Credit risk and disaster risk

François Gourio*

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Abstract

Corporate credit spreads are large, volatile, countercyclical, and significantly larger than expected losses, but existing macroeconomic models with financial frictions fail to reproduce these patterns, because they imply small and constant aggregate risk premia. Building on the idea that corporate debt, while safe in normal times, is exposed to the risk of economic depression, this paper embeds a trade-off theory of capital structure into a real business cycle model with a small, time-varying risk of large economic disaster. This simple feature generates large, volatile and countercyclical credit spreads as well as novel business cycle implications. In particular, financial frictions substantially amplify the effect of shocks to the disaster probability.

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1 Introduction

The large widening of credit spreads during the recent crisis has drawn attention to their important allocative role: for many large corporations, the bond market is the "marginal source of finance". Macroeconomic models such as Bernanke, Gertler and Gilchrist (1999) emphasize the role of credit spreads: as a firm's net worth falls, its probability of default rises and the credit spread increases, leading to a decline of capital expenditures. The importance of this financial accelerator mechanism is underscored by some recent estimation exercises.[1] However, this model, like most macroeconomic models with financial frictions, is at odds with several well-documented patterns of credit spreads, known as the "credit spread puzzle" in the empirical finance literature.[2] In the Bernanke, Gertler and Gilchrist model, the average return on a portfolio of corporate bonds is essentially the risk-free rate, because aggregate risk premia are small. Equivalently, the credit spread corresponds exactly to the probability

*Department of Economics, Boston University and NBER. Address: 270 Bay State Road, Boston MA 02215, email: fgourio@bu.edu, phone: (617) 353 4534. I thank Hui Chen, Simon Gilchrist, Nobu Kiyotaki, Harald Uhlig, Karl Walentin, Vlad Yankov, and participants in presentations at Boston University, ECB, Paris school of Economics, the CEPR-EABCN December 2010 conference, and SED 2010 for comments. Michael Siemer provided outstanding research assistance. NSF funding under grant SES-0922600 is gratefully acknowledged.

[1]See Christiano, Motto and Rostagno (2009), and Gilchrist, Ortiz and Zakrajek (2009).

[2]See Huang and Huang (2003), Hackbarth, Miao and Morellec (2006), Chen (2008), Chen, Collin-Dufresne and Goldstein (2009), among others.

of default. In contrast, in the data the probability of default of an investment grade bond is much smaller than the spreads: the probability is about 0.4% per year (and there is substantial recovery upon default, around 50%), but spreads average around 100bp.[3] These large spreads suggest the importance of a large, potentially time-varying risk premium. These spreads are moreover quite volatile, with a standard deviation around 40bp per year, and they are countercyclical. While the level of spreads was particularly elevated during the recent financial crisis, the cyclicality of spreads is a recurring feature of U.S. business cycles.[4][5]

This paper studies the effect of financial frictions in a model that reproduces the key features of credit spreads. By their very nature, corporate bonds are sensitive to the risk of large economic recessions. Building on this idea, I embed a simple trade-off model of capital structure, where the choice of defaultable debt is driven by taxes and bankruptcy costs, into a real business cycle (RBC) model, and assume that there is a small, exogenously time-varying risk of large economic disaster, following the work of Rietz (1988), Barro (2006), Gabaix (2007), and Gourio (2010). The risk of disaster captures the possibility of a large recession such as the Great Depression.[6] The capital structure choice modifies the standard RBC model equilibrium in two ways. First, the standard Euler equation is adjusted to reflect that investment is financed using both debt and equity, and the user cost of capital hence takes into account expected discounted bankruptcy costs as well as the tax savings generated by debt finance. Second, an additional equation determines the optimal leverage choice, by equating the marginal expected discounted (tax) benefits and (bankruptcy) costs of debt. The model remains highly tractable and intuitive, which allows to evaluate the role of defaultable debt and leverage choice on quantities and prices in a transparent fashion. In particular, the model encompasses the standard real business cycle model as a special (limiting) case.

The first result is that time-varying disaster risk generates large, volatile and countercyclical credit spreads, which are significantly larger than default probabilities. The second main result is that financial frictions amplify substantially – by a factor of about three – the response of the economy to a shock to the disaster probability. Consistent with the extant literature, this amplification effect does not arise if the economy is subjected to TFP shocks. Hence, it is the *interaction* between the trade-off model, a staple of corporate finance, and time-varying disaster risk which generates novel, quantitatively appealing implications for both asset prices and quantities.

The key mechanism is as follows. When the probability of economic disaster exogenously increases, the probability of default rises (holding constant the leverage policy). A higher probability of default

[3] This is the spread of a BAA-rated corporate bond over a AAA-rated corporate bond (rather than a Treasury), so as to net out differences in liquidity.

[4] Philippon (2008), Gilchrist, Yankov and Zakrajek (2009), Mueller (2009), among others, show that credit spreads are highly correlated with, and forecast, investment and output.

[5] Some researchers argue that the variation in credit spreads during the 2008 financial crisis is driven by the deteriorating balance sheets of banks and other financial institutions, who may be the marginal investors in these markets. However, corporate bonds are not exotic assets: any household can buy directly a mutual fund or an ETF of corporate bonds.

[6] The probability of economic disaster can be interpreted either as a rational, objective belief, but an alternative "behavioral" interpretation is that the probability of disaster reflects time-varying pessimism. This simple modeling device captures the idea that aggregate uncertainty is sometimes high, and that some asset price changes are not obviously related to current or future productivity, i.e. "bubbles", "animal spirits".

directly raises expected discounted bankruptcy costs. However, expected discounted bankruptcy costs also rise through a second channel: agents anticipate that defaults are now more systematic, i.e. more likely to be triggered by a bad aggregate shock rather than a bad idiosyncratic shock. This higher systematic default risk increases the risk premium on corporate debt, making it more expensive ex-ante to raise funds for investment. Overall, higher expected discounted bankruptcy costs increase the user cost of capital, leading to a reduction in investment. In equilibrium, firms also cut back on debt and substitute for equity, but since debt is cheaper due to the tax advantage, the user cost of capital has to rise. To sum up, higher disaster risk worsens financial frictions because debt is not efficient when disaster risk is high.

The model has several implications. First, eliminating the deductibility of interest expenses from taxable corporate income leads to a reduction in macroeconomic volatility and hence to significant welfare gains. Second, making debt payments contingent on disaster realizations (as has been recently suggested by several commentators) reduces volatility substantially: this simple change eliminates the amplification effect of financial frictions. Third, a high level outstanding debt makes the economy more fragile, as any negative shock is likely to lead a significant share of firms into default, which is inefficient. A consequence is that a low perceived risk of economic disaster, which leads to higher leverage, makes the economy less resilient to shocks – consistent with a widely held view regarding the recent recession.

In contrast to most of the literature, which focuses on small entrepreneurial firms which cannot raise equity easily and rely on bank financing, this model is designed to capture the richer margins that large US corporations use to raise capital. In my model, firms always pay dividends (unless they default), and no borrowing constraint binds. The relative attractiveness of debt and equity finance varies over time, leading to variation in the user cost of capital. My model thus is not subject to a standard critique of financial frictions models, that most firms do pay dividends and are "thus" unconstrained. Nor does my model rely on a significant heterogeneity between small, productive, constrained firms on the one hand, and large, unproductive, unconstrained firms on the other hand. Incorporating these realistic elements would of course be interesting, but it is not required. This suggests that the model mechanism is quite robust. My model is also at least qualitatively consistent with several stylized facts on the correlation of corporate defaults: first, the "excess clustering" documented byDas et al. (2007), and second the significant probability of large default losses on portfolios of corporate bonds estimated by Duffie et al. (2009). Last, it is important to note that while many firms do not access the corporate bond market directly and instead rely on bank loans, a significant fraction of these loans are securitized (e.g. through CLOs) and hence trade on a market that is similar to the corporate bond market.

Organization of the paper

The rest of the introduction discusses the related literature. Section 2 sets up the model. Section 3 studies its quantitative implications. Section 4 considers some implications and extensions of the baseline model. Section 5 concludes. An online appendix provides additional robustness results and details the numerical method.

Related literature

This paper is related to four different branches of literature. First, the paper draws from the recent literature on "disasters" or rare events (Rietz (1988), Barro (2006), Gabaix (2007), Wachter (2008), and the criticisms of Julliard and Ghosh (2008) and Backus, Chernov and Martin (2009)). In particular, the model is a direct, but significant, extension of Gourio (2010), who studied a frictionless real business cycle model with time-varying disaster risk.

Second, the paper builds on the large macroeconomic literature studying general equilibrium business cycle models with financing constraints (Bernanke and Gertler (1989) and Kiyotaki and Moore (1997)). Some recent studies in this vein are Chugh (2010), Gomes and Schmid (2008), Jermann and Quadrini (2008), Mendoza (2010), Miao and Wang (2010), and Liu, Wang and Zha (2009). In contrast to many studies such as Bernanke, Gertler and Gilchrist (1999) that completely shut down equity financing and focus on the accumulation of internal funds, in my model firms are able to raise equity costlessly. (Amdur (2010), Covas and Den Haan (2009), and Hennessy and Levy (2007) also study the business cycle behavior of capital structure.) Several of these papers analyze linearized DSGE models, where asset prices are much less volatile than in the data, and aggregate risk premia are small and nearly constant. Because the economic mechanism of these models often features asset prices, it seems important to examine the effect of financial frictions in a model where asset prices more closely mimic the data.

Third, the paper considers the real effects of a particular shock to uncertainty (a change in the probability of disaster). The negative effect of uncertainty on output has been studied most recently by Bloom (2009), who emphasizes the "wait-and-see" effect driven by lumpy hiring and investment behavior. My model focuses on changes in *aggregate* uncertainty and the mechanism is different: desired investment falls through a general equilibrium effect and by exacerbating financial frictions. A related mechanism has recently been explored in the studies of Arellano, Bai and Kehoe (2010) and Gilchrist, Sim and Zakrajek (2010), who consider changes in idiosyncratic uncertainty as in Bloom (2009), but in a setup with credit frictions. I compare this mechanism and my mechanism in more detail in section 4.6.

Fourth, the paper relates to the vast literature on the "credit spread puzzle" (e.g. Leland (1994), Huang and Huang (2003), Hackbardt, Miao and Morellec (2006), Chen (2010), Chen, Collin Dufresne and Goldstein (2009), and Bhamra, Kuehn and Strebulaev (2009a, 2009b)). As discussed in the introduction, this literature documents that the prices of corporate bonds are too low to be accounted for in a risk-neutral model, and considers various risk adjustments, borrowed either from the long-run risk or the habits literature, to improve the fit of prices. Perhaps surprisingly, there is, to my knowledge, no model that studies the contribution of disaster risk to the credit spread puzzle. Moreover, the literature does not consider investment and is not set in general equilibrium, making it difficult to evaluate the macroeconomic impact of the financial frictions. On the other hand, this literature studies the asset pricing implications in more detail and incorporates long-term debt.

2 Model

I first present the household problem, then the firm problem, and finally define the equilibrium and asset prices.

2.1 Household

The representative household has recursive preferences over consumption and leisure, following Epstein and Zin (1989):

$$U_t = \left((1-\beta)(C_t^{\upsilon}(1-N_t)^{1-\upsilon})^{1-\psi} + \beta E_t\left(U_{t+1}^{1-\gamma}\right)^{\frac{1-\psi}{1-\gamma}}\right)^{\frac{1}{1-\psi}}. \tag{1}$$

Here ψ is the inverse of the intertemporal elasticity of substitution (IES) over the consumption-leisure bundle, and γ measures risk aversion towards static gambles over the bundle. When $\psi = \gamma$, the model collapses to expected utility. While the additional flexibility of recursive utility is useful in calibrating the model, the key qualitative results can be obtained with standard CRRA preferences (See section 4.5).

The household supplies labor in a competitive market, and trades stocks and bonds issued by the corporate sector.[7] The budget constraint reads

$$C_t + n_t^s P_t + q_t B_t \leq W_t N_t + \varrho_t B_{t-1} + n_{t-1}^s\left(P_t + D_t\right) - T_t, \tag{2}$$

where W_t is the real wage, B_{t-1} is the quantity of debt issued by the corporate sector in period $t-1$ at price q_{t-1}, each unit of which is redeemed in period t for ϱ_t, n_t^s is the quantity of equity shares, P_t is the price of equity, D_t is the dividend, and T_t is a lump-sum tax. The number of equity shares n_t^s is normalized to one. In the absence of default, $\varrho_t = 1$, but $\varrho_t < 1$ if some bonds are not repaid in full. The household takes the process of ϱ_t as given, but it is determined in equilibrium by default decisions of firms, as we will see later.

