3.967090	3.303930	2.708130	2.947780	1.062470	1.637700	3.671750	2.416060	1.372830
1982	5.359320	3.697660	2.482050	4.247750	3.311590	2.688420	2.966550	1.053740	1.642300	4.062040	2.450940	1.394930
1983	5.509280	3.718020	2.465780	4.258860	3.300330	2.712890	2.957010	1.030810	1.633450	4.289910	2.425830	1.392140
1984	5.684700	3.768130	2.469310	4.387830	3.329390	2.778820	2.956490	1.009500	1.626160	3.808370	2.408160	1.383930
1985	5.797680	3.781160	2.472410	4.536430	3.330400	2.779590	2.960750	0.971180	1.611660	3.792120	2.406190	1.354450
1986	5.823630	3.760630	2.418010	4.228220	3.326640	2.743710	2.933880	0.914340	1.602450	3.590090	2.381550	1.314900
1987	5.681080	3.810870	2.412800	4.524080	3.329350	2.680790	2.899250	0.865750	1.591110	3.502070	2.397490	1.306960
1988	5.732290	3.883720	2.419200	4.363200	3.361990	2.735470	2.918020	0.845190	1.590540	3.466040	2.412670	1.303470
1989	5.910210	4.055390	2.472360	4.576510	3.440520	2.838800	2.986900	0.845100	1.598220	3.492650	2.430030	1.289330
1990	6.052380	4.220370	2.541240	4.653630	3.546790	2.902360	3.064640	0.844450	1.602710	3.556730	2.484700	1.291110
1991	5.875480	4.339070	2.575550	4.596550	3.622740	2.967660	3.120240	0.826190	1.606970	3.725850	2.537150	1.294960
1992	5.645630	4.401090	2.533580	4.570020	3.675280	2.970680	3.116070	0.805300	1.616940	3.679010	2.533720	1.245950
1993	5.393090	4.436340	2.576890	5.016420	3.666660	2.966510	3.103460	0.789470	1.615220	3.854190	2.540680	1.246530
1994	5.175650	4.439310	2.484800	4.816510	3.608760	2.915560	3.070990	0.763550	1.602780	3.714920	2.507750	1.223200
1995	4.607230	4.456350	2.460790	4.647550	3.589890	2.922410	3.041240	0.722340	1.591470	3.707580	2.485980	1.220550
1996	4.841410	4.446780	2.509200	4.796040	3.558000	2.880110	3.010340	0.669960	1.569100	3.672760	2.456230	1.204530
1997	5.006680	4.494470	2.566140	4.928620	3.569510	2.899350	3.038440	0.646310	1.573940	3.756050	2.478940	1.195080
1998	5.043500	4.444300	2.576230	4.610880	3.596360	2.890280	3.032570	0.628530	1.570380	3.841270	2.478720	1.184780
1999	5.310730	4.365580	2.479030	4.637770	3.551260	2.831410	2.975160	0.595910	1.553810	3.664570	2.457200	1.160100
2000	5.370230	4.382820	2.453700	4.642980	3.546550	2.818890	2.960560	0.566250	1.550790	3.579210	2.467160	1.142180
2001	5.346700	4.319450	2.418090	4.589740	3.533600	2.792250	2.922370	0.515200	1.524590	3.434580	2.448600	1.118300
2002	5.271300	4.244090	2.376880	4.497430	3.489400	2.753260	2.862470	0.478200	1.492950	3.494610	2.405850	1.084100
2003	5.465680	4.252820	2.363010	4.497180	3.471790	2.751480	2.824660	0.442490	1.474210	3.628940	2.379480	1.041710
2004	5.800240	4.295870	2.360650	4.533270	3.460920	2.788660	2.808280	0.416080	1.467550	3.752950	2.360170	1.020550
2005	5.791830	4.342500	2.379370	4.507180	3.500240	2.848110	2.807690	0.391420	1.461490	3.828880	2.348200	1.004110
2006	5.694080	4.428860	2.412450	4.648140	3.565960	2.880390	2.810330	0.379420	1.462480	3.861720	2.335950	1.000780

Table C7: Prices of Waiting Capital Services

	W_{KW1}^{t}	W_{KW2}^{t}	W_{KW3}^{t}	W_{KW4}^{t}	W_{KW5}^{t}	W_{KW6}^{t}	W_{KW7}^{t}	W_{KW8}^{t}	W_{KW9}^{t}	W_{KW10}^{t}	W_{KW11}^{t}	W_{KW12}^{t}
1955	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1956	1.009270	1.074030	1.002220	1.020920	1.034280	1.098800	1.108370	0.970160	0.949820	1.265030	1.022010	0.971160
1957	1.119030	1.137240	1.119080	1.165550	1.137620	1.252610	1.422200	1.114870	1.004040	1.496780	1.076280	0.990410
1958	1.009500	1.004300	1.046670	1.036570	1.016920	1.081450	1.431740	1.148200	0.972120	1.289630	1.041000	0.901990
1959	1.196660	1.153220	1.208980	1.259350	1.222650	1.217600	1.637870	1.217760	1.139260	1.510440	1.184050	1.044060
1960	1.553490	1.374880	1.379490	1.569800	1.502990	1.524120	2.006180	1.445030	1.293610	1.736460	1.442920	1.260090
1961	1.877770	1.677120	1.643410	2.040240	1.813870	1.739160	2.267970	1.607460	1.409780	1.914120	1.576970	1.459520
1962	1.782140	1.614370	1.565880	1.922260	1.802020	1.581300	2.228260	1.522000	1.315460	1.825860	1.513670	1.403610
1963	2.018020	1.706720	1.683240	2.072030	1.895910	1.630950	2.246990	1.455940	1.351480	1.872840	1.615750	1.500240
1964	2.116840	1.833640	1.779250	2.176660	2.113190	1.754530	2.382740	1.468120	1.402220	1.984310	1.883680	1.631570
1965	2.160770	1.769120	1.716710	2.016140	2.015150	1.678240	2.253430	1.405980	1.348000	1.915460	1.750050	1.535780
1966	2.693750	1.945860	1.498260	2.273520	2.095550	1.786250	2.384570	1.433330	1.457120	2.085030	1.856000	1.644090
1967	3.307520	2.095120	1.899610	2.562930	2.161970	1.914300	2.525980	1.451580	1.500970	2.258770	1.948930	1.728410
1968	4.040590	2.247880	2.056490	2.811280	2.274520	2.015620	2.715920	1.549450	1.568840	2.311170	2.062940	1.870830
1969	4.331910	2.310820	2.187910	2.818200	2.186330	1.996540	2.730750	1.553040	1.535670	2.378750	2.119420	1.843920
1970	4.326150	2.327930	1.955030	2.788750	2.228820	1.979070	2.698260	1.524660	1.479730	2.399540	2.128110	1.776880
1971	3.419380	1.983810	1.714130	2.306260	1.872920	1.658820	2.341660	1.255140	1.317240	2.151390	1.812110	1.552680
1972	3.058020	1.916400	1.547680	2.416390	1.786170	1.525210	2.219870	1.140420	1.319850	2.054380	1.686860	1.502530
1973	3.224920	2.198120	1.643780	3.186530	2.046210	1.591400	2.300470	1.093890	1.373670	2.514430	1.674360	1.516840
1974	3.006490	2.291750	1.639690	3.059560	2.314470	1.925720	2.554760	1.144460	1.658140	2.314220	1.795680	1.791000
1975	2.793790	2.030420	1.426450	2.505810	1.919350	1.604940	2.267240	1.005890	1.636760	2.429840	1.556410	1.589150
1976	3.394860	2.370630	1.718200	2.962800	2.185050	1.853790	2.586700	1.130570	1.920170	2.758140	1.782470	1.824760
1977	3.457990	2.384320	1.707630	2.849200	2.226040	1.864890	2.646010	1.129970	2.037390	3.072670	1.762480	1.810800
1978	3.831670	2.700130	1.824240	2.868050	2.384440	2.068430	2.900970	1.223490	2.209320	3.325780	1.908290	2.105700
1979	4.203990	3.146980	2.175790	3.763480	2.797090	2.293840	3.247080	1.355670	2.365200	3.671890	2.128260	2.389680
1980	3.979260	3.155060	2.048180	3.655360	2.730410	2.275540	3.247170	1.321320	2.315120	3.959180	2.025910	2.392040
1981	3.936760	3.007910	1.935300	3.129970	2.538500	2.136670	3.144070	1.260660	2.227160	3.853320	1.873610	2.201420
1982	3.880660	2.882400	1.846920	3.160800	2.402420	2.000480	3.015900	1.201270	2.163530	4.047820	1.791960	2.117700
1983	3.731760	2.725550	1.710250	2.953920	2.234500	1.881640	2.898840	1.132840	2.153580	4.114890	1.653950	1.969500
1984	4.275050	3.027710	1.895200	3.367680	2.497630	2.132750	3.151000	1.226220	2.299340	3.926910	1.816510	2.202840
1985	4.625730	3.203300	2.007030	3.682540	2.646040	2.256380	3.268910	1.258350	2.306150	4.007300	1.919300	2.404090
1986	4.587530	3.140110	1.927190	3.369960	2.597200	2.186780	3.170450	1.199320	2.310220	3.643090	1.868600	2.314670
1987	4.322260	3.097370	1.847470	3.464060	2.496680	2.052660	3.025570	1.125480	2.265260	3.386640	1.806190	2.209210
1988	4.482800	3.222160	1.894450	3.416780	2.577510	2.142120	3.078080	1.129550	2.272180	3.327140	1.857070	2.284140
1989	4.687900	3.405020	1.954490	3.617890	2.655020	2.244170	3.034940	1.114650	2.203820	3.208340	1.886310	2.364300
1990	4.519230	3.350720	1.882920	3.448090	2.553560	2.150500	2.902870	1.046330	2.126250	3.054090	1.808290	2.260750
1991	4.033760	3.176150	1.743760	3.112060	2.381980	2.009230	2.731490	0.951880	1.983700	3.007390	1.687040	2.096370
1992	3.809250	3.080720	1.674910	3.021160	2.365680	1.963870	2.653200	0.914160	1.943490	2.906990	1.644960	2.014470
1993	3.224350	2.779630	1.499130	2.918350	2.081000	1.725790	2.403370	0.809200	1.826020	2.832930	1.451570	1.725290
1994	3.058270	2.720030	1.418250	2.749120	2.016320	1.664120	2.430840	0.794160	1.837090	2.822710	1.406030	1.626270
1995	2.765130	2.764450	1.415600	2.673550	2.024060	1.681150	2.455190	0.781460	1.863870	2.906920	1.404340	1.604610
1996	3.078410	2.938130	1.537160	2.938110	2.137520	1.764390	2.544090	0.793970	1.867510	3.015140	1.477810	1.707510
1997	3.113620	2.891510	1.544790	2.966980	2.106670	1.745380	2.511420	0.767560	1.791640	3.024730	1.465830	1.685990
1998	2.599540	2.443380	1.290890	2.310410	1.765170	1.448260	2.186720	0.648720	1.633870	2.762140	1.218940	1.378340
1999	2.744340	2.415160	1.250310	2.339080	1.755700	1.428030	2.162620	0.634120	1.636760	2.630940	1.216340	1.370440
2000	2.959490	2.554150	1.324600	2.506460	1.875230	1.521740	2.242920	0.652440	1.658560	2.660260	1.306690	1.473670
2001	2.729000	2.365540	1.209010	2.294810	1.730540	1.396080	2.092160	0.586630	1.587790	2.458140	1.200760	1.326620
2002	2.921200	2.516170	1.290310	2.441460	1.854360	1.494620	2.158680	0.601160	1.576010	2.605150	1.280710	1.425320
2003	3.210640	2.627160	1.359730	2.587790	1.954700	1.583270	2.212280	0.604950	1.563690	2.770080	1.342620	1.474870
2004	3.920430	3.028170	1.563010	3.001520	2.240540	1.846400	2.431560	0.657070	1.595140	3.076050	1.532290	1.710530
2005	4.047260	3.166480	1.628730	3.085260	2.344040	1.949590	2.489530	0.658330	1.595870	3.179830	1.576360	1.760010
2006	4.071540	3.270770	1.689780	3.255760	2.448640	2.017550	2.521140	0.659410	1.578860	3.213010	1.603460	1.808370