Intertemporal choices are determined by the stochastic discount factor (a.k.a. marginal rate of substitution), which prices all assets:

$$M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{\upsilon(1-\psi)-1}\left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-\upsilon)(1-\psi)}\frac{U_{t+1}^{\psi-\gamma}}{E_t\left(U_{t+1}^{1-\gamma}\right)^{\frac{\psi-\gamma}{1-\gamma}}}. \tag{3}$$

The labor supply decision is governed by the familiar condition:

$$W_t = \frac{1-\upsilon}{\upsilon}\frac{C_t}{1-N_t}. \tag{4}$$

2.2 Firms

I first describe the general structure of the firm problem, then we fill in the details.

2.2.1 Summary

There is a continuum of mass one of perfectly competitive firms, which are all identical ex-ante and differ ex-post only in their realization of an idiosyncratic shock. For simplicity, we assume that firms live only for two periods. Firms purchase capital at the end of period t in a competitive market, for use in period $t+1$. This investment is financed through a mix of equity and debt. In period $t+1$, the aggregate shocks and the idiosyncratic shock are revealed, firms decide on employment and production, and then sell back

[7] It is possible to introduce government bonds as well. If the government finances this debt using lump-sum taxes and transfers, Ricardian equivalence holds, and government policy does not affect the equilibrium allocation and prices.

their capital. Two cases arise at this point: (1) the firm value is larger than outstanding debt: the debt is then repaid in full and the residual value goes to shareholders as dividends; or (2) the firm value is smaller than outstanding debt: in this case the firm declares default, equityholders receive nothing, and bondholders capture the firm's value, net of some bankruptcy costs. In all cases, the firms disappear after production in period $t+1$ and new firms are created, which will raise funds and invest in period $t+1$, and operate in period $t+2$.[8]

The timing assumption clarifies the mechanism, because a default *realization* does not affect employment, output and profits. Ex-ante however, default *risk* affects the cost of capital to the firm and hence its investment decision. This investment decision in turns affects employment and output, and in general equilibrium all quantities and prices. In section 4.1, we consider an extension where default affects employment and production.

Since firms are ex-ante identical, they will all make the same choices. Because both production and financing technologies exhibit constant return to scales, the size distribution of firms is indeterminate, and has no effect on aggregate outcomes.

2.2.2 Production

All firms operate the same constant returns to scale Cobb-Douglas production function using capital and labor. The output of firm i is

$$Y_{it} = K_{it}^{\alpha}(z_t N_{it})^{1-\alpha},$$

where z_t is aggregate total factor productivity (TFP), K_{it} is the individual firm capital stock, and N_{it} is labor. Both input and output markets are competitive and frictionless.

2.2.3 Productivity shocks

To model the possibility of large recessions, I assume that the aggregate TFP process in this economy is driven not only by the usual "small" normally distributed shocks standard in RBC theory, but also by rare large negative shocks.[9] Formally,

$$\log z_{t+1} = \log z_t + \mu + \sigma e_{t+1} + x_{t+1} \log(1 - b_{tfp}),$$

where $\{e_{t+1}\}$ is *i.i.d.* $N(0,1)$, and x_{t+1} is an indicator equal to 1 if a disaster happens, and 0 otherwise. I will also assume that the realization of disaster directly affects the capital stock (see the next paragraph). The probability of a disaster at time $t+1$ is denoted p_t. This probability of disaster p_t follows itself a Markov chain with transition matrix Q. The *three* aggregate shocks $\{e_{t+1}, x_{t+1}, p_{t+1}\}$ are assumed to be independent, conditional on p_t.

[8] The assumption that firms live two periods, while obviously unrealistic, leads to substantial simplification of the analysis, which is useful to solve the model but also to clarify its implications. An important direction of future research is to incoporate long-lived firms and long-term debt in the model. Based on section 3.1 below, I conjecture that the model mechanism would still be quantitatively relevant.

[9] For parsimony and tractability, these rare disasters are modeled as one-time permanent jump in TFP; Gourio (2011) considers various extensions and shows that the key results are largely unaffected if disasters are modeled as smaller shocks that are persistent, and are followed by recoveries, provided that risk aversion is increased somewhat.

2.2.4 Depreciation shocks

Firms decide on investment at time t, but the actual quantity of capital that they will have to operate at time $t+1$ is random, and is affected both by realizations of aggregate disasters x_{t+1} as well as an idiosyncratic shock ε_{it+1}. Specifically, if a firm i picks $K^w_{i,t+1}$ at time t (where w stands for wish), it actually has $K_{it+1} = K^w_{it+1}(1 - x_{t+1}b_k)\varepsilon_{it+1}$ to operate in period $t+1$, and $(1-\delta)K_{it+1}$ units of capital to resell. The idiosyncratic shock ε_{it+1} is *i.i.d.* across firms and across time, and drawn from a cumulative distribution function H, with mean unity.

2.2.5 Discussion of the assumptions regarding disasters

Barro (2006) and Barro and Ursua (2008) identify numerous large negative macroeconomic shocks in a cross-section of countries, which are usually caused by wars or economic depressions. In a standard neoclassical model there are two simple ways to model macroeconomic disasters – as destruction of the capital stock, or as a reduction in total factor productivity. My formulation allows for both.

TFP appears to play an important role during economic depressions (Kehoe and Prescott, 2007). While economists do not understand well the sources of fluctuations in total factor productivity, large and persistent declines in TFP may be linked to poor government policies, such as expropriation, confiscatory taxes, or trade policies. They may also be caused by disruptions in financial intermediation, if these lead to inefficient capital allocation.

Capital destruction is clearly realistic for wars or natural disasters, but it can also be interpreted more broadly. Perhaps it is not the physical capital but the intangible capital (customer and employee value) that is destroyed during prolonged economic depressions.

At the heart, the model mechanism requires two ingredients: (1) that disasters are clearly bad events, with high marginal utility of consumption; (2) that the return on capital is low during disasters. These assumptions are certainly realistic. Introducing a large TFP shock is the simplest way to obtain (1) in a neoclassical model, and introducing a depreciation shock is the simplest way to obtain (2). An alternative to depreciation shocks is to introduce steep adjustment costs: since investment falls significantly during disasters, the price of capital would also fall, generating endogenously a low return on capital during disasters.

2.2.6 Capital structure choice

The choice of equity versus debt is driven by a standard trade-off between default (bankruptcy) costs and the tax advantage of debt. Specifically, I assume that bondholders recover a fraction θ of the firm value upon default, where $0 < \theta < 1$. Moreover, a firm which issues debt at a price q receives χq, where $\chi > 1$. That is, for each dollar that the firm raises in the bond market, the government gives a subsidy $\chi - 1$ dollar. For simplicity, I assume that the subsidy takes place at issuance.[10]

The bond price q is determined at time of issuance, taking into account default risk, and hence depends on the firm's choice of debt and capital as well as the economy's state variables. Equity

[10] In reality, interest on corporate debt is deductible from the corporate income tax, hence the implicit subsidy takes place when firms' earnings are taxed.

issuance is assumed to be costless. When $\chi = \theta = 1$, the capital structure is indeterminate and the Modigliani-Miller theorem holds. When $\chi = 1$, the firm finances only through equity, since debt has no advantage. As a result, there is no default, and we obtain the standard RBC model. When $\theta = 1$, or more generally $\theta\chi \geq 1$, the firm finances only through debt, since default is not costly enough. I will assume $\chi\theta < 1$, a necessary assumption to generate an interior choice for the capital structure.

2.2.7 Employment, Output, Profits, and Firm Value

To solve the optimal financing choice, we first need to determine the profits and the firm value. (The distribution of firm value determines the probability of default and hence the lending terms the firm can obtain ex-ante.) The labor choice is determined through static profit maximization, given the realized values of both productivity and capital stock, and given the aggregate wage:

$$\pi\left(K_{it}, z_t; W_t\right) = \max_{N_{it} \geq 0} \left\{K_{it}^{\alpha}(z_t N_{it})^{1-\alpha} - W_t N_{it}\right\},$$

which leads to the labor demand

$$N_{it} = K_{it}\left(\frac{z_t^{1-\alpha}(1-\alpha)}{W_t}\right)^{\frac{1}{\alpha}}, \tag{5}$$

and the output supply

$$Y_{it} = K_{it}^{\alpha}(z_t N_{it})^{1-\alpha} = K_{it}\left(\frac{(1-\alpha)}{\frac{W_t}{z_t}}\right)^{\frac{1-\alpha}{\alpha}}.$$

These equations can then be aggregated. Define aggregates through $K_t = \int_0^1 K_{it} di$, $Y_t = \int_0^1 Y_{it} di$, etc., we obtain that $Y_t = K_t^{\alpha}(z_t N_t)^{1-\alpha}$, i.e. an aggregate production function exists, and it has exactly the same shape as the microeconomic production function. Aggregating equation (5) shows that the wage satisfies the usual condition $W_t = (1-\alpha)\frac{Y_t}{N_t}$. The law of motion for capital is obtained by summing over i the equation $K_{it+1} = K_{it+1}^{w}(1 - x_{t+1}b_k)\varepsilon_{it+1}$. Since all firms are identical ex-ante, and they will make the same investment choice $K_{it+1}^{w} = K_{t+1}^{w}$, and since ε_{it+1} has mean unity, idiosyncratic shocks average out and the aggregate capital is

$$K_{t+1} = K_{t+1}^{w}(1 - x_{t+1}b_k).$$

Profits at time $t+1$ are given by

$$\pi_{it+1} = Y_{it+1} - W_{t+1}N_{it+1} = \alpha Y_{it+1} = \alpha K_{it+1}\left(\frac{(1-\alpha)}{\frac{W_{t+1}}{z_{t+1}}}\right)^{\frac{1-\alpha}{\alpha}} = K_{it+1}\alpha\frac{Y_{t+1}}{K_{t+1}},$$

i.e. each firm receives factor payments proportional to the quantity of capital it has, and to the aggregate marginal product of capital $\alpha\frac{Y_{t+1}}{K_{t+1}}$. The total firm value at the end of the period is

$$V_{it+1} = \pi_{it+1} + (1-\delta)K_{it+1} = K_{it+1}\left(1 - \delta + \alpha\frac{Y_{t+1}}{K_{t+1}}\right). \tag{6}$$

Define the aggregate return on capital as $R_{t+1}^{K} = (1 - x_{t+1}b_k)\left(1 - \delta + \alpha\frac{Y_{t+1}}{K_{t+1}}\right)$. The individual return on capital is $R_{it+1}^{K} = \varepsilon_{it+1}R_{t+1}^{K}$. The firm value is thus

$$V_{it+1} = R_{it+1}^{K}K_{t+1}^{w} = \varepsilon_{it+1}R_{t+1}^{K}K_{t+1}^{w}.$$

From ease of notation, I will from now on abstract from the firm subscript i, since all firms are identical and differ only ex-post in their realization of ε.

2.2.8 Investment and Financing Decisions

All firms make the same choices for capital, debt, and hence equity issuance, which are linked through the budget constraint $\chi q_t B_{t+1} + S_t = K_{t+1}^w$. To find the optimal choice of investment and financing, we first need to find the likelihood of default, and the loss-upon-default, for any possible choice of investment and financing. This determines the price of corporate debt. Taking as given this bond price schedule, the firm can then decide on optimal investment and financing.

More precisely, the firm will default if its realized value V_{t+1}, which is the sum of profits and the proceeds from the sale of undepreciated capital, is too low to repay the debt B_{t+1}. This will occur if the firm's idiosyncratic shock ε is smaller than a cutoff value, which itself depends on the realization of aggregate states $(e_{t+1}, p_{t+1}, x_{t+1})$. Mathematically, at time $t+1$, the value of firms which finish operating is $V_{t+1} = \varepsilon_{t+1} R_{t+1}^K K_{t+1}^w$, hence default occurs if and only if

$$\varepsilon_{t+1} < \frac{B_{t+1}}{R_{t+1}^K K_{t+1}^w} \overset{def}{=} \varepsilon_{t+1}^*.$$

If a disaster is realized ($x_{t+1} = 1$), the return on capital is lower and the default threshold ε_{t+1}^* is higher, and more firms default. Given this default rule, the bond issue is priced ex-ante using the representative agent's stochastic discount factor:

$$q_t = E_t \left(M_{t+1} \left(\int_{\varepsilon_{t+1}^*}^{\infty} dH(\varepsilon) + \frac{\theta}{B_{t+1}} \int_0^{\varepsilon_{t+1}^*} \varepsilon R_{t+1}^K K_{t+1}^w dH(\varepsilon) \right) \right).$$

In this equation, the first integral gives the value of the debt in the full repayment states. These states depend on the realization of shocks occurring at time $t+1$, notably disasters, through the threshold for default ε_{t+1}^*. The second term gives the average recovery in default states, divided among all the bondholders and net of bankruptcy costs. The bond price can be rewritten as

$$q_t = E_t \left(M_{t+1} \left(1 - H\left(\varepsilon_{t+1}^*\right) + \frac{\theta R_{t+1}^K K_{t+1}^w}{B_{t+1}} \Omega\left(\varepsilon_{t+1}^*\right) \right) \right), \tag{7}$$

where $\Omega(x) = \int_0^x s dH(s)$. Note the following properties of Ω, which follow from the fact that H is a c.d.f. with mean unity: (i) $\Omega(x) = 1 - \int_x^\infty s dH(s)$; (ii) $\lim_{x\to\infty} \Omega(x) = 1$; (iii) $\Omega'(x) = x h(x)$.

We can now set up the firm's problem at time t : it must decide how much to invest, how much debt to issue (and hence how much of the investment is financed through equity), so as to maximize the expected discounted equity value:

$$\max_{B_{t+1}, K_{t+1}^w, S_t} E_t \left(M_{t+1} \max\left(V_{t+1} - B_{t+1}, 0\right)\right) - S_t, \tag{8}$$

subject to:

$$\chi q_t B_{t+1} + S_t = K_{t+1}^w, \tag{9}$$

$$V_{t+1} = \varepsilon_{t+1} R_{t+1}^K K_{t+1}^w. \tag{10}$$

Equation (9) is the funding constraint: investment must come out of equity S_t, or the sale of bonds (including the subsidy) $\chi q_t B_{t+1}$. The objective function (8) takes into account the option of default for equityholders. Given that the firm defaults if $\varepsilon_{t+1} < \varepsilon^*_{t+1}$, we can rewrite this problem as:

$$\max_{B_{t+1},K^w_{t+1}} E_t\left(M_{t+1}\left(\begin{array}{c} R^K_{t+1}K^w_{t+1} + (\chi\theta-1)\,R^K_{t+1}K^w_{t+1}\Omega(\varepsilon^*_{t+1}) \\ +(\chi-1)\,B_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right)\end{array}\right)\right) - K^w_{t+1}, \quad (11)$$

$$s.t. \quad : \qquad \varepsilon^*_{t+1} = \frac{B_{t+1}}{R^K_{t+1}K^w_{t+1}}.$$

In this expression, the first term is the expected discounted firm value, $E_t\left(M_{t+1}R^K_{t+1}K^w_{t+1}\right)$; the second term (which is negative since $\chi\theta < 1$) is expected discounted bankruptcy costs; and the third term is the expected discounted tax shield. The last term K^w_{t+1} is simply the cost of investment. By contrast, in a frictionless model, the firm would simply maximize $E_t\left(M_{t+1}R^K_{t+1}K^w_{t+1}\right) - K^w_{t+1}$. The difference is that the firm also takes into account the value of tax subsidies and default costs in making its decisions. Default costs are born by debt holders ex-post, but expected default costs are passed on into debt prices ex-ante, implying that equity holders actually bear the costs of default.

To solve this program, we simply take the first-order conditions with respect to K^w_{t+1} and B_{t+1}. The first-order condition with respect to K^w_{t+1} yields,

$$E_t\left(M_{t+1}R^K_{t+1}\left(1+(\chi\theta-1)\,\Omega(\varepsilon^*_{t+1}) + (\chi-1)\,\varepsilon^*_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right)\right)\right) = 1. \quad (12)$$

Recall that $R^K_{t+1} = (1-x_{t+1}b_k)\left(1-\delta+\alpha\frac{Y_{t+1}}{K_{t+1}}\right)$ is the familiar expression for the unlevered physical return on capital, adjusted to reflect the possibility of disasters. In a model without financial frictions, the standard Euler equation implies $E_t\left(M_{t+1}R^K_{t+1}\right) = 1$; here, equation (12) is modified to take into account the bankruptcy costs (the second term), which raise the cost of capital, and the tax shield (the third term), which reduces it. When $\chi = \theta = 1$, we return to the standard equation, corresponding to the case of an unlevered firm. Overall the firm has always access to cheaper financing than in the frictionless (all-equity financed) model, since it always has the possibility to not take any debt. As a result, the steady-state capital stock is always higher when $\chi > 1$ than in the frictionless version.

The first order condition with B_{t+1} is

$$(1-\theta)\,E_t\left(M_{t+1}\varepsilon^*_{t+1}h\left(\varepsilon^*_{t+1}\right)\right) = \left(1-\frac{1}{\chi}\right)E_t\left(M_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right)\right). \quad (13)$$

This equation determines the optimal financing choice between debt and equity.[11] The left-hand side is the marginal cost of debt, i.e. an extra dollar of debt will increase the likelihood of default, and the associated bankruptcy costs. The right-hand side is the marginal benefit of debt, i.e. the higher tax shield in non-default states. Importantly, both the marginal cost and the marginal benefit are discounted using the stochastic discount factor M_{t+1}. The importance of this risk-adjustment is consistent with the empirical work by Almeida and Philippon (2007), who note that corporate defaults are more frequent in "bad times" and as a result the ex-ante marginal cost of debt is higher than a risk-neutral calculation would suggest. This risk-adjustment will play a substantial role in the analysis below: for a given debt level, an increase in the probability of disaster increases expected discounted default costs, not

[11] A second order condition is required to ensure that this condition is sufficient. Some regularity condition must be imposed on the distribution H, e.g. the function $z \rightarrow \frac{zh(z)}{1-H(z)}$ is increasing. Bernanke, Gertler and Gilchrist (1999) make the same assumption in the context of a related model. Most distributions (such as the log-normal distribution) satisfy this assumption.

only because defaults become more likely, but also because they are more likely to occur during bad aggregate times.

Define desired leverage $L_{t+1} = B_{t+1}/K^w_{t+1}$, which is decided at time t. The firm defaults if $\varepsilon R^K_{t+1} < L_{t+1}$ i.e. if the return on capital is low relative to the leverage.

2.3 Equilibrium

The equilibrium definition is standard. First, the labor market clears:

$$(1-\alpha)\frac{Y_t}{N_t} = W_t = \frac{(1-\upsilon)C_t}{\upsilon\,(1-N_t)}. \tag{14}$$

Second, the goods market clears, i.e. total consumption plus investment plus bankruptcy costs equals output,

$$C_t + I_t + (1-\theta)\Omega(\varepsilon^*_t)V_t = Y_t. \tag{15}$$

This equation implies that a wave of defauls leads to large bankruptcy costs and induces a negative wealth effect. In order to clarify the mechanism, I initially abstract from this effect, by assuming that the default cost is a tax, i.e. it is transferred to the government, which then rebates it to household using lump-sum transfers (T_t in equation 2). Then, the resource constraint is simply

$$C_t + I_t = Y_t. \tag{16}$$

Under this simplification, equations (12) and (13) are the only departures of our model from the standard real business cycle model: first, the Euler equation needs to be adjusted to reflect the tax shield and bankruptcy costs; second, the optimal leverage is determined by the trade-off between costs and benefits of debt finance. To summarize, the equilibrium is characterized by the equations (14), (16), as well as (12) and (13) and the definition of the stochastic discount factor (1) and (3).

2.3.1 Recursive Representation

It is useful, both for conceptual clarity and to implement a numerical algorithm, to present a recursive formulation of this equilibrium. This can be done in three steps. First, make the simplifying assumption that the bankruptcy cost is a tax, instead a of a real resource cost. Second, note that the equilibrium can be entirely characterized from time t onwards given the values of the realized aggregate capital stock K_t, the probability of disaster p_t, and the level of total factor productivity z_t, i.e. these are the three state variables.[12] Hence, the model has the same states as the frictionless real business cycle (RBC) model. Third, examination of the first-order conditions shows that they can be rewritten solely as a function of the detrended capital $k_t = K_t/z_t$ and p_t. This is a standard simplification in the stochastic growth model when technology follows a unit root, which also applies to our framework.

As a result the equilibrium policy functions can be expressed as functions of two state variables only, k and p. Compared to the standard RBC model, we have an additional equilibrium policy function to

[12] The level of outstanding debt B_t at the beginning of period is not a state variable, since it does not affect production or investment possibilities. It does affect default, but because defaults do not affect production, and bankruptcy costs are not in the resource constraint, the realization of default does not matter in itself – what matters is the possibility of default going forward. Here we rely on two assumptions: (1) the default cost is a tax; (2) default takes place after production.

solve for, the desired leverage $L(k,p)$, and correspondingly, we have an additional first-order condition (equation (13)). Last, the first-order condition determining optimal investment, i.e. the standard Euler equation (equation 12)), is modified to take into account the marginal financing costs. The full list of equations of this recursive representation is in appendix.

2.3.2 Asset Prices

Any payoff can be priced using the stochastic discount factor, given by the representative agent's marginal rate of substitution. I focus here on four assets: a pure risk-free asset, a short-term government bond which may default during disasters, the corporate bond, and the equity. All these assets last only one period. The price of the risk-free asset can be calculated as the expectation of the stochastic discount factor, $P_t^{rf} = E_t\left(M_{t+1}\right)$. Following Barro (2006), the government bond is assumed to default by a factor Δ during disasters, and hence its price is $P_t^{gov} = E_t\left(M_{t+1}\left(1-x_{t+1}\Delta\right)\right)$. The payoff to a diversified portfolio of corporate bonds, used in the household budget constraint (equation (2)), is

$$\varrho_{t+1} = 1 - H\left(\varepsilon_{t+1}^{*}\right) + \frac{\theta R_{t+1}^{K} K_{t+1}^{w}}{B_{t+1}} \Omega\left(\varepsilon_{t+1}^{*}\right), \tag{17}$$

and the corporate bond price is $P_t^{corp} = q_t = E_t\left(M_{t+1}\varrho_{t+1}\right)$. Last, the equity value is (equation (11)):

$$P_t^{eq} = E_t\left(M_{t+1}\left(R_{t+1}^{K}K_{t+1}^{w}\left(1-\Omega\left(\varepsilon_{t+1}^{*}\right)\right) - B_{t+1}\left(1-H\left(\varepsilon_{t+1}^{*}\right)\right)\right)\right).$$

Given constant return to scale and no equity issuance costs, the equity price satisfies a free entry condition: $P_t^{eq} = S_t$.

3 Quantitative results

This section studies the implications of the model presented in the previous section. First, I present a combination of analytical results and comparative statics to illustrate the workings of the model. Then, a parametrized version of the model is solved numerically so as to delineate its predictions for business cycle quantities, for asset returns, an in particular for the level and volatility of credit spreads, and their relation with investment and GDP.[13]

3.1 Steady-state comparative statics

To better understand the model, it is useful to perform a "steady-state" analysis, as is commonly done in macroeconomics, but one that takes into account the risk of disaster. The first step is the following result.

Proposition 1 *Assume that $b_k = b_{tfp}$, i.e. capital and productivity fall by the same factor in a disaster Then, a disaster leads consumption, investment, output to also drop by the same factor $b_k = b_{tfp}$, while hours do not change. The return on physical capital is reduced by the same factor. There is no further effect of the disaster on quantities or prices, i.e. all the effect is on impact.*

[13] Given the nonlinear form of the model, and the focus on risk premia, it is important to use a nonlinear solution method. The policy functions $c(k,p), N(k,p), g(k,p)$, and $L(k,p)$, are approximated using Chebychev polynomials and solved for using projection methods. The appendix details the computational method.

Proof. The equilibrium is characterized by the policy functions $c(k,p), i(k,p), N(k,p), L(k,p)$ and $y(k,p) = k^{\alpha}N(k,p)^{1-\alpha}$ which express the solution as a function of the probability of disaster p (the exogenous state variable) and the detrended capital k (the endogenous state variable). The detrended capital evolves according to the shocks ε', x', p' through

$$k' = \frac{(1 - x'b_k)\left((1-\delta)k + i(k,p)\right)}{(1 - x'b_{tfp})\, e^{\mu+\sigma e'}}.$$

Since $b_k = b_{tfp}$,

$$k' = \frac{\left((1-\delta)k + i(k,p)\right)}{e^{\mu+\sigma e'}},$$

is independent of the realization of disaster x'. As a result, the realization of a disaster does not affect c, i, N, y, L since k is unchanged, and hence it leads consumption $C = cz$, investment $I = iz$, and output $Y = yz$ to drop, like z, by a factor $b_k = b_{tfp}$ on impact. Furthermore, once the disaster has hit, it has no further effect since all the endogenous dynamics are captured by k, which is unaffected. The statement regarding returns follows from the expression $R^K_{t+1} = (1 - x_{t+1}b_k)\left(1 - \delta + \alpha\frac{y_{t+1}}{k_{t+1}}\right)$. ■

To obtain further results, we consider a simplified version of the model, where we shut down the shocks to the probability of disaster and the TFP shocks e_{t+1}. As a result, the only source of shocks are disaster realizations, which makes it possible to solve for the path of quantities and returns.