Table C8: Quantities of Investment Goods

	Y_{I1}^t	Y_{I2}^t	Y_{I3}^t	Y_{I4}^t	Y_{I5}^t	Y_{I6}^t	Y_{I7}^t	Y_{I8}^t	Y_{I9}^t	Y_{I10}^t	Y_{I11}^t	Y_{I12}^t
1955	13.31	1127.17	1.36	0.33	3.61	7.84	191.6	77.64	123.58	46.57	19	6.53
1956	15.41	1321.52	1.82	0.43	4.72	9.91	273.11	115.65	160.94	54.84	22.91	7.97
1957	17.87	1556.83	2.33	0.55	6.23	12.18	329.58	156.46	217.02	74.66	29.88	9.8
1958	19.62	1594.4	2.5	0.72	7.39	11.51	329.5	170.66	258.94	99.6	30.78	11.07
1959	20.26	1957.15	3.09	0.86	8.82	13.97	395.72	234.4	237.09	91.92	39.05	15.22
1960	15.2	2523.88	4.88	1.26	12.71	18.79	559.97	330.79	376.38	137.31	53.16	18.35
1961	17.89	2896.81	6.76	1.55	17.54	31.2	774.78	434.83	536.51	196.97	75.7	27.16
1962	23.97	3211.32	8.99	2.04	22.24	41.7	854.5	509.06	611.19	222.5	86.76	42.22
1963	25.63	3616.49	11.46	2.41	28.86	50.13	917.57	543.91	701.46	257.25	111.14	45.34
1964	34.83	4182.81	15.55	3.44	36.38	66.28	996.48	657.55	878.3	316.36	123.23	52.93
1965	39.48	4352.3	17.28	4.05	41.47	83.41	950.8	700.54	895.1	338.7	138.06	73.64
1966	36.23	4856.22	25.14	4.76	59.83	86.52	1099.18	837.89	1130.29	392.39	160.22	83.56
1967	36.97	5485.82	24.26	5.64	84.32	92.38	1471.96	1121.52	1374.32	416.32	205.22	130.29
1968	32.65	6282.25	26.46	6.49	117.38	108.57	1878.51	1437.94	1708.52	492.27	243.8	233.38
1969	30.72	7200.59	27.28	7.47	172.08	163.21	2250.56	1857.71	2046.25	513.89	278.62	293.24
1970	31.34	8149.89	30.91	8.6	216.46	212.76	2739.22	2439.49	2334.42	520.56	310.39	346.22
1971	33.15	8599.94	29.85	8.32	218.45	198.58	2749.95	2586.09	2318.14	503.48	356.33	455.95
1972	37.43	9509.51	33.25	7.99	215.37	213.03	2683.29	3050.3	2590.81	566.52	418.25	529.31
1973	39.36	10524.4	35.01	6.96	205.48	271.97	3221.84	3766.99	2741.18	475.2	516.41	541.41
1974	32.02	10284.79	34.48	7.05	171.3	238.19	2919.41	3716.83	2024.94	431.2	528.76	517.19
1975	26.89	10477.63	30.8	6.58	154.7	222.55	2588.4	3568.89	2050.73	367.61	529.78	539.45
1976	45.41	10745.5	33.05	7.32	177.79	213.96	2685.96	3882.86	2035.17	374.64	562.15	788.96
1977	38.51	11020.3	34.04	7.78	176.59	219.3	2689.41	4337.08	2125.31	371.05	576.46	730.96
1978	23.34	11616.12	35.72	7.96	183.22	233.76	2774.56	5061.44	2548.48	467.36	614.17	1130.96
1979	32.13	12033.37	40.76	8.15	211.02	278.79	2986.79	5460.94	2793.01	558.19	687.16	1134.87
1980	33.03	11899.16	41.58	7.69	207.86	248.55	3013.99	5919.69	2754.88	497.41	698.73	1071.33
1981	32	11837.65	50.89	7.41	205.09	260.33	3361.1	6358.72	2748.64	574.22	707.87	1190.94
1982	33.02	11767.59	60.88	6.36	210.02	242.12	3487.19	6940.57	2490.05	507.33	707.72	1323.1
1983	30.6	11300.33	66.07	5.4	197.73	231.32	3536.13	7430.42	2478.86	555.78	723.41	1460.81
1984	31.42	11232.55	77.72	4.74	202.1	242.29	4087.09	8942.98	2326.53	684.74	766.38	1596.98
1985	35.3	11720.86	96.1	4.26	224.54	244.36	4418.71	11244.79	2597.56	810.69	869.08	1769.21
1986	35.35	12415.71	107	4.62	248.4	236.66	4453.08	12393.72	2804.9	844.32	924.34	2203.22
1987	37.33	13364.85	118.96	4.65	287.82	211.41	4532.37	13732.43	3231.38	945.51	984.46	2374.03
1988	38.87	14658.46	146.5	5.32	349.36	229.07	5385.86	15780.72	3596.63	893.23	1088.22	3144.82
1989	42.95	15698.06	170.35	5.64	418.15	226.45	6171.23	17293.44	4122.26	882.45	1198.65	3874.34
1990	43.28	16771.48	198.34	5.99	485.88	218.2	7100	18611.7	4772.73	847.61	1312.6	4041.83
1991	42.04	17186.52	195.66	6.36	457.69	208.95	7348.48	19389.53	4591.9	757.88	1380.23	4711.3
1992	40.61	16808.47	213.06	6.41	426.8	187.45	6869.43	19047.24	4595.64	700.76	1402.5	5030.3
1993	42.18	16597.09	197.34	5.39	364.59	166.74	6052.29	18846.8	4435.1	589.87	1274.54	5020.06
1994	42.33	16483.61	190.4	5.09	312.53	147.87	5703.26	18527.61	4376.74	556.7	1200.02	5064.57
1995	42	15477.06	186.58	5.01	274.93	142.24	6309.77	21813.88	4997.97	578.16	1226.66	5161.61
1996	39.24	15583.72	169.28	4.63	263.43	153.47	6801.42	26559.08	5273.38	591.51	1404.73	5304.91
1997	36.57	15449.48	172.65	4.8	270.47	155.39	7259.99	27212.56	5145.99	573.85	1503.32	5257.9
1998	34.81	14220.84	144.04	4.73	241.06	150.98	6831.66	25124.37	4237.95	454.88	1525.15	5802.81
1999	32.14	13862.71	147.62	4.41	225.24	157.14	6659.38	25962.21	4641.87	517.11	1536.68	5837.14
2000	35.12	13423.6	142.34	4.4	217.26	164.5	6889.77	27384.46	4921.56	539.34	1516.06	6198.2
2001	35.41	13017.41	133.44	4.35	209.43	164.48	6923.07	28403.62	5020.97	605.61	1534.74	6541.86
2002	34.08	12642.58	132.24	4.18	202.73	138.22	6124.06	25101.46	5274.53	636.25	1453.39	6649.7
2003	31.05	12187.11	134.47	3.91	194.36	131.54	6167.31	26961.09	5550.62	640.41	1466.72	6758.98
2004	28.54	12182.13	132.88	3.84	191.59	127.37	6821.78	28356.53	5289.34	580.96	1502.48	6869.98
2005	29.88	11994.6	137.88	4.03	198.46	132.24	7721.56	31976.57	5716.31	581.64	1587.16	7280.61
2006	29.67	12061.59	141.15	4.08	203.19	137.75	7965.23	33396.38	5612.65	569.32	1663.2	7399.95