Proposition 2 *Assume that $b_k = b_{tfp}$, that $\sigma = 0$, and that $p_t = p$. The economy has a balanced growth path where $k_t, c_t, i_t, y_t, L_t, N_t$, the risk-free rate, the expected return on capital, and the probability of default, and the credit spread are constant, equal to k^*, c^*, i^*, etc. Along this balanced growth path, the level of capital, consumption, investment and output K_t, C_t, I_t, Y_t, are obtained by multiplying k^*, c^*, i^*, y^* by z_t, which is evolves as $z_{t+1} = z_t e^{\mu + x_{t+1}\log(1-b_z)}$.*

Proof. Given that $\sigma = 0$, and p is constant, we can conjecture an equilibrium of the form described in proposition, and it is easy to check that it satisfies the first-order conditions. Along this balanced growth path, k is constant since it is unaffected by disaster realizations; the policy functions $c(k), i(k), N(k), L(k)$ then imply that these variables are also constant if $k = k^*$. Given this, consumption growth and other variables are iid, implying that expected returns and credit spreads are constant. ■

A graphical illustration of this result, is that macroeconomic quantities simply grow along constant trends, without any shocks except for occasional large downward jumps. During these jumps, realized returns on bonds and equity are low, but the dynamics of quantities are unaffected. The discount factor for this simplified version of the model depends only on the disaster realization:

$$M(x') = \frac{\beta e^{\mu((1-\psi)\upsilon-1)}(1 - x'b_{tfp})^{(1-\gamma)\upsilon-1}}{\left(1 - p + p(1-b_{tfp})^{(1-\gamma)\upsilon}\right)^{\frac{\psi-\gamma}{1-\gamma}}},$$

and the economy's steady-state capital-labor ratio k/N and leverage $L = B/K^w$ are determined by the two equations:

$$\begin{aligned}
&\frac{\beta e^{\mu((1-\psi)\upsilon-1)}}{\left(1 - p + p(1-b_{tfp})^{(1-\gamma)\upsilon}\right)^{\frac{\psi-\gamma}{1-\gamma}}}\left(1 - \delta + \alpha\left(\frac{k}{N}\right)^{\alpha-1}\right) \qquad (18)\\
= \;& (1-p)\left(1 + (\chi\theta - 1)\,\Omega\left(\varepsilon^*_{nd}\right) + (\chi - 1)\,\varepsilon^*_{nd}\left(1 - H\left(\varepsilon^*_{nd}\right)\right)\right)\\
&+ p\left(1 - b_{tfp}\right)^{\upsilon(1-\gamma)-1}(1 - b_k)\left(1 + (\chi\theta - 1)\,\Omega\left(\varepsilon^*_{d}\right) + (\chi - 1)\,\varepsilon^*_{d}\left(1 - H\left(\varepsilon^*_{d}\right)\right)\right).
\end{aligned}$$

and

$$
\begin{aligned}
0 &= (1-p)\left(\chi(\theta-1)\varepsilon_{nd}^{*}h\left(\varepsilon_{nd}^{*}\right)+(\chi-1)\left(1-H\left(\varepsilon_{nd}^{*}\right)\right)\right) \\
&\quad +p\left(1-b_{tfp}\right)^{\upsilon(1-\gamma)-1}\left(\chi(\theta-1)\varepsilon_{d}^{*}h\left(\varepsilon_{d}^{*}\right)+(\chi-1)\left(1-H\left(\varepsilon_{d}^{*}\right)\right)\right),
\end{aligned} \tag{19}
$$

with $\varepsilon_d^* = \frac{L}{(1-b_k)\phi}$ and $\varepsilon_{nd}^* = \frac{L}{\phi}$, and $\phi = 1-\delta+\alpha\left(\frac{k}{N}\right)^{\alpha-1}$ is the standard marginal product of capital.

While these expressions initially appear complicated, they provide significant intuition. First, note that they are recursive: equation (19) first determines the ratio of leverage to the marginal product $\frac{L}{\phi}$, and equation (18) then determines the marginal product of capital ϕ and hence $\frac{k}{N}$.[14] When there is neither disaster risk nor financial frictions, i.e. $p = 0$ and $\chi = \theta = 1$, the first equation collapses to the standard user cost equation,

$$
\beta e^{\mu((1-\psi)\upsilon-1)}\left(1-\delta+\alpha\left(\frac{k}{N}\right)^{\alpha-1}\right)=1.
$$

When there is disaster risk but no financial frictions (as in Gourio (2010)), the steady-state capital is determined as

$$
\beta e^{\mu((1-\psi)\upsilon-1)}\left(1-\delta+\alpha\left(\frac{k}{N}\right)^{\alpha-1}\right)\left(1-p+p\left(1-b_{tfp}\right)^{\upsilon(1-\gamma)}\right)^{\frac{1-\psi}{1-\gamma}}=1.
$$

Simple algebra shows that a higher probability of disaster p induces to a lower capital stock provided that the IES is greater than unity: agents are reluctant to invest in the more risky capital stock. Consider now the case of financial frictions but no disaster risk, equation (19) reflects simply the trade-off between the default costs and tax benefits of leverage:

$$
\chi(1-\theta)\varepsilon^{*}h\left(\varepsilon^{*}\right)=(\chi-1)\left(1-H\left(\varepsilon^{*}\right)\right).
$$

Last, in the full model, disaster risk affects the amount of desired leverage for two reasons. First, it changes the distribution of payoffs to the investment. Second, it changes the discount rates which multiply this distribution of payoffs (the term $\left(1-b_{tfp}\right)^{\upsilon(1-\gamma)-1}$ in equation (19)).

3.1.1 The determinants of optimal leverage and investment

Figure 1 uses this simplified version of the model to illustrate the effect of several key parameters on the steady-state values of capital, leverage, default probability and credit spreads. Each column of this figure corresponds to one parameter; the first column shows the effect of idiosyncratic volatility σ_ε. Holding debt policy constant, higher idiosyncratic risk leads to more default and hence higher credit spreads, increasing the user cost of capital. This leads firms to reduce investment. In equilibrium, firms also endogenously reduce leverage, which mitigates the increase in default and in credit spreads, but makes firms rely more heavily on equity issuance, which is more costly.

[14] Labor supply and the scale of the economy are then determined by preferences in the standard way. First, note that

$$
c=k^{\alpha}N^{1-\alpha}-\delta k=N\left(\left(\frac{k}{N}\right)^{\alpha}-\delta\frac{k}{N}\right),
$$

and second the MRS = MPL condition implies $\frac{1-\upsilon}{\upsilon}\frac{c}{1-N}=(1-\alpha)\left(\frac{k}{N}\right)^{\alpha}$. Since $\frac{k}{N}$ is known, this is one equation in one unknown N.

The second column shows the effect of the tax subsidy χ. A higher χ directly reduces the user cost of capital, since holding debt policy constant, the firm is able to raise more capital. Second, a higher χ makes debt relatively more attractive than equity, leading firms to take on more debt and increase leverage. This higher leverage leads to a higher probability of default and higher credit spreads.

Finally, the third column shows the effect of increasing the recovery rate parameter θ. Since the expected cost of bankruptcy falls, the user cost of investment falls and investment rises. Holding debt policy constant, a higher θ leads to a lower credit spread, since the recovery value is higher. However, since firms take on more debt, the probability of default and credit spreads go up.

3.1.2 User cost, financial frictions and probability of disaster

Turning now to the effect of the probability of disaster, figure 2 displays the effect of a rise in p on capital, leverage, credit spreads and the user cost $\alpha \left(\frac{k}{N}\right)^{\alpha-1}$, which is $r+\delta$ in the standard neoclassical model. Higher disaster risk leads to a reduction in leverage in equation (19), and hence an increase in the user cost (adjusted for the tax shield and bankruptcy costs) in equation (18) and a lower capital-labor ratio. The figure compares the frictionless model ($\chi = \theta = 1$, i.e. the firm is only equity-financed) and the model with the friction ($\chi > 1$). The percentage response of the steady-state capital stock to a change in the probability of disaster is substantially larger in the model with the financial friction, reflecting that the user cost is much more affected by an increase in disaster risk. An increase in disaster risk in itself increases the probability of default, but also makes the risk of default more likely to be driven by a bad *aggregate* realization, hence increases the cost of debt significantly, as reflected by the credit spread.[15] Overall, the probability of disaster p has an effect similar to that of σ_ε, which is the shock considered by Arellano, Bai and Kehoe (2010) or Gilchrist, Sim and Zakrajek (2010) in very recent studies. I return to this comparison in section III.F.

3.2 Parametrization

Parameters are listed in Table 1. The period is one year. Many parameters follow the business cycle literature (Cooley and Prescott (1995)). The risk aversion parameter is four, in order to get a reasonable level for the equity premium. Note that this is the risk aversion over the consumption-hours bundle. Since the share of consumption in the utility index is .3, the effective risk aversion to a consumption gamble is 1.33 (Swanson (2010)), a very low number by the standards of the asset pricing literature.

The intertemporal elasticity of substitution of consumption (IES) is set at 2. There is a large debate regarding the value of the IES. Most direct estimates using aggregate data find low numbers (e.g. Hall (1988)), but this view has been challenged by several authors (see among others Bansal and Yaron (2004), Gruber (2006), Mulligan (2004), Vissing-Jorgensen (2002)). As emphasized by Bansal and Yaron (2004), a low IES has the counterintuitive effects that higher expected growth lowers asset prices, and higher uncertainty increases asset prices. Section 4.5 analyzes how the results are affected by the intertemporal elasticity of substitution.

[15] For high values of the probability of disaster p, the credit spread is decreasing in p. This counterintuitive result simply reflects that for very high p, firms reduce debt significantly to avoid bankruptcy and associated costs.

One crucial element of the calibration is the probability and size of disaster, which follow Barro (2006, 2009) and Barro and Ursua (2008) closely. The probability of a disaster is 1.7% per year on average. For computational simplicity, I summarize the historical distribution of disasters using a five-point distributions, with disaster sizes ranging from 15% to 57%.[16] While these disaster sizes may seem very large, they are the ones estimated by Barro and Barro and Ursua (2007) in a large international panel data set. The results of the paper are largely unchanged if the disaster size is set to be smaller – e.g., perhaps the US faces smaller disasters than most other countries – but risk aversion is correspondingly increased.

The second crucial element is the persistence and volatility of movements in this probability of disaster. I assume that the log of the probability follows an AR(1) process:

$$\log p_{t+1} = \rho_p \log p_t + (1 - \rho_p) \log \overline{p} + \sigma_p \varepsilon_{p,t+1},$$

where $\varepsilon_{p,t+1}$ is *i.i.d.* $N(0,1)$.[17] The parameter $\overline{p}$ is picked so that the average probability is .017 per year, and I set $\rho_p = .75$ and the unconditional standard deviation $\frac{\sigma_p}{\sqrt{1-\rho_p^2}} = 1.50$ in order to roughly match the volatility of credit spreads.

As is standard, I use a log-normal distribution for H, the distribution of idiosyncratic shocks. The three remaining parameters determine the leverage choice: χ, θ and σ_ε, the variance of idiosyncratic shocks. Following the corporate finance literature, I set $\theta = 0.4$, consistent with estimates of recovery rates in "bad times". The parameters σ_ε and χ are then picked to match a target average probability of default and leverage. The target for the probability of default is 0.5% per year. I also set a target for leverage equal to 0.55. In the data leverage is somewhat smaller, perhaps 0.45. Targeting a leverage of 0.45 leads to an unrealistically large variance of idiosyncratic shocks σ_ε. This likely reflects that firm values are more volatile in the model than in the data. Higher volatility may be driven by fixed costs of production, which are equivalent to a higher target for leverage. Alternatively, the distribution of idiosyncratic shocks may exhibit skewness and/or kurtosis.[18]

3.3 Impulse response functions

I first illustrate the dynamics of the model in response to the three aggregate shocks: the standard TFP shock, the disaster realization, and a shock to the probability of disaster. I next discuss how the model fits both quantities and price data.

[16] The data from Barro and Ursua refers to consumption or output, but my model requires to parametrize the capital and TFP destruction. It would be interesting to gather further evidence on disasters, and measure b_k and b_{tfp} directly. This is beyond the scope of this paper. I concentrate on the parsimonious benchmark case $b_k = b_{tfp}$. Given this assumption, to match a drop of, say, 25% in consumption, requires exactly a drop of 25% of capital and z, hence the Barro and Ursua distribution of GDP losses leads directly to the distribution of capital and productivity losses. (Because TFP $= z^{1-\alpha}$, the drop in total factor productivity is smaller than 25%.)

[17] This equation allows the probability to be greater than one, however I will approximate this process with a finite Markov chain, which ensures that $0 < p_t < 1$.

[18] The targets are not exactly matched in the full model because the calibration is done using the "steady-state" version of the model, studied in the previous section.

3.3.1 The effect of a TFP shock

Figure 3 displays the response of quantities and returns to a one standard-deviation shock to the level of total factor productivity. (For clarity, this picture, as well as the ones following, assumes that no other shock is realized.) The response of quantities is similar to that of the standard real business cycle model: investment rises as firms desire to accumulate more capital, employment rises because of the higher labor demand, and consumption adjusts gradually, leading to temporarily high interest rates. The equity return is high on impact, reflecting the sensitivity of firms' dividends to TFP shocks due to leverage, but corporate bonds are largely immune to small TFP shocks - the default and recovery rates are barely affected.[19] As a result, the path for the bond return mirrors that of the risk-free return. There is essentially no change in leverage or credit spreads, since the trade-off determining optimal leverage is hardly affected by the slightly higher TFP.

3.3.2 The effect of a disaster

Figure 4 shows the response of quantities and returns to a disaster which hits at $t = 5$. The disaster realization leads capital and TFP to fall by the factors b_k and b_{tfp} respectively. The calibration assumes that these parameters are equal, and in this simulation, $b_k = b_{tfp} = 25\%$. As a result, the transitional dynamics are very simple, as seen in the figure, and as proved in proposition 1: output, consumption and investment drop on impact by the same factor, and hours do not change. The return on capital is also -25%, and is divided among equity and debt. But it is also further reduced by default, which leads to losses since $\theta < 1$. In this simulation, approximately 12% of firms are in default, the realized equity return is roughly -52% and the realized bond return is -4.5%. (The returns we compute are the average across all the firms, as defined in section 2.3 there are always some firms with very high idiosyncratic shocks which do not default.) Figure 4 illustrates that both equity and corporate debt are risky assets, since their returns are very low precisely in the states (disasters) when marginal utility is high, i.e. consumption growth is low. Consistent with proposition 1, the figure confirms that a disaster does not generate any transitional dynamics in quantities, leverage, credit spreads, interest rates, or risk premia: a disaster leads to a once-and-for-all shift in steady-states.

3.3.3 The effect of an increase in the probability of a disaster

The important shock in this paper is the shock to the probability of disaster – i.e. an increase in perceived risk. Figure 5 presents the responses to an unexpected increase in the probability of disaster at time $t = 5$. The higher risk leads to a sharp reduction in investment. Simultaneously, the higher risk pushes down the risk-free interest rate, as demand for precautionary savings increases. This lower interest rate decreases employment through an intertemporal substitution effect. Hence, output decreases because employment decreases, even though there is no change in current or future total factor productivity, and even though the capital stock adjusts slowly. Intuitively, there is less demand for investment and this reduces the need for production.

[19] The default rate is defined as the share of firms in default. Because some of the capital is recovered in defaults, this is not the realized loss for debholders.

Consumption increases on impact since households want to invest less in the now more risky capital. Consumption then falls over time. Qualitatively, these dynamics are similar to that in the frictionless version, but the quantitative results are quite different. To illustrate this clearly, figure 6 superimposes the responses to a shock to the probability of disaster for the frictionless model ($\chi = \theta = 1$) and for the current model. The response of macro quantities on impact is approximately three times larger in the model with financial frictions.

As argued in section 3.1, the mechanism through which disaster risk affects the economy is by changing the expected discounted bankruptcy costs. These become significantly higher, since default is (i) more likely and (ii) more likely to occur in "bad times". This increases the user cost for a given financial policy, leading firms to cut back on investment. Moreover, firms also adjust their financial policy, reducing debt and leverage.

Because risk increases, risk premia rise as the economy enters this recession: the difference between equity returns and risk-free returns becomes larger, and the spread of corporate bonds over risk-free bonds also rises (see the bottom panel of figure 5). This last result is not fully general, however. The equilibrium level of credit spreads depends on the *endogenous* quantity of debt, or leverage that firms decide to take on. For certain parameter values, the endogenous decrease in leverage leads, paradoxically, to lower credit spreads in response to a higher probability of disaster. However, for the parameter values that we use, firms do not decide to cut back on debt too much, and spreads rise with the probability of disaster. The model hence generates the required negative correlation between credit spreads and investment output. More generally, the model implies that risk premia are larger in recessions, consistent with the data.

3.4 Business cycle and financial statistics

Tables 2, 3 and 4 report standard business cycle and asset return statistics as well as default rates and leverage ratios.[20] To illustrate the role of disaster risk and time-varying disaster risk, I solve the model with the benchmark parameter values, under different assumptions regarding the structure of shocks: (i) only TFP shocks, (ii) TFP shocks and disasters, but a constant probability of disaster; (iii) TFP shocks and disasters, with a time-varying risk of disaster. I also consider three variant of the model: (a) with the financial friction, (b) with constant leverage, and (c) with no financial friction. The benchmark model results (a-iii) are indicated in bold in these tables. The variant with constant leverage adds the constraint that $B_{t+1} = \overline{L} K^w_{t+1}$, i.e. firms must pick debt and capital so that their ratio is constant (and equal to the average leverage in the benchmark model).

The models with only TFP shocks (rows 1 through 3) generate a decent match for quantity dynamics, as is well known from the business cycle literature. This model, however, generates rather small spreads for corporate bonds, and these spreads simply account for the average default of corporate bonds, because aggregate risk premia are very small. The spread is 51bp, twice below the data, whereas the probability of default is 79bp, larger than the data. Moreover, these spreads are essentially constant.

[20] The leverage and default probability data are taken from Chen, Collin-Dufresne, and Goldstein (2009). The other data (GDP, consumption, investment, and credit spreads) are from FRED. I use BAA-AAA as the credit spread measure, and obtain similar results as Chen, Collin-Dufresne, and Goldstein. All series are annualized.

The risk premium for equity is also very small and equity returns are not volatile. Note that except for investment, which is somewhat less volatile in the model with financial friction, the quantity moments are largely unchanged as we go from row 1 to row 3. Hence, financial frictions do not amplify the response to TFP shocks.[21] The smaller volatility of investment in the model with financial frictions is apparently driven by the higher steady-state capital stock (as in Santoro and Wei (2010)).

When constant disaster risk is added to the model (rows 4 through 6), the quantity dynamics are unaffected (table 2). Table 3 reveals that credit spreads are significantly larger however, because defaults are much more likely during disasters, when marginal utility is high. The model generates a higher equity risk premium and a plausible credit spread: the average spread is 129bp, and the probability of default is 50bp. However, the volatility of spreads is still close to zero. This motivates turning to the model with time-varying risk of disaster.

Rows 7 through 9 display the results for the models with time-varying disaster risk. The variation in the disaster risk does indeed lead to volatile credit spreads, roughly in line with the data. The equity premium is too low, but it is significant, and similar to that of the model with constant probability of disaster. Introducing the time-varying risk of disaster also generates new quantity dynamics: output and especially investment become more volatile. Moreover, credit spreads are countercyclical. Overall, the model fits well many stylized facts.

It is noteworthy that the model can generate volatile spreads only when disaster risk is time-varying. This suggests that variation in aggregate risk is important and plays a role in shaping business cycles.[22]

The amplification effect of disaster risk shock through financial frictions is visible in table 2: while the financial friction model exhibits less volatility than the RBC model when disaster risk is constant, it has more volatility than the RBC model when disaster risk is added. This is especially true for investment volatility, which nearly doubles as time-varying disaster risk is introduced.

The model with constant leverage generates even more volatility of quantities. Because firms cannot delever easily when the probability of disaster rises, the model generates more movements in spreads and investment. Finally, the model implies some volatility of leverage, but it falls somewhat short of the data. However, the one-period nature of firms in this model makes it difficult to interpret this statistic: the flow and stock of debt are equal in the model, while they behave differently in the data (Jermann and Quadrini (2009), Covas and Den Haan (2009)).

It is interesting to quantify the increase in *systematic* risk that occurs when the disaster probability rises. Figure 7 presents the correlation of defaults that is expected given the probability of disaster today, i.e. $Corr_t\left(def_{i,t+1}, def_{j,t+1}\right)$ for any two firms i and j in the model economy. In normal times, the probability of disaster is low, and defaults are largely idiosyncratic since aggregate TFP shocks do not create much variation in default rates. Hence, this correlation is low. The correlation becomes much higher, however, when the probability of disaster rises. This is because defaults are now much

[21] The appendix presents a comparison of the impulse response functions to a TFP shock for the different models, which confirms this result.

[22] The key mechanism of the model is time-varying aggregate uncertainty. This time-varying aggregate uncertainty comes here from a time-varying probability of disaster, but the model implications are similar if the uncertainty takes the form of normally distributed shocks. If the shocks are small, risk aversion needs to be correspondingly higher.

more likely to be simultaneously triggered by the realization of a disaster. This higher correlation would show up in some asset prices such as CDO or CLO (collateralized debt or loan obligations). This higher correlation stems directly from the increase in aggregate uncertainty, holding idiosyncratic uncertainty constant. This correlation is affected by firms' choices, however, since they decide on how much debt to take which affects their default likelihood: for very large p, firms cut back on debt so much that this correlation may fall.

Overall, the model has two main deficiencies: first, the correlation of consumption and output is too low; second, the equity return is not volatile enough. The latter point is also driven by the fact that equities are only a one-period asset here, implying that the conditional volatility of equity returns equals the conditional volatility of dividends (i.e. there is only a cash flow effect and no discount rate effect).

4 Extensions and Robustness

This section considers some implications and extensions of the baseline model, and the sensitivity of the quantitative results to parameter changes.

4.1 Default crises and time-varying resilience of the economy

For the purpose of analytical clarity, the benchmark model assumes that default does not affect output: (i) bankruptcy costs are a tax rather than a real resource cost, and (ii) a firm in default is as productive as a firm in good standing. This section relaxes these two assumptions: (i) in reality, bankruptcies are costly: costs include legal fees as well as the loss of intangible capital such as customer goodwill; (ii) firms in default are likely less productive as they need to reorganize and are constrained in their relations with suppliers and customers. Relaxing either of these assumptions implies that an economy with a high level of outstanding debt is prone to "default crises": any negative shock may drive many firms into default, which further degrades the economy. The exact effect of (i) and (ii) is however different: (i) is a pure wealth effect, while (ii) reduces productivity and hence labor demand. Neither (i) nor (ii) affects the default decision ex-post, since the outside option of equity holders is zero.

An important implication of this extension of the model is that the economy's sensitivity to shocks (or resilience) is time-varying. For instance, as discussed in the previous section, a low probability of disaster leads firms to pick a high leverage. This makes the economy less resilient, i.e. its investment and output will fall more should a bad shock occur. This is consistent with a widely held view that during the 2000s, perception of risk fell, leading firms to increase leverage and making the 2008 recession worse.

Formally, we make the following two changes to the model. The first is to assume that a share ω of the bankruptcy costs is a real resource cost. The second is that firms in default have lower productivity, by a factor $(1-\zeta)^{\alpha}$. These two changes do not affect the expression for the default threshold $\varepsilon^*_{t+1} = \frac{B_{t+1}}{K^w_{t+1} R^K_{t+1}}$.Total output, taking into account the lower productivity of firms in default, is now

$$Y_t = (K_t)^{\alpha} (z_t N_t)^{1-\alpha} \left(\left(1-(1-\zeta)\Omega(\varepsilon^*_{t+1})\right)\right)^{\alpha}.$$

The resource constraint now reads

$$C_t + I_t + (1-\theta)\omega\Omega\left(\varepsilon_{t+1}^*\right) R_t^K K_t = Y_t.$$

We also need to modify consequently the firm value and bond price equations and the associated first order conditions; these equations are available in the appendix. As a result of this change, the quantity of debt B is now an additional state variable.

Figure 8 illustrates the negative effect of outstanding debt on the economy for the case $\zeta = 0.5$ and $\omega = 0$, i.e. firms in default are more productive. (The appendix presents an examples for the case of $\zeta = 0$ and $\omega = 0.5$, i.e. bankruptcies have real resource costs.) Ceteris paribus, a larger amount of debt increases default rates, and reduces output, employment, investment and consumption.