Table C9: Quantities of Capital Services

	X_{K1}^t	X_{K2}^t	X_{K3}^t	X_{K4}^t	X_{K5}^t	X_{K6}^t	X_{K7}^t	X_{K8}^t	X_{K9}^t	X_{K10}^t	X_{K11}^t	X_{K12}^t
1955	85.33	1430.70	3.20	0.16	8.80	20.57	256.78	60.04	44.95	134.66	10.30	15.28
1956	64.05	1448.85	2.60	0.21	8.10	19.87	256.39	66.34	70.44	120.05	12.01	11.79
1957	50.01	1485.96	2.38	0.29	7.71	19.51	273.48	79.34	102.99	111.96	14.42	10.00
1958	40.99	1544.64	2.45	0.38	7.65	19.49	298.88	98.78	144.60	110.19	17.72	9.31
1959	35.36	1597.51	2.58	0.49	7.78	19.38	318.23	119.43	187.54	113.39	20.10	9.12
1960	31.84	1687.78	2.87	0.62	8.18	19.62	348.36	150.42	215.48	115.92	23.60	9.64
1961	28.09	1865.08	3.77	0.84	9.16	20.49	406.50	195.32	279.13	126.32	28.95	10.82
1962	26.12	1994.37	5.08	1.08	10.78	22.97	495.97	254.14	375.53	143.32	37.24	13.02
1963	26.37	2142.88	6.72	1.41	12.92	26.66	585.83	319.87	471.20	160.34	44.62	16.69
1964	27.08	2307.36	8.64	1.77	15.73	31.18	674.42	382.53	568.57	180.11	54.12	20.72
1965	30.10	2499.17	11.63	2.33	19.49	37.51	765.58	459.22	698.14	204.01	62.30	25.55
1966	33.80	2691.31	14.23	2.94	23.56	45.62	835.33	535.91	802.73	226.48	70.86	32.20
1967	36.08	2882.99	18.78	3.61	29.77	53.46	922.60	628.76	946.31	254.44	79.37	41.35
1968	38.23	3130.47	21.45	4.37	39.05	61.42	1072.04	767.02	1128.33	284.20	92.85	55.62
1969	38.89	3405.93	23.96	5.20	52.18	70.90	1276.36	952.17	1356.62	321.78	107.98	79.38
1970	39.50	3733.37	26.02	6.11	74.00	86.98	1515.02	1197.16	1625.77	359.82	123.05	111.88
1971	40.54	4082.44	28.64	7.12	101.05	108.42	1814.09	1529.95	1904.14	390.97	138.01	151.69
1972	42.06	4448.46	29.88	7.82	125.49	126.09	2072.56	1826.99	2100.64	411.45	156.07	203.57
1973	44.69	4845.87	31.53	8.26	146.39	144.21	2281.19	2187.84	2273.98	438.17	175.22	299.64
1974	47.38	5332.15	32.70	8.30	161.59	168.78	2566.99	2596.38	2406.15	444.03	201.14	365.09
1975	46.04	5774.01	33.11	8.36	170.34	186.66	2743.31	2953.54	2376.95	455.32	225.12	410.02
1976	43.83	6228.43	32.04	8.27	175.31	200.88	2842.01	3238.77	2378.60	456.51	240.20	465.98
1977	48.49	6656.65	32.03	8.41	183.12	212.71	2926.11	3527.56	2346.29	451.06	254.87	580.47
1978	48.70	7081.64	32.27	8.65	189.27	224.24	3012.82	3874.41	2343.05	453.93	273.68	690.15
1979	44.27	7519.16	32.97	8.88	196.33	236.70	3101.39	4324.16	2495.89	475.38	295.82	852.96
1980	44.06	7959.03	35.41	9.12	206.02	254.25	3214.64	4721.83	2638.96	500.25	318.67	993.21
1981	44.67	8388.15	39.63	9.16	214.84	266.20	3324.61	5184.01	2838.72	513.76	347.16	1071.70
1982	43.63	8796.46	46.05	9.12	222.41	278.73	3476.82	5616.34	2939.23	541.16	372.87	1195.66
1983	43.71	9166.97	54.17	8.79	230.58	287.72	3626.21	6076.23	2974.63	546.89	392.65	1366.45
1984	43.11	9534.95	61.54	8.26	236.35	294.47	3765.12	6576.93	3019.63	563.09	416.51	1556.60
1985	42.80	10193.62	71.02	7.97	247.44	302.33	4008.37	7781.29	3084.00	597.51	447.41	1760.07
1986	44.23	10634.37	84.47	7.38	258.11	309.78	4258.57	9072.51	3200.68	647.97	491.14	2003.33
1987	45.31	11212.70	97.76	6.96	270.35	326.25	4478.57	10189.07	3341.75	740.73	539.18	2388.96
1988	46.81	11733.75	111.05	6.65	287.83	330.32	4684.14	11386.40	3568.61	808.48	592.13	2733.69
1989	48.39	12220.39	130.63	6.60	312.84	333.11	5023.30	12795.04	3845.83	855.51	654.54	3198.15
1990	50.93	12784.89	152.88	6.66	347.49	335.31	5449.35	14218.29	4229.34	884.78	727.17	3819.23
1991	52.35	13398.23	178.52	6.80	388.27	336.11	5970.94	15615.26	4685.29	900.41	811.33	4259.11
1992	52.52	14107.92	194.22	7.02	417.84	335.68	6456.13	16914.28	4998.04	899.85	893.84	4845.83
1993	52.24	14581.28	211.37	7.19	439.99	332.36	6770.31	17509.78	5240.49	882.37	957.21	5425.35
1994	52.49	15063.94	216.34	7.04	447.56	326.52	6861.65	17982.41	5373.15	846.41	978.31	5755.96
1995	53.12	15478.78	216.84	6.83	446.01	318.62	6880.58	18492.73	5486.88	815.87	977.79	5926.21
1996	53.36	15785.76	215.65	6.65	437.55	310.54	7015.93	19593.80	5749.04	795.60	986.88	6046.92
1997	52.46	16090.43	208.01	6.41	427.58	304.64	7220.87	21483.86	6030.31	783.00	1034.02	6259.92
1998	51.02	16306.47	204.36	6.27	420.07	299.63	7485.04	23057.45	6227.03	769.44	1082.30	6582.59
1999	49.25	16535.69	190.64	6.14	408.75	294.46	7641.50	23623.55	6150.79	736.82	1130.58	7270.61
2000	46.91	16770.98	183.09	5.96	394.50	290.56	7742.13	23971.79	6198.08	719.13	1163.38	7908.00
2001	46.35	16874.29	176.09	5.81	382.76	287.98	7860.95	25441.19	6316.06	709.12	1183.49	8502.46
2002	46.04	17023.40	167.99	5.68	370.70	285.59	7950.02	26557.09	6436.69	711.73	1208.75	9158.55
2003	45.29	17225.37	162.20	5.54	358.18	279.82	7901.27	26406.99	6605.18	722.29	1208.23	9824.96
2004	43.62	17415.24	159.31	5.35	345.93	273.66	7875.96	27190.85	6834.61	732.41	1214.45	10322.71
2005	41.54	17615.07	156.79	5.19	334.60	267.46	7978.86	27933.67	6959.51	731.65	1233.24	10734.51
2006	40.61	17867.91	157.13	5.12	326.23	262.47	8232.61	29280.86	7183.83	731.96	1263.74	11186.19

Table C10: Quantities of Inventory Services, Land Services and Labour Inputs

	X_{KIV1}^t	X_{KIV2}^t	X_{KIV3}^t	X_{KIV4}^t	X_{LD1}^t	X_{LD2}^t	X_{LD3}^t	X_{LD4}^t	X_{LB1}^t	X_{LB2}^t	X_{LB3}^t
1955	75.44	14.53	130.5	29.89	1020.84	59.87	194.3	74.34	637.74	224.27	2877.08
1956	85.55	16.95	128.93	36.79	1020.84	59.87	194.3	76.98	647.11	215.8	3194.8
1957	95.53	22.45	127.28	46.25	1019.82	60.39	196.25	80.81	666.08	214.66	3451.28
1958	106.8	26.49	125.43	55.68	1019.32	61.07	198.58	84.17	647.68	209.95	3618.58
1959	115.9	24.22	124.22	57.48	1017.92	62.23	202.89	88.33	649.17	203.86	3767.36
1960	128.51	28.71	122.57	66.59	1018.5	62.24	204.18	91.73	656.23	198.58	4023.32
1961	147.67	34.44	120.69	73.87	1018.36	64.33	211.85	97.56	628.18	191.59	4207.68
1962	178.03	44.65	118.54	87.16	1016.4	66.39	219.68	102.81	579.27	181.19	4480.87
1963	195.39	48.46	116.77	91.12	1016.33	68.13	226.05	108.54	569.23	171.99	4568.48
1964	215.25	54.68	114.82	102.68	1016.38	69.87	232.5	114.47	568.61	167.62	4718.43
1965	241.18	64.19	111.75	111.2	1016.8	78.6	234.35	121.69	555.17	159.28	4894.8
1966	266.37	66.64	107.94	118.04	1017.93	78.6	273.82	128.87	561.19	155.07	5117.88
1967	292.88	74.68	104.5	127.03	1017.63	78.6	313.61	135.57	575.24	154.92	5265.48
1968	334.28	90.32	101.67	142.49	1019.48	87.33	314.66	143.24	579.44	150.93	5342.57
1969	385.33	100.98	100.02	153.97	1019.94	87.33	353.45	150.44	584.91	147.88	5414.86
1970	428.77	119.64	98.65	171.81	1020.4	96.07	354.52	162.71	573.35	140.36	5570.05
1971	477.08	139.41	96.62	182.48	1021.88	104.8	354.26	173.68	555.38	129.47	5685.35
1972	492.58	146.21	94.91	177.63	1022.93	113.54	354.95	186.69	557.72	123.79	5751.78
1973	503.83	150.31	92.4	177.46	1022.29	113.54	394.64	200.24	567.35	114.85	5955.78
1974	531.64	174.59	90.73	205.61	1021.81	118.78	415.5	211.8	550.49	108.09	5848.43
1975	585.47	188.26	90.21	215.79	1022.04	123.14	437.29	224.55	531.23	105.48	5699.43
1976	578.26	179.63	89.4	211.26	1022.24	124.02	453.07	233.76	533.72	104.19	5988.87
1977	595.1	185.11	89.21	213.82	1022.08	126.64	487.55	241.16	540.82	104.96	6130.52
1978	592.25	181.12	87.83	212.3	1021.35	126.64	510.47	249.05	556.28	107.16	6186.72
1979	597.29	171.72	87.13	198.75	1020.83	126.64	526.67	257.06	553.53	103.93	6305.62
1980	660.7	181.27	86.37	228.69	1020.2	127.51	546.13	263.93	541.27	98.69	6456.34
1981	704.32	206.48	85.49	234.66	1019.43	127.51	553.09	270.46	533.05	96.55	6603.2
1982	720.17	220.29	84.52	229.88	1018.59	128.38	568.35	276.05	534.65	95.58	6700.19
1983	738.43	215.17	83.88	231.71	1017.88	130.13	579.01	281.9	526.45	93.51	6953.82
1984	727.86	214.67	83.71	221.84	1018.24	130.13	578.57	285.34	514.58	91.76	7034.67
1985	739.41	230.08	83.52	225.94	1019.67	130.13	607.49	286.59	512.6	89.83	7139.19
1986	752.1	235.93	82.62	229.91	1018.6	130.13	606.81	290.5	512.68	88.12	7243.95
1987	747.67	225.91	81.4	222.94	1018.24	130.13	622.01	292.56	513.33	87.94	7368.89
1988	754.07	225.34	80.25	235.79	1017.88	138.8	636.24	291.61	508.61	86.16	7574.04
1989	769.2	234.58	78.63	232.13	1017.16	138.8	649.41	293.99	499.87	84.93	7744.14
1990	815.56	250.54	76.37	238.03	1017.52	138.8	662.92	298.46	484.44	82.3	7861.59
1991	823.16	281.1	74.21	251.44	1017.88	138.8	662.51	305.85	467.64	76.85	8001.62
1992	841.84	300.37	73.75	236.16	1018.06	143.14	676.78	309.57	452.15	71.77	7999.44
1993	851.58	307.09	72.09	221.58	1018.24	147.48	690.38	313.82	428.95	66.17	7983.4
1994	840.34	306.1	71.14	199.63	1018.96	147.48	710.18	316.47	420.61	64.11	8003.58
1995	837.66	298.06	70.27	180.27	1019.67	147.48	716.23	322.18	414.01	62.28	8087.25
1996	858.69	313.76	69.17	184.2	1020.03	147.48	730.3	327.49	402.31	59.6	8129.86
1997	896.01	324.54	67.62	190.31	1020.39	147.48	744.08	332.65	402.32	57.78	8100.96
1998	938.9	325.22	66.48	196.15	1020.39	147.48	758.97	340.67	388.04	55.76	8059.33
1999	931.18	343	66.73	185.75	1021.47	147.48	759.91	345.76	383.34	53.96	8003.24
2000	858.92	355.67	65.97	173.49	1021.83	147.48	774.69	349.8	372.88	51.36	8171.31
2001	843.34	385.73	66.45	172.02	1021.47	147.48	790.17	351.28	353.88	48.82	8053.7
2002	846.27	313.05	67.48	168.9	1021.47	147.48	790.88	355.34	343	46.33	7971.28
2003	827.87	303.81	67.1	163.88	1021.83	147.48	805.39	359.53	334.11	43.96	7964.25
2004	816.72	269.22	66.78	159.71	1022.55	138.8	805.61	363.67	332.73	42.79	8020.87
2005	776.4	263.59	66.93	170.37	1023.26	138.8	805.68	364.86	329.32	41.29	8032.98
2006	863.51	251.54	63.87	188.48	1024.93	138.8	817.52	371.71	317.59	35.07	8152.53