4.2 State-contingent debt

The defining characteristic of debt is that it is not state contingent. In the aftermath of the 2008 financial crisis, several economists have proposed that debt should be conditioned on large aggregate shocks. This section evaluates this proposal by allowing firms in the model to issue debt which repayments are contingent on the disaster realization x'.

The model is easily modified; first, the budget constraint now reads,

$$K_{t+1}^w = S_t + \chi q_t^{nd} B_{t+1}^{nd} + \chi q_t^d B_{t+1}^d,$$

where B_{t+1}^{nd} (resp. B_{t+1}^d) is the face value of the debt to be repaid in non-disaster (resp. disaster) states, and q_t^{nd} (resp. q_t^d) the associated price:

$$q_t^{nd} = E_t\left((1-x_{t+1})M_{t+1}\left(\int_{\varepsilon_{t+1}^*}^{\infty} dH(\varepsilon) + \frac{\theta}{B_{t+1}}\int_0^{\varepsilon_{t+1}^*} \varepsilon R_{t+1}^K K_{t+1}^w dH(\varepsilon)\right)\right),$$

where $(1-x_{t+1})$ is a dummy equal to 1 if no disaster happens, and similarly for q_t^d.Taking first-order conditions leads to the following characterization of the equilibrium: first, the Euler equation is

$$E_t\left(M_{t+1}R_{t+1}^K\left(\begin{array}{c} 1+(\chi-1)\,L_{t+1}^{nd}(1-x_{t+1})\left(1-H\left(\varepsilon_{t+1}^*\right)\right) \\ +(\chi-1)\,L_{t+1}^{d}x_{t+1}\left(1-H\left(\varepsilon_{t+1}^*\right)\right) \\ +(\theta\chi-1)\,\Omega(\varepsilon_{t+1}^*) \end{array}\right)\right) = 1,$$

and second, optimal debt is determined through the two equations:

$$\begin{aligned} \frac{\chi-1}{\chi}E_t\left((1-x_{t+1})M_{t+1}\left(1-H\left(\varepsilon_{t+1}^*\right)\right)\right) &= (1-\theta)\,E_t\left(M_{t+1}\Omega'(\varepsilon_{t+1}^*)\,(1-x_{t+1})\right), \\ \frac{\chi-1}{\chi}E_t\left(x_{t+1}M_{t+1}\left(1-H\left(\varepsilon_{t+1}^*\right)\right)\right) &= (1-\theta)\,E_t\left(M_{t+1}\Omega'(\varepsilon_{t+1}^*)x_{t+1}\right). \end{aligned}$$

The Euler equation interpretation is similar to that of the benchmark model; the investor takes into account the total user cost of debt, which now must take into account the different leverage in disaster vs. non-disaster states. The optimal leverage condition simply says that, rather than equating expected discounted marginal costs and benefits of debt over all the states together, the firm can now equate these expected marginal costs and benefits *conditional on the disaster happening or not*. This added flexibility will lead the firm to issue little debt that is payable in disaster states, since bankruptcy is much more likely and costly in these states. As a useful special case, suppose that there are no TFP shocks or shocks to p, then the expectations are just expectations over the idiosyncratic shocks ε, and the first-order condition states, if we denote default cutoff in non-disaster states by ε_{t+1}^{nd*} and in disasters by ε_{t+1}^{d*}:

$$\frac{\chi-1}{\chi}\left(1-H\left(\varepsilon_{t+1}^{d*}\right)\right)=(\theta-1)\,\Omega'(\varepsilon_{t+1}^{d*}),$$
$$\frac{\chi-1}{\chi}\left(1-H\left(\varepsilon_{t+1}^{nd*}\right)\right)=(\theta-1)\,\Omega'(\varepsilon_{t+1}^{nd*}),$$

implying that $\varepsilon_{t+1}^{d*}=\varepsilon_{t+1}^{nd*}$, i.e. $\frac{B_{t+1}^{d}}{K_{t+1}^{w}(1-b_k)}=\frac{B_{t+1}^{nd}}{K_{t+1}^{w}}$ or $B_{t+1}^{d}=B_{t+1}^{nd}(1-b_k)$. Hence, the firm targets the *same* default probability, conditional on a disaster happening, and conditional on no disaster happening. This implies a much lower face value of debt in disasters.

Figure 9 compares the response of the model with state-contingent debt to an increase in disaster risk, with the response of the benchmark model. The amplification effect largely disappears, and the model implies now no more volatility in investment than the frictionless RBC model. Hence, while the assumption that private contracts are not made contingent on aggregate realizations is made in many models (such as Bernanke, Gertler and Gilchrist (1999) or Kiyotaki and Moore (1997)), this result suggest that it is far from innocuous. Krishnamurthy (2003) similarly found that allowing for conditionality reduces or eliminate the amplification effect of financial frictions.

The benefits of debt conditionality in reducing volatility in response to shocks to disaster risk, comes on top of the obvious advantage that, should a disaster happen, there will be fewer defaults, which are likely to be costly (as in the previous section). This suggests that debt conditionality is likely valuable, provided that disasters can be well defined in a contract.

4.3 Welfare cost of the tax shield

Following a large literature in corporate finance, the model features as a prime determinant of capital structure the tax subsidy to debt, or tax shield. The tax shield is inefficient in the model for two reasons. First, the tax shield lowers the user cost of capital and hence encourages capital accumulation. However, the competitive equilibrium of the model without taxes is already Pareto optimal, hence the subsidy leads to *overaccumulation* of capital. Second, the tax shield also amplifies fluctuations in aggregate quantities, including consumption, and hence reduces welfare. Table 8 illustrates this effect by displaying the volatility of output, investment and employment, for various values of χ. Both in terms of steady-states and in terms of fluctuations then, the tax subsidy generates deadweight losses. A figure in appendix gives the welfare cost of the tax subsidy, as a function of χ. For our benchmark calibration of $\chi=1.062$, removing the tax shield entirely would increase welfare substantially, equivalent to a permanent increase of consumption of approximately 3.52%.

4.4 Capital adjustment costs

While the benchmark model abstracts from adjustment costs in the interest of simplicity, introducing them is useful to generate further volatility in the value of capital. In particular, the model implies that an increase in the probability of disaster has essentially no effect on realized equity returns or bond returns.[23] This implication is overturned if there are adjustment costs, because the price of capital then falls following an increase in the probability of disaster, since investment and marginal Q fall. It is simplest to consider an external adjustment cost formulation. Suppose that capital goods are produced by a competitive investment sector which takes I_t consumption goods at time t, and K_t capital goods at time t, and generates $K_{t+1} = (1-\delta)K_t + \Phi\left(\frac{I_t}{K_t}\right) K_t$ capital goods next period. These capital goods are then sold in a competitive market to final goods producing firms at a price given by: $P_t^K = \frac{1}{\Phi'\left(\frac{I_t}{K_t}\right)}$.The same formulas as in the model then apply, with the proviso that the return on capital R_{t+1}^K is now

$$R_{t+1}^K = \left(\frac{(1-\delta)\, P_{t+1}^K + \alpha \frac{Y_{t+1}}{K_{t+1}}}{P_t^K}\right)(1 - x_{t+1} b_k),$$

and $V_t = K_t R_{t+1}^K P_t^K$, and $\varepsilon_{t+1}^* = \frac{B_{t+1}}{R_{t+1}^K K_{t+1}^w P_t^K} = \frac{L_{t+1}}{R_{t+1}^K P_t^K}$. Following Jermann (1998), I set $\Phi(x) = a_0 + a_1 \frac{x^{1-\eta}}{1-\eta}$, where a_0 and a_1 are picked to make the steady-state investment rate and marginal Q independent of η.

Tables 5 through 7 report model moments for two values of η, and a figure in appendix compares the impulse response function of the benchmark model (without adjustment costs) and the model with adjustment costs ($\eta = .1$), when the shock is an increase in the probability of disaster. As expected, adjustment costs smooth the response of investment and output. The qualitative dynamics, as well as the asset prices, remain similar. When the probability of disaster rises, the return on equity is now lower, and the return on the corporate bond is also slightly lower, reflecting the fall in the resale value of capital and the ensuing higher default rate.

4.5 Role of the IES and risk aversion

While the households are assumed to have recursive utility, the model can also be solved in the special case of expected utility. When the elasticity of substitution is kept equal to 2, and the risk aversion is lowered to .5 to reach expected utility, the *qualitative* implications are largely unaffected. Tables 5 through 7 report the model moments with this specification. Because risk aversion is lower, all risk premia are lower, and the response of quantities to a probability of disaster shock is also smaller since agents care less about risk.[24]

In contrast, when the elasticity of substitution is small, a shock to the probability of disaster may lead to different *qualitative* effects. When the IES is low enough, investment, output and employment rise (rather than fall) as the probability of disaster rises. The intuition is that higher risk makes people save more, despite the fact that the capital is more risky. In the frictionless model, the threshold value for the IES is exactly unity. In the model of this paper, higher uncertainty has a more negative effect on investment demand, and hence the threshold value for the IES is lower than unity. Hence, for a

[23] Technically, the only effect is through a decrease in the supply for labor which pushes the wage up, leading to slightly lower profits and hence slightly higher default rates.

[24] There is one qualitative change, but it is hard to discern in most statistics. A shock to the probability of disaster increases consumption, hence with expected utility it is a "good state", i.e. low marginal utility of consumption state. This is not the case with Epstein-Zin utility, since the future value is lower, making a high probability of disaster state a "bad state" (high marginal utility of consumption). This in turn implies that assets which pay off well in that state have higher risk premia rather than lower risk premia.

certain range of values of IES below unity, the financial friction model implies that higher disaster risk lowers economic activity, while the frictionless model implies the opposite – an extreme example of the potential importance of financial frictions. Tables 5 through 7 report the model moments with a low IES (.25), which generates the opposite comovement. This specification is unattractive, since it implies that risk premia are procyclical, contrary to the data.

4.6 Comparison with idiosyncratic uncertainty shocks

Following Bloom (2009), several recent studies consider the effect of an increase in idiosyncratic uncertainty, σ_ε in our notation. While Bloom (2009) focused on the transmission of this shock through adjustment costs frictions, Arellano, Bai and Kehoe (2009), and Gilchrist, Sim and Zakrajek (2010) use default risk frictions, similar to my model. The shock to disaster risk is also an increase in uncertainty, and hence has a qualitatively similar effect. For instance, comparing figures 1 and 2 shows that the two parameters p and σ_ε have similar effects on steady-states. However, the channel through which the mechanism operates is somewhat different in my model, because an increase in aggregate uncertainty makes defaults more systematic and hence affects the bond risk premium.

To illustrate the differences in the mechanism, we can think of three experiments. First, the response of the economy to a shock to σ_ε is essentially unaffected by the coefficient of risk aversion. In contrast, as shown in section III.E, the response to an increase in disaster risk in my model is stronger when risk aversion is larger. Second, in the frictionless version, an increase in disaster risk leads to a recession, whereas an increase in idiosyncratic risk has no effect on economic activity.[25] Finally, suppose that we consider a shock to disaster risk, such that high disaster risk states have low idiosyncratic volatility, making the total quantity of risk constant over time. In essence, we are changing only the relative importance of aggregate and idiosyncratic risk, and hence the *correlation* across firms. This shock reduces investment and output, if risk aversion is positive, even though total risk does not change at the microeconomic level. The appendix produces the impulse responses corresponding to these three experiments.

The aim of this discussion is not to argue that idiosyncratic uncertainty shocks are unimportant, but that the channel through which they operate is different than the channel through which aggregate uncertainty shock operate, at least in this model. The two approaches have different strengths: my model connects well with the evidence on the behavior of credit spreads, correlation risk and aggregate risk premia. In contrast, the studies of Arellano et al. and Gilchrist et al. focus on more realistic microeconomic heterogeneity, and take into account the effect of uncertainty on reallocation and on the labor wedge among other issues.

[25] In some models, an increase in uncertainty would lead to a boom by leading to labor reallocation among firms with decreasing return to scale. But in the model of this paper, idiosyncratic shocks literally wash out because of the combined assumptions of constant return to scale and frictionless labor market.

4.7 Samples with disasters

So far the results reported are calculated in samples which do not include disasters. Large excess returns arise for two reasons: first, a standard risk premium; second, a sample selection ("Peso problem") since the sample does not include the lowest possible return realizations. To quantify the importance of the second effect, tables 5 through 7 report the model moments in the benchmark model if the sample includes disasters. Quantities and returns are of course more volatile since they include some large realizations. The average excess returns on equities is 1.38% (vs. 2.30% in a sample without disasters). Similarly, the average return on corporate bonds is 0.44% (vs. 0.60% in a sample without disasters (unreported in tables)). The dynamics of credit spreads and leverage are completely unaffected.