Table C11: Quantities of Depreciations

	Y_{DEP1}^t	Y_{DEP2}^t	Y_{DEP3}^t	Y_{DEP4}^t	Y_{DEP5}^t	Y_{DEP6}^t	Y_{DEP7}^t	Y_{DEP8}^t	Y_{DEP9}^t	Y_{DEP10}^t	Y_{DEP11}^t	Y_{DEP12}^t
1955	-72.66	-557.04	-2.8	-0.13	-6.45	-12.98	-175.24	-40.43	-32.51	-92.99	-8.46	-12.28
1956	-54.36	-564.94	-2.27	-0.18	-5.91	-12.54	-174.22	-44.95	-50.96	-81.8	-9.85	-9.1
1957	-42.27	-578.03	-2.08	-0.24	-5.6	-12.31	-185.62	-54.26	-74.54	-75.55	-11.82	-7.37
1958	-34.47	-598.33	-2.14	-0.31	-5.52	-12.3	-202.75	-68.47	-104.57	-73.77	-14.53	-6.57
1959	-29.57	-618.25	-2.25	-0.41	-5.57	-12.23	-215.62	-84.05	-135.52	-75.52	-16.47	-6.15
1960	-26.49	-648.18	-2.51	-0.51	-5.82	-12.38	-236.32	-107.61	-155.58	-77.21	-19.33	-6.19
1961	-23.26	-693.53	-3.29	-0.69	-6.45	-12.93	-276.68	-141.44	-201.29	-84.37	-23.72	-6.74
1962	-21.52	-747.86	-4.43	-0.89	-7.5	-14.5	-338.85	-185.78	-270.44	-95.24	-30.52	-7.93
1963	-21.59	-806.43	-5.88	-1.16	-8.92	-16.83	-401.13	-236.26	-339.13	-105.85	-36.58	-9.94
1964	-22.04	-871.75	-7.55	-1.46	-10.79	-19.68	-462.85	-285.92	-408.27	-118.27	-44.39	-12.31
1965	-24.33	-951.35	-10.16	-1.92	-13.31	-23.68	-525.88	-345.25	-500.28	-132.62	-51.14	-15.25
1966	-27.15	-1026.89	-12.44	-2.43	-16.01	-28.8	-573.56	-406.09	-574.11	-145.79	-58.17	-19.15
1967	-28.88	-1109.92	-16.41	-2.98	-20.14	-33.74	-634.39	-481.46	-675.68	-163.2	-65.2	-25.81
1968	-30.54	-1209.37	-18.74	-3.6	-26.31	-38.77	-740.29	-592.91	-803.75	-182.53	-76.35	-36.05
1969	-31.02	-1324.33	-20.93	-4.29	-34.99	-44.75	-885.54	-744.81	-962.69	-207.68	-88.87	-51.26
1970	-31.67	-1461.43	-22.73	-5.04	-50.37	-54.9	-1053.39	-947.06	-1148.93	-234.33	-101.21	-74.27
1971	-32.75	-1618.26	-25.02	-5.88	-69.72	-68.43	-1267.39	-1224.27	-1340.09	-255.12	-113.51	-103.96
1972	-34.2	-1776.83	-26.1	-6.45	-87.18	-79.59	-1451.56	-1466.65	-1473.81	-268.13	-128.35	-142.52
1973	-36.54	-1945.91	-27.55	-6.81	-102.21	-91.03	-1601.38	-1771.25	-1590.7	-286.3	-144.15	-230.64
1974	-38.9	-2136.05	-28.57	-6.85	-113.03	-106.53	-1814.2	-2106.71	-1679.56	-288.57	-165.39	-285.69
1975	-37.79	-2325.95	-28.93	-6.9	-119.47	-117.82	-1940.56	-2401.74	-1657.67	-296.13	-184.99	-320.62
1976	-36.03	-2510.19	-27.99	-6.82	-123.15	-126.79	-2014.14	-2642.35	-1655.78	-297.78	-197.38	-368.28
1977	-39.93	-2688.29	-27.99	-6.94	-128.8	-134.26	-2072.15	-2887.95	-1631.02	-293.12	-209.04	-472.92
1978	-40.05	-2859.3	-28.19	-7.13	-133.14	-141.54	-2136.92	-3186.57	-1630.23	-296.87	-224.44	-580.07
1979	-36.41	-3025.83	-28.81	-7.33	-138.16	-149.41	-2203.67	-3576.15	-1740.12	-314.31	-242.53	-734.58
1980	-36.27	-3208.46	-30.94	-7.52	-144.95	-160.48	-2290.34	-3918.18	-1840.58	-330.94	-261.17	-866.77
1981	-36.86	-3388.45	-34.63	-7.56	-151.24	-168.02	-2375.75	-4320.82	-1984.5	-339.97	-284.43	-932.08
1982	-35.95	-3560.57	-40.24	-7.52	-156.72	-175.93	-2493.39	-4699.9	-2058.35	-359.62	-305.26	-1047.25
1983	-36.01	-3727.45	-47.33	-7.25	-162.65	-181.61	-2608.75	-5117.62	-2085.24	-362.01	-321.33	-1213.49
1984	-35.54	-3877.73	-53.77	-6.81	-166.96	-185.87	-2717.12	-5570.84	-2117.66	-372.49	-340.83	-1400.92
1985	-35.29	-4148.66	-62.05	-6.57	-175.01	-190.83	-2905.42	-6664.87	-2167.27	-395.81	-366.15	-1605.41
1986	-36.53	-4342.47	-73.81	-6.09	-182.86	-195.53	-3096.71	-7856.59	-2255.69	-429.97	-401.94	-1856.97
1987	-37.5	-4591.72	-85.42	-5.74	-191.77	-205.93	-3265.03	-8877.46	-2354.91	-486.43	-441.19	-2264.31
1988	-38.82	-4797.35	-97.03	-5.49	-204.44	-208.5	-3424.63	-9977.8	-2508.39	-530.38	-484.36	-2624.26
1989	-40.22	-4998.48	-114.14	-5.45	-222.21	-210.26	-3688.82	-11293.44	-2695.04	-565.45	-535.16	-3099.43
1990	-42.44	-5238.85	-133.58	-5.49	-247.01	-211.65	-4016.74	-12620.26	-2964.23	-586.43	-594.36	-3740.52
1991	-43.67	-5502.06	-155.99	-5.61	-275.89	-212.15	-4412.19	-13909.58	-3274.32	-597.34	-663.01	-4167.92
1992	-43.81	-5769.98	-169.7	-5.79	-296.6	-211.88	-4778.39	-15089.08	-3486.61	-597.09	-730.35	-4753.36
1993	-43.57	-5995.68	-184.69	-5.93	-312.54	-209.79	-5016.41	-15591.52	-3652.05	-583.79	-782.07	-5332.22
1994	-43.77	-6180.28	-189.03	-5.81	-318.11	-206.1	-5084.04	-16008.81	-3730.75	-557.58	-799.27	-5634.65
1995	-44.33	-6342.11	-189.47	-5.64	-317.35	-201.11	-5097.41	-16476.8	-3804.6	-536.15	-798.71	-5759.14
1996	-44.55	-6465.93	-188.43	-5.49	-311.68	-196.01	-5200.58	-17508.7	-3973	-523.02	-805.96	-5826.89
1997	-43.81	-6569.83	-181.76	-5.29	-304.86	-192.29	-5356.96	-19319.18	-4152.62	-515.04	-844.24	-5997.71
1998	-42.63	-6695.59	-178.57	-5.17	-299.76	-189.13	-5557.52	-20812.77	-4276.43	-505.78	-883.51	-6303.77
1999	-41.16	-6789.93	-166.58	-5.07	-291.94	-185.86	-5677.76	-21315.96	-4216.2	-482.06	-922.75	-7013.72
2000	-39.2	-6858.6	-159.98	-4.91	-281.84	-183.4	-5759.71	-21620.75	-4241.07	-470.22	-949.22	-7680.68
2001	-38.75	-6920.64	-153.86	-4.79	-273.62	-181.77	-5849.54	-23050.23	-4312.47	-464.63	-965.5	-8299.28
2002	-38.49	-6977.54	-146.78	-4.69	-265.17	-180.27	-5912.65	-24146.82	-4386.41	-466.88	-985.92	-8989.7
2003	-37.87	-7032.14	-141.72	-4.57	-256.36	-176.62	-5879.47	-24009.63	-4497.48	-475.04	-985.2	-9706.27
2004	-36.46	-7080.48	-139.2	-4.41	-247.71	-172.74	-5864.29	-24842.29	-4653.83	-483.17	-990.07	-10243.19
2005	-34.72	-7146.74	-137	-4.28	-239.71	-168.82	-5947.14	-25639.42	-4731.35	-484.28	-1005.3	-10690.22
2006	-33.95	-7210.91	-137.3	-4.22	-233.81	-165.67	-6144.88	-27048.64	-4883.98	-486.5	-1029.98	-11179.98

Table C12: Quantities of Waiting Capital Services

	X_{KW1}^t	X_{KW2}^t	X_{KW3}^t	X_{KW4}^t	X_{KW5}^t	X_{KW6}^t	X_{KW7}^t	X_{KW8}^t	X_{KW9}^t	X_{KW10}^t	X_{KW11}^t	X_{KW12}^t
1955	12.66	873.66	0.40	0.03	2.35	7.59	81.54	19.62	12.44	41.68	1.84	3.00
1956	9.69	886.46	0.33	0.04	2.18	7.33	82.17	21.39	19.48	38.24	2.16	2.69
1957	7.74	907.44	0.30	0.05	2.11	7.19	87.85	25.11	28.45	36.34	2.60	2.61
1958	6.52	937.64	0.31	0.07	2.13	7.19	96.11	30.44	40.04	36.30	3.20	2.70
1959	5.77	969.78	0.33	0.09	2.20	7.15	102.55	35.72	52.03	37.69	3.63	2.89
1960	5.31	1016.55	0.36	0.11	2.35	7.23	112.06	43.73	59.91	38.54	4.27	3.28
1961	4.76	1085.46	0.48	0.15	2.68	7.56	130.17	55.72	77.81	41.84	5.23	3.81
1962	4.50	1164.39	0.64	0.19	3.21	8.47	158.03	71.45	104.97	47.79	6.72	4.68
1963	4.63	1251.19	0.85	0.25	3.90	9.83	186.11	88.41	131.88	53.93	8.05	6.13
1964	4.85	1348.38	1.09	0.31	4.79	11.50	213.62	103.67	159.87	60.98	9.75	7.63
1965	5.50	1460.00	1.47	0.41	5.98	13.83	242.20	123.24	197.10	69.96	11.20	9.37
1966	6.28	1570.90	1.80	0.52	7.27	16.83	264.41	141.87	227.49	78.62	12.73	11.84
1967	6.77	1695.40	2.37	0.63	9.25	19.72	291.52	163.49	268.99	88.70	14.23	14.62
1968	7.21	1843.90	2.71	0.77	12.20	22.65	336.99	196.28	322.04	98.93	16.60	19.06
1969	7.36	2013.10	3.02	0.91	16.40	26.15	398.90	238.78	389.69	111.38	19.26	27.28
1970	7.37	2214.88	3.28	1.07	22.77	32.08	472.16	294.13	470.34	123.19	21.99	37.52
1971	7.40	2436.87	3.61	1.25	30.41	39.99	561.36	366.93	555.15	133.49	24.66	49.18
1972	7.49	2665.63	3.77	1.37	37.25	46.50	638.65	434.75	616.25	140.77	27.90	64.29
1973	7.77	2918.01	3.98	1.45	42.98	53.18	699.96	509.03	671.06	149.25	31.27	82.00
1974	8.07	3199.62	4.13	1.45	47.24	62.25	776.76	600.08	713.12	152.71	35.99	96.94
1975	7.85	3471.20	4.18	1.46	49.41	68.84	828.31	677.58	705.80	156.34	40.43	109.00
1976	7.42	3759.24	4.04	1.45	50.60	74.08	854.17	734.56	708.85	155.81	43.14	121.19
1977	8.11	4026.57	4.04	1.47	52.66	78.45	881.02	790.81	700.98	155.09	46.26	141.73
1978	8.21	4303.32	4.07	1.52	54.41	82.70	903.90	855.61	698.93	154.19	49.70	157.64
1979	7.46	4571.73	4.16	1.56	56.38	87.30	926.99	939.21	742.06	158.37	53.80	185.27
1980	7.39	4844.60	4.47	1.60	59.19	93.77	955.47	1015.27	784.11	166.49	58.06	209.54
1981	7.38	5112.48	5.00	1.61	61.62	98.17	981.72	1098.89	840.17	170.90	63.36	227.99
1982	7.29	5369.82	5.81	1.60	63.58	102.80	1018.14	1174.42	867.24	178.53	68.37	249.64
1983	7.30	5619.46	6.84	1.54	65.67	106.11	1053.65	1241.22	876.09	181.87	72.16	273.93
1984	7.17	5865.47	7.77	1.45	66.98	108.60	1085.76	1316.22	888.70	187.50	76.58	299.69
1985	7.11	6256.33	8.96	1.40	69.84	111.50	1144.15	1498.32	904.51	198.44	82.22	326.19
1986	7.26	6537.86	10.66	1.29	72.47	114.25	1206.46	1681.08	934.26	214.48	90.26	354.48
1987	7.35	6916.96	12.34	1.22	75.60	120.32	1260.94	1846.64	975.58	250.33	99.16	393.81
1988	7.48	7243.58	14.02	1.17	80.13	121.82	1309.54	2020.29	1046.28	273.79	109.11	431.35
1989	7.61	7558.15	16.49	1.16	87.08	122.85	1388.70	2209.02	1133.41	285.22	120.93	488.28
1990	7.88	7918.30	19.30	1.17	96.48	123.66	1491.13	2399.52	1246.11	293.15	134.61	561.24
1991	8.04	8314.04	22.54	1.19	107.95	123.96	1621.80	2594.00	1387.85	297.66	150.38	627.90
1992	8.08	8725.55	24.52	1.23	116.64	123.80	1744.71	2790.22	1485.64	297.32	165.80	707.43
1993	8.03	9048.33	26.68	1.26	122.46	122.58	1822.85	2915.15	1560.87	293.78	177.66	785.09
1994	8.08	9341.86	27.31	1.23	124.24	120.42	1847.50	2997.28	1612.46	285.04	181.63	849.52
1995	8.12	9609.83	27.37	1.20	123.20	117.51	1853.42	3069.43	1651.01	276.45	181.79	906.54
1996	8.12	9834.31	27.22	1.17	120.26	114.53	1886.37	3207.21	1741.23	269.35	183.77	961.30
1997	7.96	10014.83	26.26	1.12	117.04	112.35	1936.13	3414.09	1839.08	264.71	192.91	1019.50
1998	7.71	10173.64	25.80	1.10	114.53	110.51	2001.03	3593.32	1909.79	260.59	202.20	1074.46
1999	7.43	10326.17	24.07	1.08	110.92	108.60	2037.23	3688.98	1893.93	252.69	211.58	1143.20
2000	7.08	10460.96	23.11	1.04	106.90	107.16	2054.37	3751.88	1915.61	247.00	218.30	1201.27
2001	6.98	10544.33	22.23	1.02	103.38	106.21	2083.96	3890.88	1960.91	242.25	222.32	1258.26
2002	6.92	10657.15	21.21	1.00	99.79	105.33	2111.89	3989.92	2006.39	242.41	227.44	1315.36
2003	6.80	10765.80	20.47	0.97	96.16	103.20	2094.93	3967.86	2062.46	244.37	227.88	1365.32
2004	6.56	10878.78	20.11	0.94	92.67	100.93	2083.78	4003.82	2133.98	245.93	229.38	1405.20
2005	6.25	10984.12	19.79	0.91	89.47	98.64	2103.90	4040.37	2179.87	243.72	233.06	1438.85
2006	6.10	11075.81	19.84	0.90	87.08	96.80	2161.16	4134.19	2250.02	241.39	239.09	1476.94

References

Archibald, R.B. (1977), "On the Theory of Industrial Price Measurement: Output Price Indexes", *Annals of Economic and Social Measurement* 6, 57-72.

Balk, B.M. (1998), *Industrial Price, Quantity and Productivity Indices*, Boston: Kluwer Academic Publishers.

Balk, B.M. (2003), "The Residual: on Monitoring and Benchmarking Firms, Industries and Economies with Respect to Productivity", *Journal of Productivity Analysis* 20, 5-47.

Balk, B.M. (2007), "Measuring Productivity Change without Neoclassical Assumptions: A Conceptual Analysis", paper presented at the Sixth Annual Ottawa Productivity Workshop, Bank of Canada, May 14-15, 2007.