5 Conclusion

There are two main contributions. First, the paper embeds the standard capital structure trade-off theory, in a tractable equilibrium business cycle model. The trade-off model is a well established theory in corporate finance, and is a promising financial friction for macroeconomics, because it applies to all firms, large and small, and does not rely on binding borrowing constraints. Second, the paper studies the reaction of the economy to an increase in disaster risk. Time-varying disaster risk is essential to replicate the level, volatility and countercyclicality of credit spreads, and the fact that credit spreads are much larger than expected losses (or expected probabilities of default). Moreover, the trade-off friction substantially amplifies, by a factor of about three, the response of macroeconomic aggregates to disaster risk. The key mechanism is that defaults are expected to be more systematic, increasing risk-adjusted bankruptcy costs and hence the user cost of capital.

A natural direction in which to extend the analysis is to consider long-lived firms which may issue long-term debt. In the current version, firms are able to readjust leverage costlessly each period. In reality, it may be difficult to restructure the debt. This would likely generate endogenous persistence through firms' net worth.

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Parameter	Symbol	Value
Capital share	α	.3
Depreciation rate	δ	.08
Share of consumption in utility	υ	.3
Discount factor	β	.98
Trend growth of TFP	μ	.01
Standard deviation of TFP shock	σ	.02
Intertemporal elasticity of substitution	$1/\psi$	2
Risk aversion	γ	4
Mean probability of disaster		.017
Distribution of $b_{tfp} = b_k$: values		(.15,.25,.35,.45,.57)
Distribution of $b_{tfp} = b_k$: probabilities		(.333,.267,233,.033,.133)
Persistence of $\log(p)$	ρ_p	.75
Unconditional std. dev. of $\log(p)$	$\frac{\sigma_p}{\sqrt{1-\rho_p^2}}$	1.5
Idiosyncratic shock volatility	σ_ε	0.2267
Tax subsidy	$\chi - 1$	0.0616
Recovery rate	θ	0.4

Table 1: **Parameter values for the benchmark model.** The time period is one year.

		$\sigma(\Delta \log Y)$	$\frac{\sigma(\Delta \log C)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log I)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log N)}{\sigma(\Delta \log Y)}$	$\rho_{C,Y}$	$\rho_{I,Y}$
	Data	2.78	0.65	2.52	0.96	0.61	0.80
No disaster risk	Benchmark	1.80	0.55	1.88	0.33	0.97	0.99
	Constant leverage	1.80	0.55	1.89	0.33	0.97	0.99
	RBC	1.82	0.56	2.47	0.35	0.96	0.98
Constant	Benchmark	1.81	0.55	1.97	0.34	0.96	0.99
Disaster risk	Constant leverage	1.81	0.55	1.97	0.34	0.96	0.99
	RBC	1.83	0.56	2.50	0.35	0.96	0.98
Time-varying	**Benchmark**	**2.11**	**0.77**	**3.38**	**0.83**	**0.12**	**0.86**
Disaster risk	Constant leverage	2.46	0.89	4.62	1.05	-0.23	0.86
	RBC	1.86	0.60	2.89	0.46	0.79	0.91

Table 2: **Business cycle statistics (annual).** Second moments implied by the model, for different versions of the model. The statistics are computed in a sample without disasters. rho(A,B) is the correlation of the growth rate of time series A and B. The benchmark model is in bold.

		$E(R_f)$	$E(R_e)$	E(spread)	σ(spread)	ρ(Spread,GDP)	$\sigma(R_f)$	$\sigma(R_e)$
	Data	0.80	7.60	0.94	0.41	-0.37	2.50	16.20
No disaster risk	Benchmark	2.46	2.44	0.51	0.00	-0.56	0.23	0.39
	Cst leverage	2.46	2.39	0.55	0.01	0.64	0.23	0.39
	RBC	2.59	2.54	-0.01	0.00	0.11	0.27	0.33
Constant	Benchmark	1.21	3.84	1.29	0.00	-0.65	0.22	0.41
Disaster risk	Cst leverage	1.21	3.81	1.32	0.01	0.65	0.22	0.41
	RBC	1.32	2.65	-0.01	0.00	-0.03	0.26	0.33
Time-varying	**Benchmark**	**1.31**	**3.60**	**0.98**	**0.46**	**-0.53**	**2.33**	**1.18**
Disaster risk	Cst leverage	1.31	3.85	1.21	1.40	-0.80	2.58	1.88
	RBC	1.41	2.65	-0.01	0.00	-0.07	2.05	0.34

Table 3: **Financial Statistics, 1.** Mean and standard deviation of the risk-free return, the equity return, and the spread between the corporate bonds and the risk-free bond. The statistics are calculated in a sample without disasters. The correlation is the correlation between the spread BAA-AAA and HP-filtered GDP.

		E(Lev)	Std(Lev)	E(ProbDef)	Std(ProbDef)
	Data	0.45	0.09	0.39	NA
No disaster risk	Benchmark	0.56	0.00	0.79	0.01
	Constant leverage	0.57	0.00	0.86	0.03
	RBC	0.00	0.00	0.00	0.00
Constant	Benchmark	0.54	0.00	0.50	0.01
Disaster risk	Constant leverage	0.55	0.00	0.53	0.02
	RBC	0.00	0.00	0.00	0.00
Time-varying	**Benchmark**	**0.54**	**0.04**	**0.58**	**0.23**
Disaster risk	Constant leverage	0.54	0.00	0.49	0.02
	RBC	0.00	0.00	0.00	0.00

Table 4: **Financial Statistics, 2.** Mean and volatility of leverage and of probability of default. The statistics are calculated in a sample without disasters. Data from Chen, Collin-Dufrense and Goldstein (2009).

	$\sigma(\Delta \log Y)$	$\frac{\sigma(\Delta \log C)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log I)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log N)}{\sigma(\Delta \log Y)}$	$\rho_{C,Y}$	$\rho_{I,Y}$
Data	2.78	0.65	2.52	0.96	0.61	0.61
Benchmark model	2.11	0.77	3.38	0.83	0.12	0.86
Samples with disasters	5.47	1.00	1.58	0.32	0.81	0.80
Adjustment costs ($\eta = .1$)	1.75	0.83	2.60	0.62	0.54	0.83
Adjustment costs ($\eta = .2$)	1.58	0.90	2.11	0.48	0.73	0.80
IES = .5	1.59	0.78	1.58	0.23	0.96	0.96
IES = .25	1.57	0.94	2.11	0.51	0.70	0.80
Risk aversion = .5	1.97	0.69	2.83	0.67	0.39	0.88

Table 5: **Extensions of the model: business cycle statistics (annual).**

	$E(R_f)$	$E(R_e)$	E(spread)	σ(spread)	ρ(Spread,GDP)	$\sigma(R_f)$	$\sigma(R_e)$
Data	0.80	7.60	0.94	0.41	-0.37	2.50	16.20
Benchmark model	1.31	3.60	0.98	0.46	-0.53	2.33	1.18
Samples with disasters	1.30	2.68	0.98	0.46	-0.52	2.34	7.05
Adjustment costs ($\eta = .1$)	1.31	3.62	0.98	0.46	-0.46	2.17	1.75
Adjustment costs ($\eta = .2$)	1.32	3.62	0.98	0.46	-0.37	2.09	2.07
IES = .5	1.61	3.90	0.98	0.47	-0.10	2.36	1.11
IES = .25	1.97	4.27	0.98	0.46	0.21	2.40	1.11
Risk aversion = .5	2.28	3.68	0.77	0.28	-0.63	1.50	0.91

Table 6: **Extensions of the model: Financial Statistics, 1.**

	E(Lev)	Std(Lev)	E(ProbDef)	Std(ProbDef)
Data	0.45	0.09	0.39	0.51
Benchmark model	0.54	0.04	0.58	0.23
Samples with disasters	0.54	0.04	0.81	2.55
Adjustment costs ($\eta = .1$)	0.54	0.04	0.58	0.24
Adjustment costs ($\eta = .2$)	0.54	0.05	0.58	0.24
IES = .5	0.54	0.04	0.58	0.23
IES = .25	0.54	0.04	0.58	0.23
Risk aversion = .5	0.55	0.03	0.65	0.18

Table 7: **Extensions of the model: Financial Statistics, 2.**

	$\sigma(\Delta \log Y)$	$\frac{\sigma(\Delta \log I)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log N)}{\sigma(\Delta \log Y)}$	$E(\text{Lev})$
$\chi = 1.062$ (Benchmark)	2.11	3.38	0.83	0.54
$\chi = 1.06$	2.11	3.41	0.82	0.54
$\chi = 1.05$	2.05	3.41	0.77	0.53
$\chi = 1.04$	2.00	3.37	0.71	0.51
$\chi = 1.03$	1.96	3.30	0.64	0.50
$\chi = 1.02$	1.91	3.14	0.57	0.47
$\chi = 1.01$	1.89	3.00	0.51	0.43
$\chi = 1.005$	1.87	2.94	0.48	0.39
$\chi = 1.002$	1.86	2.92	0.47	0.34
$\chi = 1.001$	1.86	2.91	0.46	0.30
$\chi = 1$ (RBC)	1.86	2.89	0.43	0.00

Table 8: **Effect of tax shield parameter on mean leverage and volatilities of quantities.**

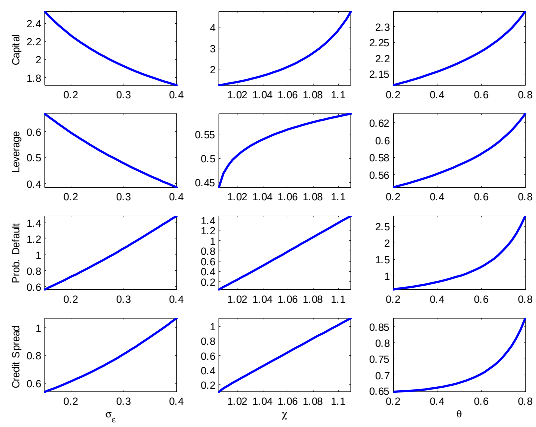

Figure 1: **Comparative statics on steady-state.** Effect of idiosyncratic volatility σ_ε, tax subsidy χ, and recovery rate θ, on capital, leverage, probability of default (in %), and credit spread (in %).

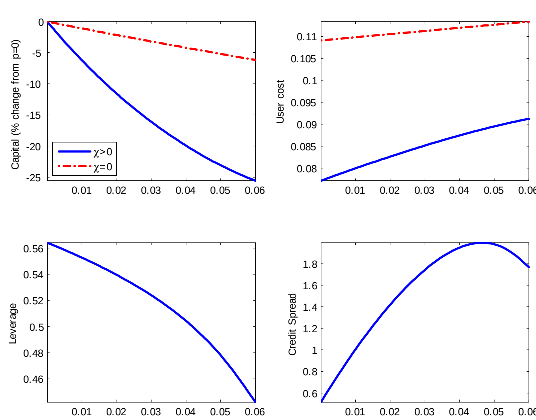

Figure 2: **Comparative statics on steady-state.** Effect of an increase in the probability of disaster on capital, leverage, credit spreads (in %), and the user cost of capital, for the frictionless model ($\chi = 0$, red dot-dashed line) and the benchmark model ($\chi > 0$, blue full line).

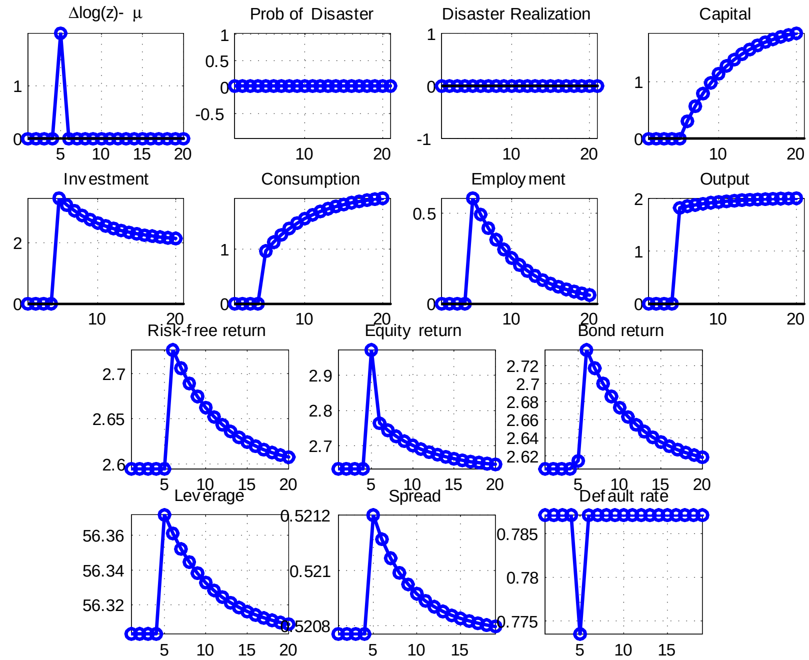

Figure 3: **Impulse response function of model quantities and returns to a one standard deviation shock to total factor productivity**. Quantity responses are shown in % deviation from balanced growth path. Returns, default rates, credit spreads, leverage and the probability of disaster are annual, in % per year.

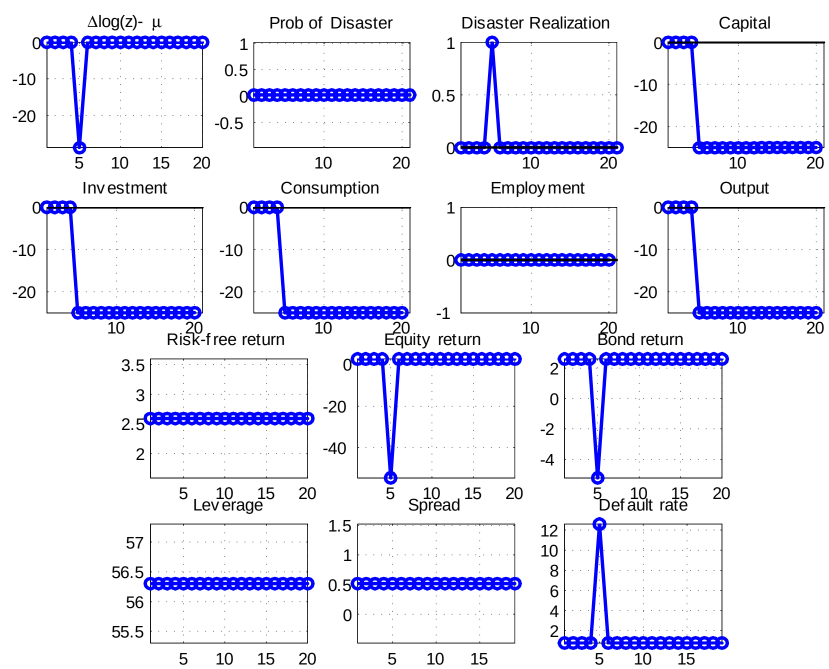

Figure 4: **Impulse response function of model quantities and returns to a disaster realization**. Quantity responses are shown in % deviation from balanced growth path. Returns, default rates, credit spreads, leverage and the probability of disaster are annual, in % per year.

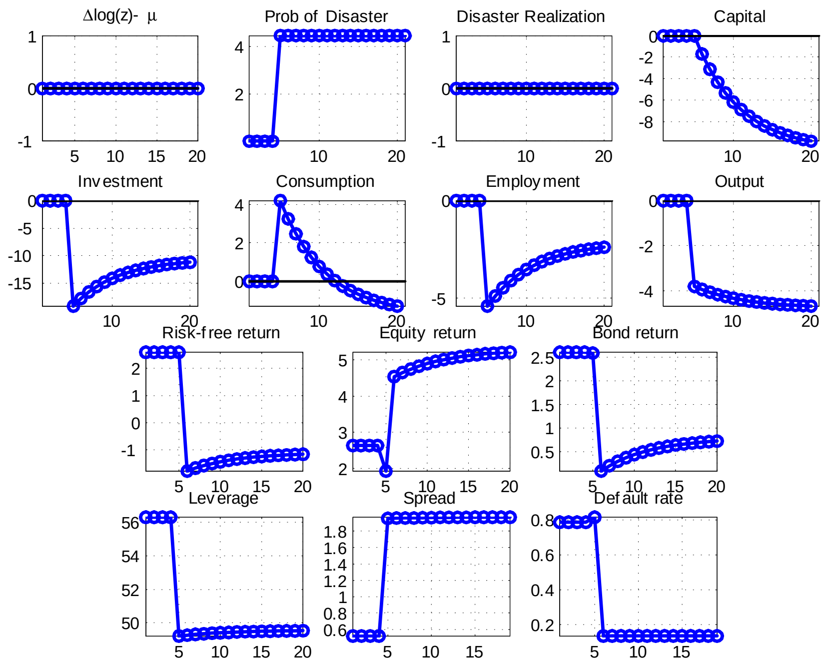

Figure 5: **Impulse response function of model quantities and returns to a shock to the probability of disaster**. Quantity responses are shown in % deviation from balanced growth path. Returns, default rates, credit spreads, leverage and the probability of disaster are annual, in % per year.

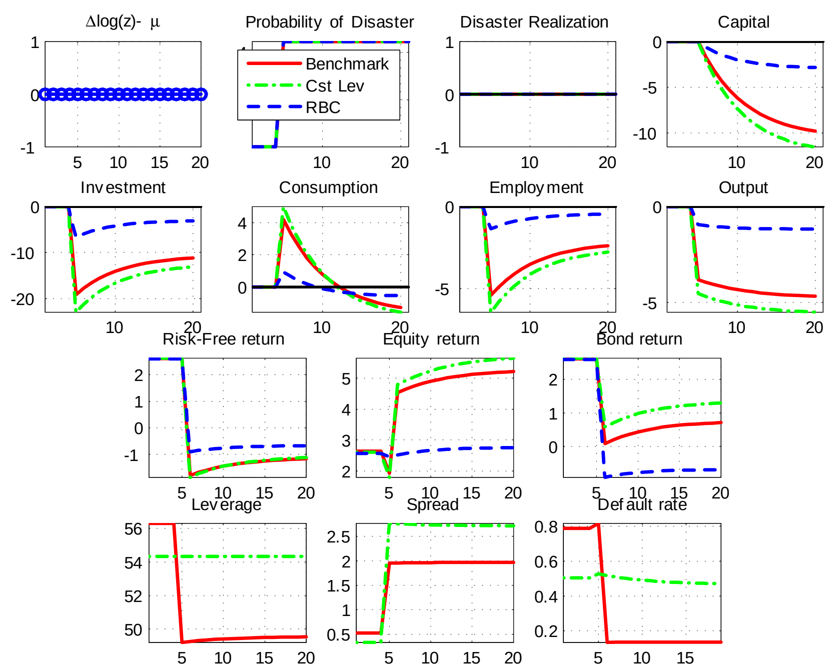

Figure 6: **Comparison of RBC model with and without financial friction.** This figure compares the impulse response of three models to a probability of disaster shock: the benchmark model (red full line), the model with constant leverage (green dot-dashed line), and the frictionless RBC model (blue dashed line). Quantity responses are shown in % deviation from balanced growth path. Returns, default rates, credit spreads, leverage and the probability of disaster are annual, in % per year.

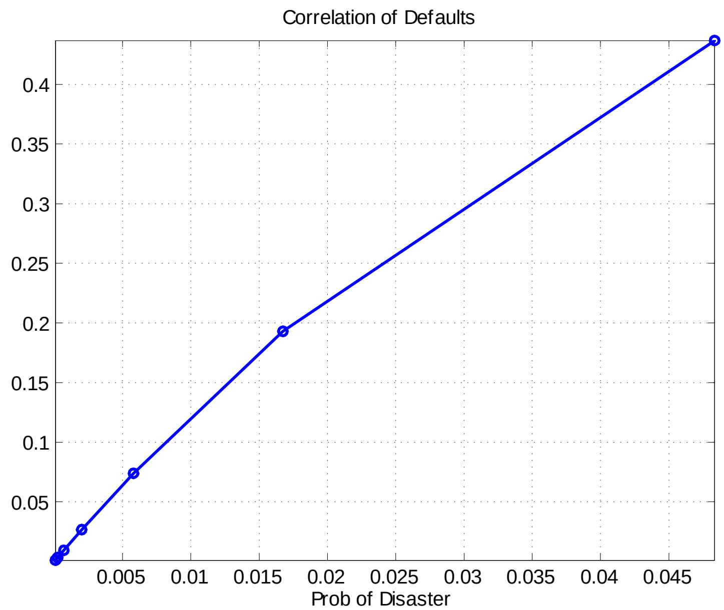

Figure 7: **Time-varying systematic risk: Correlation of defaults in the model.** This picture plots the correlation of default indicator between any two firms next period, i.e. $Corr_t(def_{it+1}, def_{jt+1})$, as a function of the disaster probability p.

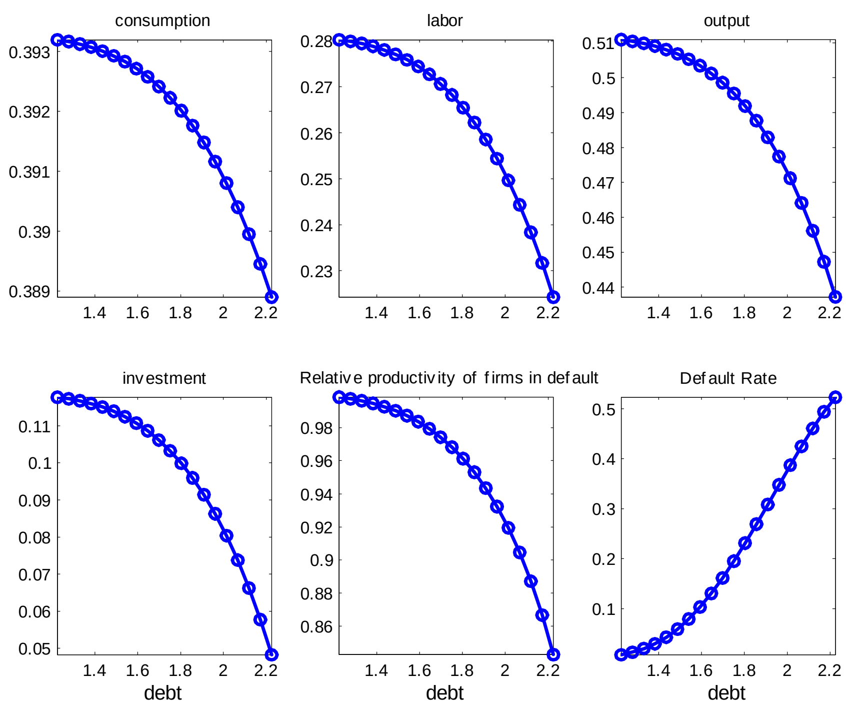

Figure 8: **Effect of outstanding debt on quantities, when firms in default are less productive.** The figure plots the policy functions for consumption, $c(k, b, p)$, employment $N(k, b, p)$, output $y(k, b, p)$, investment $i(k, b, p)$, the relative productivity of firms in default relative to firms not in default, and the share of firms in default, as a function of outstanding debt b (holding k and p fixed).

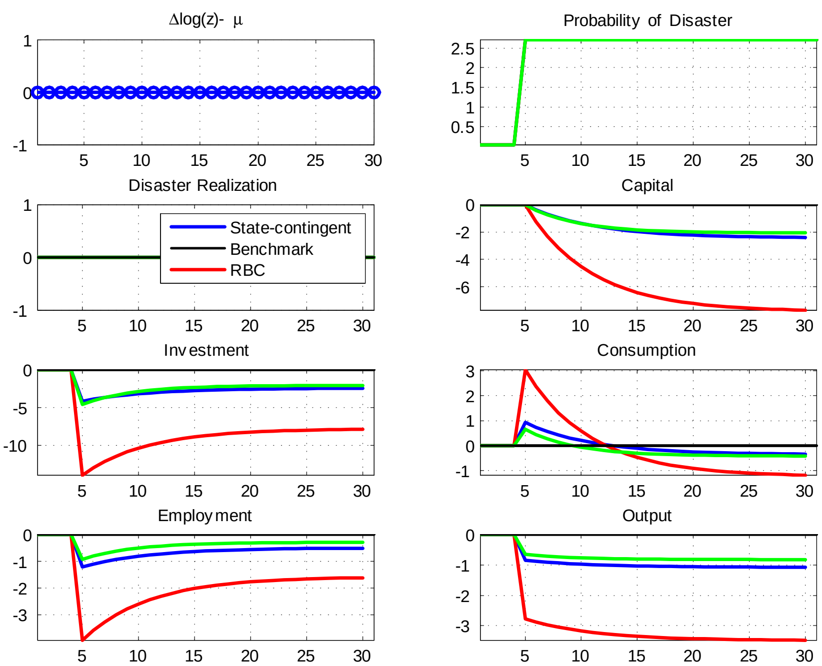

Figure 9: **Role of state-contingent debt.** The figure plots the impulse response function of model quantities to a shock to the probability of disaster. Blue full line = state-contingent debt, red line = benchmark model, green line = RBC frictionless model. Quantity responses are shown in % deviation from balanced growth path.