Balk, B.M., R. Färe and S. Grosskopf (2004), "The Theory of Economic Price and Quantity Indicators", *Economic Theory* 23, 149-164.

Bennet, T.L. (1920), "The Theory of Measurement of Changes in Cost of Living", *Journal of the Royal Statistics Society* 83, 455-462.

Chambers, R.G. (2001), "Consumers' Surplus as an Exact and Superlative Cardinal Welfare Indicator", *International Economic Review* 41, 105-119.

Chambers, R.G. (2002), "Exact Nonradial Input, Output and Productivity Measurement", *Economic Theory* 20, 751-765.

Diewert, W.E. (1973), "Functional Forms for Profit and Transformation Functions", *Journal of Economic Theory* 6, 284-316.

Diewert, W.E., (1974), "Applications of Duality Theory," pp. 106-171 in M.D. Intriligator and D.A. Kendrick (ed.), *Frontiers of Quantitative Economics*, Vol. II, Amsterdam: North-Holland. http://www.econ.ubc.ca/diewert/theory.pdf

Diewert, W.E. (1976), "Exact and Superlative Index Numbers", *Journal of Econometrics* 4, 114-145.

Diewert, W.E. (1992), "Exact and Superlative Welfare Change Indicators", *Economic Inquiry* 30, 565-582.

Diewert, W.E. (2002), "The Quadratic Approximation Lemma and Decompositions of Superlative Indexes", *Journal of Economic and Social Measurement* 28, 63-88.

Diewert, W.E. (2005), "Index Number Theory Using Differences Instead of Ratios", *The American Journal of Economics and Sociology* 64:1, 311-360.

Diewert, W.E. and K.J. Fox (2005), "On Measuring the Contribution of Entering and Exiting Firms to Aggregate Productivity Growth", Discussion Paper 15-02, Department of Economics, University of British Columbia, Vancouver, Canada, V6T 1Z1.

Diewert, W.E. and D. Lawrence (2006), *Measuring the Contributions of Productivity and Terms of Trade to Australia's Economic Welfare*, Report by Meyrick and Associates to the Productivity Commission, Canberra, Australia.

Diewert, W.E., H. Mizobuchi and K. Nomura (2005), "On Measuring Japan's Productivity, 1955-2003", Discussion Paper 05-22, Department of Economics, University of British Columbia, Vancouver, B.C., Canada, V6T 1Z1, December . http://www.econ.ubc.ca/discpapers/dp0522.pdf

Diewert, W.E. and C.J. Morrison (1986), "Adjusting Output and Productivity Indexes for Changes in the Terms of Trade", *The Economic Journal* 96, 659-679.

Diewert, W.E. and A.O. Nakamura (2003), "Index Number Concepts, Measures and Decompositions of Productivity Growth", *Journal of Productivity Analysis* 19, 127-159.

Diewert, W.E. and L. Ostensoe (1988), "Flexible Functional Forms for Profit Functions and Global Curvature Conditions", pp. 43-51 in *Dynamic Econometric Modeling*, W.A. Barnett, E.R. Berndt and H. White (eds.), Cambridge: Cambridge University Press.

Diewert, W.E. and T.J. Wales (1987), "Flexible Functional Forms and Global Curvature Conditions", *Econometrica* 55, 43-68.

Diewert, W.E. and T.J. Wales (1988a), "Normalized Quadratic Systems of Consumer Demand Functions", *Journal of Business and Economic Statistics* 6, 303-12.

Diewert, W.E. and T.J. Wales (1988b), "A Normalized Quadratic Semiflexible Functional Form", *Journal of Econometrics* 37, 327-42.

Diewert, W.E. and T.J. Wales (1992), "Quadratic Spline Models For Producer's Supply and Demand Functions", *International Economic Review* 33, 705-722.

Feenstra, R.C. (2004), *Advanced International Trade: Theory and Evidence*, Princeton N.J.: Princeton University Press.

Fisher, F.M. and K. Shell (1972), "The Pure Theory of the National Output Deflator", pp. 49-113 in *The Economic Theory of Price Indexes*, New York: Academic Press.

Fisher, I. (1922), *The Making of Index Numbers*, Houghton-Mifflin, Boston.

Fox, K.J. and U. Kohli (1998), "GDP Growth, Terms of Trade Effects and Total Factor Productivity", *Journal of International Trade and Economic Development* 7, 87-110.

Gorman, W.M. (1968), "Measuring the Quantities of Fixed Factors", pp. 141-172 in *Value, Capital and Growth: Papers in Honour of Sir John Hicks*, J.N. Wolfe (ed.), Chicago: Aldine.

Hotelling, H. (1932), "Edgeworth's Taxation Paradox and the Nature of Demand and Supply Functions", *Journal of Political Economy* 40, 577-616.

Jorgenson, D.W. and Z. Griliches (1967), The Explanation of Productivity Change", *The Review of Economic Studies* 34, 249-283.

Kohli, U. (1978), "A Gross National Product Function and the Derived Demand for Imports and Supply of Exports", *Canadian Journal of Economics* 11, 167-182.

Kohli, U. (1990), "Growth Accounting in the Open Economy: Parametric and Nonparametric Estimates", *Journal of Economic and Social Measurement* 16, 125-136.

Kohli, U. (1991), *Technology, Duality and Foreign Trade: The GNP Function Approach to Modeling Imports and Exports*, Ann Arbor: University of Michigan Press.

Kohli, U. (2003), "Growth Accounting in the Open Economy: International Comparisons", *International Review of Economics and Finance* 12, 417-435.

Kohli, U. (2004a), "An Implicit Törnqvist Index of Real GDP", *Journal of Productivity Analysis* 21, 337-353.

Kohli, U. (2004b), "Real GDP, Real Domestic Income and Terms of Trade Changes", *Journal of International Economics* 62, 83-106.

Kohli, U. (2006), "Real GDP, Real GDI and Trading Gains: Canada, 1982-2005", *Productivity Monitor*, Number 13, Fall, 46-56.

Kohli, U. (2007), "Terms of Trade, Real Exchange Rates, and Trading Gains", forthcoming in *Price and Productivity Measurement*, W.E. Diewert, B.M. Balk, D. Fixler, K.J. Fox and A.O. Nakamura (eds.), Canada: Trafford Press.

Lau, L. (1976), "A Characterization of the Normalized Restricted Profit Function", *Journal of Economic Theory*, 12:1, 131-163.

Morrison, C.J. and W.E. Diewert (1990), "Productivity Growth and Changes in the Terms of Trade in Japan and the United States", pp. 201-227 in *Productivity Growth in Japan and the United States*, Chicago: University of Chicago Press. http://www.econ.ubc.ca/diewert/terms.pdf

McFadden, D. (1978), "Cost, Revenue and Profit Functions", pp. 3-109 in *Production Economics: A Dual Approach to Theory and Applications*. Volume 1, M. Fuss and D. McFadden (eds.), Amsterdam: North-Holland.

Samuelson, P.A. (1953), "Prices of Factors and Goods in General Equilibrium", *Review of Economic Studies* 21, 1-20.

Samuelson, P.A. and S. Swamy (1974), "Invariant Economic Index Numbers and Canonical Duality: Survey and Synthesis", *American Economic Review* 64, 566-593.

Woodland, A.D. (1982), *International Trade and Resource Allocation*, Amsterdam: North-Holland.